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# POD BASED TECHNIQUE FOR 3D FLOW RECONSTRUCTION USING 2D DATA SET

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## KEYWORDS:

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**Other keywords:** *Direct Numerical Simulation*

**ABSTRACT :** *We propose a technique, based on proper orthogonal decomposition, for reconstructing a fully developed 3D flow starting from spatially restricted measurement of the flow. The POD provides a basis of spatially coherent structures for the description of the 3D flow under study. We demonstrate that the 3D reconstruction, from spatially restricted measurement, can ideally be performed without loss of information as far as the transfer matrix, computed from the POD modes, is invertible.*

## 1 General Introduction

Experimental and numerical fluid dynamics most often deal with three-dimensional velocity fields. Experimentally, however, usual measurement techniques provide at best two-dimensional (2D) velocity fields with two (2C) or three (3C) velocity components, for example using particle image velocimetry (PIV) techniques. Henceforth, a part of the knowledge about the flow is lost, and a challenging task lies in the ability to recover the fully three dimensional field. Numerically, on another hand, three dimensional velocity fields represent a huge amount of data to deal with, and it may be desirable to achieve efficient data compression such that only two dimensional restrictions of the field may suffice. In this paper, we propose a technique aimed to achieve a three dimensional reconstruction of the velocity field, from partial two dimensional knowledge about it, using a principal value decomposition based technique applied to a collection of velocity fields (proper orthogonal decomposition).

## 2. The Proper Orthogonal Decomposition

### 2.1 Definition

Proper orthogonal decomposition (POD) techniques are applied to flow mechanics since the pioneer works by Lumley *et al* in the early 80's [1]. By an appropriate change of basis in the vector space of the flow realizations, it is aimed to identify, in a collection of statistically representative velocity field realizations, the most coherent features of the field (with respect to the point to point linear correlation). The singular value decomposition based procedure then provides an empirical basis of orthonormal coherent structures for the data set description, among which any vector field can be decomposed.

Suppose we have to deal with a data set  $S = \{ \mathbf{u}(\mathbf{r}, t_k) \}_{k=1 \dots M}$  of  $M$  flow realizations. The velocity field  $\mathbf{u}(\mathbf{r}, t)$ , function of position  $\mathbf{r}$  and time  $t$ , may be two ( $x, y$ ) or three ( $x, y, z$ ) dimensional, with two ( $u_x, u_y$ ) or three ( $u_x, u_y, u_z$ ) components. Building up the data matrix  $A$  as:

$$A = \begin{pmatrix} u_x(r_1, t_1) & u_x(r_1, t_2) & \dots & u_x(r_1, t_M) \\ u_x(r_2, t_1) & u_x(r_2, t_2) & \dots & u_x(r_2, t_M) \\ \vdots & \vdots & \ddots & \vdots \\ u_x(r_N, t_1) & u_x(r_N, t_2) & \dots & u_x(r_N, t_M) \\ u_y(r_1, t_1) & u_y(r_1, t_2) & \dots & u_y(r_1, t_M) \\ \vdots & \vdots & \ddots & \vdots \\ u_y(r_N, t_1) & u_y(r_N, t_2) & \dots & u_y(r_N, t_M) \\ u_z(r_1, t_1) & u_z(r_1, t_2) & \dots & u_z(r_1, t_M) \\ \vdots & \vdots & \ddots & \vdots \\ u_z(r_N, t_1) & u_z(r_N, t_2) & \dots & u_z(r_N, t_M) \end{pmatrix} \quad (1)$$

where  $N$  is the number of pixels in the spatial grid, and  $r_n$  the  $n^{th}$  pixel in the grid, referenced by its coordinates ( $x, y, z$ ). The POD of  $A$  extracts the most coherent spatial events in the velocity field, and rank them into columns of a  $N \times N$  matrix  $U$ , from the most (point to point linearly) correlated in the first column, to the less correlated in the last column, such that:

$$A = U \cdot \Sigma \cdot V^t \quad (2)$$

The singular values of  $A$  are all positive and therefore  $U$  is an orthogonal matrix. In the  $N \times M$  rectangular matrix  $\Sigma$  are ranked, in the decreasing order, the  $N$  singular values  $\sigma_n$  associated to the  $N$  principal components (the so-called coherent structures), among which only  $M \ll N$  are non zero. One better understands why the principal components in  $U$  actually are the coherent features of the velocity field by noticing that  $U$  is the change-of-coordinate matrix of the spatial correlation operator  $K$ :

