



Macroscopic modeling of asynchronous irregular activity states

Sami El Boustani, Alain Destexhe

► To cite this version:

Sami El Boustani, Alain Destexhe. Macroscopic modeling of asynchronous irregular activity states. Deuxième conférence française de Neurosciences Computationnelles, "Neurocomp08", Oct 2008, Marseille, France. ⟨hal-00331575⟩

HAL Id: hal-00331575

<https://hal.science/hal-00331575v1>

Submitted on 17 Oct 2008

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



HAL Authorization

MACROSCOPIC MODELING OF ASYNCHRONOUS IRREGULAR ACTIVITY STATES

El Boustani S. & Destexhe A.

Unité de Neurosciences Intégratives et Computationnelles

CNRS UPR-2191

1 Avenue de la Terrasse

91198 Gif-sur-Yvette - France

{name}@unic.cnrs-gif.fr

ABSTRACT

Many efforts have been devoted to modeling asynchronous irregular (AI) activity states, which resemble the complex activity states seen in the cerebral cortex of awake animals. Most of models have considered balanced networks of excitatory and inhibitory spiking neurons in which AI states are sustained through recurrent sparse connectivity, with or without external input. We propose a mesoscopic description of such AI states. Using the master equation formalism, we derive a second-order mean-field set of ordinary differential equations describing the temporal evolution of randomly connected balanced networks. This formalism takes into account finite size effects, and is applicable to any neuron model as long as its transfer function can be characterized. We compare the predictions of this approach with numerical simulations for different network configurations and parameter spaces. Considering the randomly connected network as a unit, this approach could be used to build large-scale networks of such connected units, with an aim to model activity states constrained by macroscopic measurements, such as voltage-sensitive dye imaging which requires a spatially extended description. This formalism can further be generalized by integrating pairwise correlations among the neuron populations.

KEY WORDS

Master Equation, Markov, Phase diagram, Transfer function, Stability, Second-order statistics

1 Introduction

Cortical activity in awake animals manifests highly complex behavior, often characterized by seemingly noisy activity. At the level of single neurons, the activity in awake animals is associated with considerable subthreshold fluctuations of the membrane potential and irregular firing.

Many efforts have been devoted to the study of how such activity emerges. Balanced networks have been introduced as a possible model to generate dynamical states similar to the biological ones (Fig. 1). Two antagonistic states have been highlighted: synchronous regular states (SR) and asynchronous irregular states (AI) [1]. AI states are of particular interest because their dynamical character-

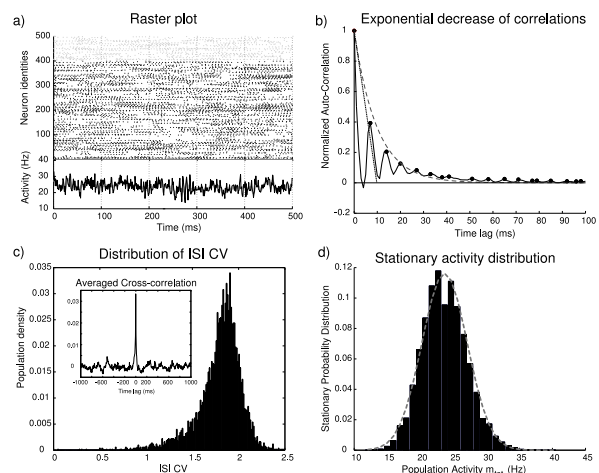


Figure 1. Example of a self-sustained asynchronous irregular (AI) state in a sparsely connected network of conductance-based neurons. The network contains 5,000 neurons with a ratio 4:1 between excitatory and inhibitory neurons and a connection probability $p^{conn} = 0.01$. a) *Top*: Raster plot for a subset of excitatory (black dots) and inhibitory (gray dots) neurons. *Bottom*: Population activity with a bin size of 1 ms (black) and 5 ms (gray). b) Auto-correlation of the total network activity. This function has an exponentially decaying envelope with a typical time constant of few millisecond. c) The population distribution of the Interspike Interval Coefficient of Variation (CV) with an average value of 1.8 which is typical of irregular firing. In the inset, the averaged pairwise cross-correlation computed with a bin size of 5 ms. The peak value at zero time lag is 0.032 so that the network exhibit almost no synchronous firing. d) Stationary activity distribution of the network activity sampled with a time bin of 5 ms. At this time scale, the activity distribution is well described by a gaussian function.

