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USE THE DISCRETE GEOMETRY TO DISCRETIZE THE RIDGELET TRANSFORM

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ABSTRACT

The Ridgelet transform allows to represent effectively rectilinear discontinuities in an image, essential information for image analysis. In this paper, we present our nD implementation, the Discrete Analytical Ridgelet Transform (DART). The innovative step is the definition of a discrete transform with the discrete analytical geometry theory by the construction of discrete analytical lines in the Fourier domain. The DART uses the Fourier strategy for the computation of the associated discrete Radon transform [2, 3]. It has a very simple forward/inverse algorithm that provides an exact reconstruction without any iterative method. In order to illustrate the potentiality of this discrete transform, we compare 3D DART denoising of colour video with reference methods.

1. THE DART

A team of the University of Stanford has proposed the ridgelet transform to effectively represent edges in an image. The continuous ridgelet theory is documented in the Ph.D. thesis of Candès [1]. A basic strategy for calculating the continuous Ridgelet transform is first to compute the Radon transform and secondly, to apply a 1-D wavelet transform to the slices. The implementation of the discrete Ridgelet transform can use the same principle. The discrete wavelet decomposition is easy to implement, is stable and invertible, and can be associated to a discrete orthogonal representation. The discretization of the Radon transform is more difficult to achieve.

Via the central slice theorem, the Radon transform can be obtained by applying the 1D inverse Fourier transform to the nD Fourier transform restricted to radial lines going through the origin. The discretization of the nD Radon transform can be obtained in computing the nD Discrete Fourier Transform, in extracting Fourier coefficients that belong to straight lines going through the origin and in computing the 1D inverse Discrete Fourier Transform on each line.

For the discrete Radon transform, we need a discrete straight line that has a central symmetry and forms a "good" approximation of the corresponding Euclidian straight line (i.e. direction). The idea is to represent each direction with an analytical nD discrete straight line in the Fourier domain. For this, discrete straight lines need to have a central symmetry and to cover all the Fourier domain to be perfectly invertible. The discrete analytical line of nD straight line L noted $\mathbb{D}(L)$ can be defined by:

$$\mathbb{D}(L) = \left\{ k \in \mathbb{Z}^n \mid d_p(k, L) \leq \frac{1}{2} \right\} = \left(L \oplus \mathbb{B}_{d_p}^\infty \left(\frac{1}{2} \right) \right) \cap \mathbb{Z}^n$$

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where \oplus is the Minkowski sum and $\mathbb{B}_{d_p}^\infty \left(\frac{1}{2} \right)$ ball of radius $\frac{1}{2}$ for the p -norm distance d_p . The choice of the distance for the construction of the discrete line defines the arithmetical thickness of the discrete line. The thickness is an important parameter that controls, among other things, the connectivity of the discrete line and the cover of the Fourier domain. The proposed Radon/Ridgelet transformation is redundant but the repetition of information depends on the type of the discrete lines used and can be adapted to the application. Moreover this method is associated with a non-iterative inverse algorithm that gives an exact reconstruction.

2. RESULTS FOR 3D

The 3D DART is compared to a 3D undecimated Daubechies wavelet and the 3D dual tree complex wavelet with a noisy video ($\sigma = 42$) in RGB colour representation. Denoising results are independently evaluated on each spatio-temporal block, corresponding to color components, with a soft thresholding scheme and the same threshold is used for each subband of the three type of transforms with scale 4 transform.

	Noisy	UDWT	Dual tree transform	DART
Red	27.42	32.09	31.81	32.44
Green	26.05	30.63	30.39	31.18
Blue	20.64	25.30	25.54	26.70

Table 1. SNR on each denoised color component (in dB).

The SNR calculations, computed after the denoising of five color videos, show that the 3D DART is better than the reference transforms (cf. table 1). This simple experimentation illustrates the fact that the 3D DART is competitive for video denoising.

3. REFERENCES

- [1] E. Candès, *Ridgelets : Theory and Applications*, Ph.D. thesis, University of Stanford, August 1998.
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