$$K = A \cdot A^t = U \cdot (\Sigma \cdot \Sigma^t) \cdot U^t \quad (3)$$

Similarly,  $V$ -columns represent the time correlated events in the temporal evolution of the velocity field, also ranked from the most to the less correlated one,  $V$  being the  $M \times M$  change-of-coordinate matrix of the temporal correlation operator  $C$ :

$$C = A^t \cdot A = V \cdot (\Sigma^t \cdot \Sigma) \cdot V^t \quad (4)$$

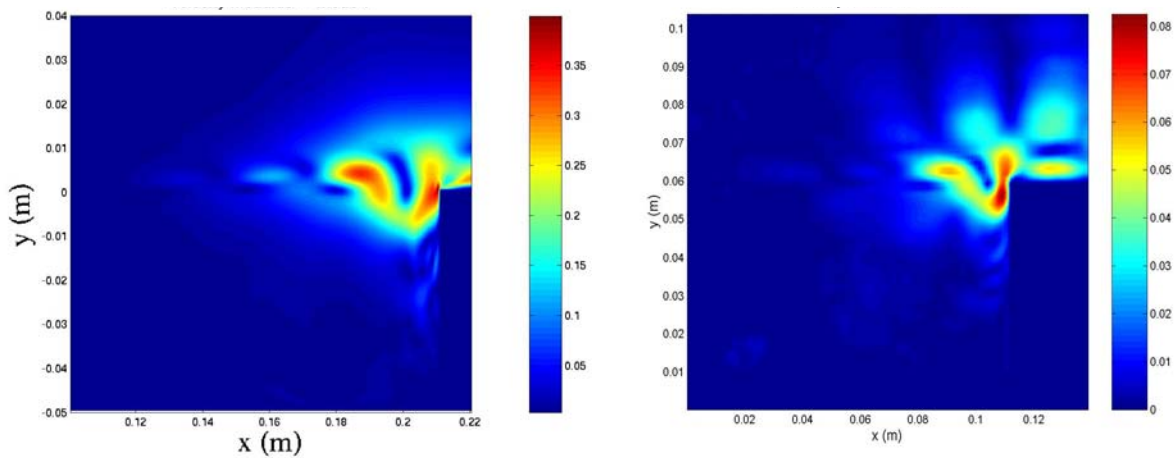
Noting  $\phi_n(\mathbf{r})$  the  $n^{th}$  column of  $U$  reshaped as a two or three dimensional spatial field, similarly to  $\mathbf{u}$ , the singular value decomposition ensures that the  $\phi_n(\mathbf{r})$  are orthonormal functions, with respect to the scalar product over the spatial domain. Henceforth, the velocity field, at a given time  $t$ , can be decomposed on the functional basis of the  $\{\phi_n(\mathbf{r})\}_{n=1 \dots M}$ :

$$u(r, t) = \sum_{n=1}^M a_n(t) \phi_n(r) \quad (5)$$

where the time dependent coefficient  $a_n(t)$ , ranked as the  $n^{th}$  column of  $V$ , is a measure of the  $n^{th}$  eigen-mode  $\phi_n(\mathbf{r})$  contribution in  $\mathbf{u}$  at time  $t$ .

## 2.2 Illustration on a benchmark case

For the sake of illustration, consider the interaction between a laminar boundary layer and a parallelepipedic rectangular cross-sectional cavity. The cavity height  $H$  is 5 cm, length  $L=10$  cm, span  $S=30$  cm. The Reynolds number, based on the cavity length  $L$ , is  $Re_L=8\,000$  (flow rate velocity  $U=1.2$  m/s). For a direct comparison of the numerical and the experimental flows, we computed the POD on the 2D plane corresponding to the PIV measurement. The first POD eigen-mode is shown in Figure 1. In both cases, numerical and experimental, it can be seen that the first POD mode identifies the coherent features in the shear layer. The wavelength is similar in both cases, as well as the frequency of oscillations contained in the time coefficient  $a_1(t)$  (13 Hz in the DNS vs 12.7 Hz in the experiment).



**Figure 1:** First POD mode in the DNS (left) and in the experiment (right).  
The color bar encodes the velocity modulus.

### 3 From 2D data to 3D flow reconstruction

The first step of the technique requires to have at disposal either 3D simulations or 3D experimental measurements of the flow, with large enough statistically independent flow realizations such that the POD basis is representative of the flow. This step is the most critical one, because this flow of reference must be equivalent to the further flows that will be subjected to the reconstruction.