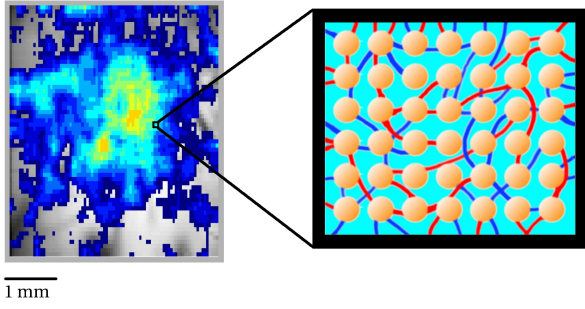


Figure 2. A conceptual representation of a pixel in Voltage-Sensitive-Dyes optical imaging. On the left, a frame taken from a VSD recording in the rat barrel cortex *in vivo*. On the right, a schematic representation of the network contained in a pixel. At this scale, a network could be approximated by a random network. A mean-field description could be appropriate as long as the underlying neuron population is statistically homogeneous. Courtesy to Diego Contreras.

istics are very similar to those observed in awake animals. For conductance-based integrate-and-fire neuron networks, they have even been observed without external stimulation [2, 3, 4].

In parallel to such studies, population measures of neural activity have also been of great interest, in particular through the emergence of new imaging techniques such as voltage-sensitive dye imaging or two-photon imaging. Although the relation of such signals to single-cell properties is still not completely clear, such measurements reveal structures and correlations over large distances (millimeters or centimeters). No model is presently able to describe neuronal dynamics in large-scale networks at such distance scales, and there is a need of theoretical models specifically designed to handle the temporal and spatial scales of optical imaging.

The type of model that seems most appropriate for such scales are mean-field approaches (Fig. 2). However, first-order mean-field approximation fails to fully describe these networks because of their inherent dynamics which can rely dramatically on activity fluctuations. Moreover, the large network limit is usually performed for randomly connected networks despite the lack of biological relevance.

Our aim is to obtain a macroscopic description of distributed neuronal activity during AI states, where the unit is not the neuron, but a small network of neurons. The difficulty, however, is to obtain a description which captures the statistics of network activity, while being consistent with single-cell behavior. For this reason, we introduce a “mesoscopic” description of neuronal activity, in which finite size effects are explicitly taken into account. We will consider networks of typical sizes of a few thousand neurons, far away from the large network limit.

To obtain such a mesoscopic model, we will use a

master equation formalism appropriate for a second-order mean-field description of network activity.

$$\partial_t P_t(\{m_\gamma\}) = \prod_{\alpha=1, \dots, K} \int_0^{1/T} dm'_\alpha \left(P_t(\{m'_\gamma\}) W(\{m_\gamma\}|\{m'_\gamma\}) - P_t(\{m_\gamma\}) W(\{m'_\gamma\}|\{m_\gamma\}) \right)$$

where m_γ are the random variables describing each neuron network for $\gamma = 1, \dots, K$ and $P_t(\{m_\gamma\})$ is the time-dependent probability distribution for each random variable. The entire master equation is characterized by the transition function $W(\{m_\gamma\}|\{m'_\gamma\})$ which must be build according to the underlying neuron network AI dynamics. The typical gaussian shape of the stationary activity distribution as well as the exponentially decaying correlation functions (Fig. 1) allow analytical developpement for time scale of few millisecond.

The AI states, characterized by low firing rates and exponential decrease of the activity auto-correlation (Fig. 2), can be naturally incorporated in such a framework. A complete description of the correlations and covariances can be extracted for time scales governed by the network time constants. This question was already addressed with a similar formalism for binary neural networks [5, 6]. The core of the theory is the transfer function, which maps the output firing rate of the neuron as a function of its input rates. Theoretical work has been done to obtain analytical transfer functions for a range of neuron models [1, 7]. However, to take advantage of such results, we have to rely on a continuous description of network activity to link the neuron statistics to the network ones.

We will consider different neuron models ranging from those for which a transfer function has been derived, to those for which an approximative model is required. For current-based integrate-and-fire models, analytical work have made some functions accessible so that they can provide accurate predictions (Fig. 3). For unsolved analytical models, we will suggest empirical models that can account for the network dynamics for a broad range of parameters. In particular, for conductance-based neurons, such transfer function can be found that gives a good qualitative description of different network regimes. For instance by using a template transfer function such as

$$\nu = \frac{1}{2\tau} \left(1 + \operatorname{erf} \left(\frac{\langle V \rangle - V^{\text{threshold}}}{\sqrt{2}\sigma(V)} + \Delta h \right) \right) \quad (1)$$

describing the neuron output firing rate according to the membrane potential statistics and containing two free parameters ($\tau, \Delta h$) we can look for the best parameter set which reproduces the stationary first and second order properties of a conductance-based neuron network (Fig. 4). This provides a better prediction than the original transfer function obtained by usual approximations to an effective currant-based model.

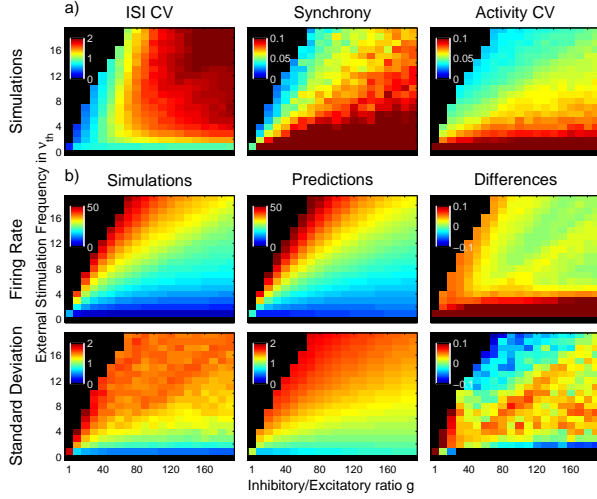


Figure 3. AI states characterization and the first and second order statistics of the excitatory population activity in the (ν_{ext}, g) parameter space, similar to [1]. The external network stimulation ν_{ext} is represented in the y-axis and the inhibitory/excitatory synaptic strength ration J_{inh}/J_{exc} in the x-axis. The network contains $N = 5,000$ neurons randomly connected with probability $p^{conn} = 0.01$ and current-based synaptic interactions. a) AI region is delimited using the mean ISI CV and the mean pairwise cross-correlation. b) Top : Mean activity estimated from numerical simulations (left) and computed using the master equation formalism (middle). In the right panel, the relative difference between measured and predicted values. Bottom : The activity standard deviation is estimated from numerical simulations and compared as well with the mean-field predictions. The predictions fit well with the numerical simulations for the first and second-order statistics in the middle region. Better prediction can be obtained for larger networks (data not shown).

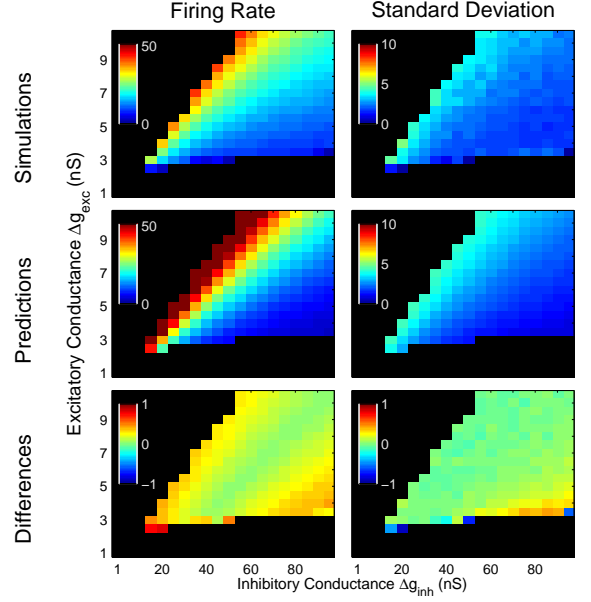


Figure 4. First and second order statistics of the excitatory population activity in the $(\Delta g_{exc}, \Delta g_{inh})$ parameter space. The excitatory synaptic strength Δg_{exc} is represented in the y-axis and the inhibitory synaptic strength Δg_{inh} in the x-axis. A network was simulated with $N = 10,000$ conductance-based neurons randomly connected with probability $p^{conn} = 0.01$. The moments are computed with a time bin of $T = 5$ ms and the analytical model was solved with the same parameter and an effective transfer function. *Top*: Mean activity and standard deviation estimated from numerical simulations. *Middle*: Mean activity and standard deviation computed from the master equation formalism. *Bottom*: Relative difference between measured and predicted values. These predictions have been obtained using an effective transfer function which could be obtained using an optimisation process. The result is in good agreement with the simulations in the middle region.