Then, consider the usual case where the measurement is done on a restricted spatial domain of a flow, whose features are those of the model 3D flow. For example, when dealing with PIV techniques, the velocity field measurement is performed in the  $(x,y)$  plane at a given spanwise position  $z=z_0$ . When dealing with LDV measurement, the measurement area is even reduced to a single point  $(x_0,y_0,z_0)$ . For the sake of the demonstration, suppose that the field is known using PIV measurements in the  $(x,y)$  plane at  $z=z_0$ . Then, Equation (5) can be rewritten:

$$u(x,y,z_0,t) = \sum_{n=1}^M a_n(t) \varphi_n(x,y,z_0) \quad (6)$$

In the reconstruction procedure, the  $a_n$ 's are the initially unknown quantities, their knowledge automatically providing the 3D flow, in pursuance of Equation (5). The known quantities, in return, are the velocity field measurement  $u(x,y,z_0,t)$  and the flow basis  $\varphi_n(x,y,z)$ . In order to achieve the desired 3D reconstruction, the  $a_n$ 's have to be determined. This is achieved by calculating the coefficients:

$$b_m = \langle \varphi_m(x,y,z_0) | u(x,y,z_0) \rangle = \sum_{n=1}^M a_n(t) \langle \varphi_m(x,y,z_0) | \varphi_n(x,y,z_0) \rangle \quad (7)$$

where the scalar product between two vectors  $u$  and  $v$  is defined, over the spatial domain  $D$ , as:

$$\langle u | v \rangle = \int_D (u_x(r) \cdot v_x(r) + u_y(r) \cdot v_y(r) + u_z(r) \cdot v_z(r)) dr \quad (8)$$

Although the 3D POD modes  $\varphi_n(x,y,z)$  are orthogonal, such is not the case for their 2D restrictions  $\varphi_n(x,y,z_0)$ . Henceforth, the  $b_m$  and  $a_n$  are related by:

$$b_m(t) = M_{mn} a_n(t) \quad (9)$$

where the transfer matrix writes:

$$M_{nm} = \int_D (\varphi_{x,n}(x,y,z_0) \varphi_{x,m}(x,y,z_0) + \varphi_{y,n}(x,y,z_0) \varphi_{y,m}(x,y,z_0)) dx dy \quad (10)$$

As a consequence, and rigorously, from the knowledge of the  $b_m$ , the  $a_n$  can be determined as far as the transfer matrix  $M_{nm}$  is invertible:

$$a_m(t) = M_{mn}^{-1} b_n(t) \quad (11)$$

### 4 Discussion and perspectives

The three dimensional POD based reconstruction proposed in this paper requires two conditions to be fulfilled by the data set. First, it must reproduce most of the experimental flow features, and second, it

must be long enough, or the statistical set must be large enough, so that any further flow realization features are actually included in the data set (including rare events). If these conditions are fulfilled (which is automatically the case when working on the data set itself [2]), Equation (9) guarantees the reconstruction from 2D to 3D to be done without lack of information. For example, in the case where the POD basis would not include the frequency competition in the shear layer oscillations (competition which is observed at different flow rate velocities [3]), then, from a campaign of measurements where such a phenomenon would occur, the reconstructed flow would only provide a unique mode of oscillation and, as a consequence, would fail in reconstructing the adequate 3D flow.

However, the reconstruction becomes impossible, whatever the flow realization is, if the transfer matrix  $M_{mn}$  is not invertible. The invertibility of the matrix is related to how much information about the flow is captured by the subdomain (here a 2D section). Indeed, if the subdomain is “large enough”, then the projection of the restrictions will be “close enough” to the identity matrix, which will guarantee that the matrix will be invertible. Limiting the measurement volume to a single point, for instance when dealing with LDV techniques, will be too restrictive and the technique should fail. Clearly, two different 3D flow realizations could give a similar value for the velocity field at the measurement point. In such a case, the transfer matrix is not invertible anymore.

To validate the technique, some criterion must be found to quantify the quality of the 3D reconstruction. This is not a trivial task since the initial 3D flow, on which was done the measurement, is unknown. A class of criterion could be based on the comparison between the 2D measured velocity flow, with the corresponding 2D plane out of the 3D reconstructed flow. Other criteria may be based on statistical properties of the reconstructed flow that may give characteristic information about the instantaneous flow features.

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