2 Conclusion

Preliminary results have shown that considering more local connectivities – instead of random schemes – does not alter significantly the master equation predictions, as long as the sparseness is strong enough (Fig. 5). Therefore, first and second order activity statistics do not require an exact description of the network structure. However, heterogeneity among the neurons can be responsible for slight discrepancies between simulations and predictions. Although our model does not take into account specific delays between neurons, for random delays of the order of T , the numerical simulations are even closer to predictions. Indeed, global oscillations are destroyed by heterogenous delays and the AI region is larger. We are currently working in a systematic study of phase diagrams and their dependency on various parameters.

Such a “mesoscopic” description constitutes a first step towards obtaining a large-scale model of cerebral cortex tissue. The typical size of the networks considered here ($N \sim 5,000$ neurons) can be thought of representing the population of cortical neurons seen under one or several “pixels” of optical imaging data. Typical values are 100×100 pixels, covering from about 3×3 mm to 3×3 cm of cortical tissue, which gives about 5 to 5,000 neurons per pixel in superficial layers according to neuronal densities published previously (Braitenberg & Shutz, 1998). Thus, constructing a 100×100 network of such populations, each described by a Master equation analogous to the present model, would be possible if the connectivity between adjacent and distant populations can be incorporated in the formalism. This important addition will require to study interconnected networks of neurons in AI states, which constitutes a natural extension of the present modeling effort.

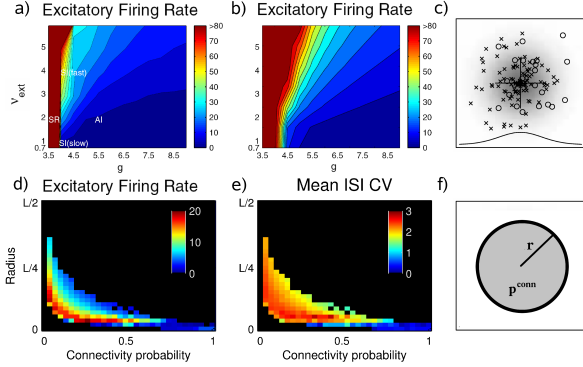


Figure 5. Effect of local network connectivity on macroscopic statistics. a-c) Comparison of mean excitatory activity for a locally randomly connected current-based network [7] in the (ν_{ext}, g) parameter space. a) Numerical simulations with gaussian distribution. b) First order mean field predictions. c) The gaussian distributed connectivity scheme where each neuron in the network is connected to its neighbour according to a gaussian probability law and with a fixed output synapse number. d-f) Effect of local uniformly random connectivity. The parameter space is described by the connectivity disc radius and the connection probability inside the disc. d) Mean excitatory activity. e) Mean excitatory interspike interval coefficient of variation. The iso-statistics curves in the phase space overlap the network configuration with identical incoming synapse number per neuron. f) The uniform local random connectivity scheme which is built as in c) but with a uniform probability profile.

3 Acknowledgements

Research supported by the CNRS, ANR and the European Community (FACETS grant FP6 15879).

References

- [1] N. Brunel. Dynamics of sparsely connected networks of excitatory and inhibitory spiking neurons. *J Comput Neurosci*, 8(3):183–208, 2000.
- [2] Tim P Vogels and L. F. Abbott. Signal propagation and logic gating in networks of integrate-and-fire neurons. *J Neurosci*, 25(46):10786–10795, Nov 2005.
- [3] Sami El Boustani, Martin Pospischil, Michelle Rudolph-Lilith, and Alain Destexhe. Activated cortical states: experiments, analyses and models. *J Physiol Paris*, 101(1-3):99–109, 2007.
- [4] Arvind Kumar, Sven Schrader, Ad Aertsen, and Stefan Rotter. The high-conductance state of cortical networks. *Neural Comput*, 20(1):1–43, Jan 2008.

- [5] Ginzburg and Sompolinsky. Theory of correlations in stochastic neural networks. *Phys Rev E Stat Phys Plasmas Fluids Relat Interdiscip Topics*, 50(4):3171–3191, Oct 1994.
- [6] Hdi Soula and Carson C Chow. Stochastic dynamics of a finite-size spiking neural network. *Neural Comput*, 19(12):3262–3292, Dec 2007.
- [7] Nicolas Fourcaud and Nicolas Brunel. Dynamics of the firing probability of noisy integrate-and-fire neurons. *Neural Comput*, 14(9):2057–2110, Sep 2002.