Multiscale nonsmooth analysis of tensegrity systems

Shéhérazade Nineb, Pierre Alart, David Dureisseix

Multiscale nonsmooth analysis of tensegrity systems

Shéhérazade Nineb, Pierre Alart, David Dureisseix*

*Laboratoire de Mécanique et Génie Civil UM2-CNRS CC048
Place Eugène Bataillon F-34095 Montpellier CEDEX 5
{nineb, alart, dureisse}@lmgc.univ-montp2.fr

Abstract
A specific feature of multicontact problems is the large number of contact or interaction conditions that leads to large scale problems. Several approaches, among them algorithm parallelization [11], have been designed to tackle the numerical difficulties arising from such problems. This work extends to strongly nonsmooth problems the LATIN (LArge Time INcrement) [5] multiscale approach. As a first step, the case of tensegrity systems is considered as a particular discrete medium.

1 Introduction

Nonsmooth systems can arise from different mechanical models. For instance granular media [6] where interactions consist of contact with friction between grains, or tensegrity structures where a large number of cables can slacken, lead to large scale nonsmooth problems. When the number of grains, or cables, becomes large, a multiscale approach including an homogenization technique can be well suited to a resolution strategy of these problems, together with advanced numerical techniques [12, 11]. We are interested here in tensegrity structures, when they are designed as an assembly of elementary modules.

2 Problem settings

Tensegrity systems are reticulated space structures constituted with rectilinear elements such as "cables" or "bars", see figure 1 (right) [10]. Bars are subjected to compression loading, while cables are subjected to traction loading. Joining elements are perfect articulations called "nodes". Later on, cables and bars will be designated by "links" between nodes. These systems allow for selfstressed states, i.e. stress states that satisfy the equilibrium without external loading. These stress states are mandatory to ensure the overall structure rigidity. The reference problem is herein related to the static behavior of such a structure, denoted by Ω, with small perturbation assumption. The loadings are a prescribed displacement U_d on a first part of the nodes, Ω_1, and a prescribed force F_d, on the complementary part of the nodes, Ω_2. For the sake of simplicity, the body force is not taken into account in this presentation. Therefore, only the final configuration is of interest, and we do not seek any evolution problem. The strain in each link j (bar or cable) is defined by $\epsilon_j = \frac{W_j}{l_j}$ where l_j is the element length and W_j its variation. The length variation vector, W, is a linear function of the displacement vector U of the nodes. This is expressed with $W = BU$. Finally, F denotes the vector of internal forces in the links. The problem is to find (U, F)
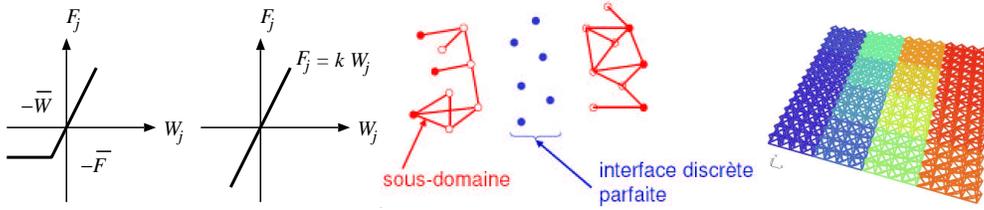
Figure 1: Cable and bar behaviors (left), discrete interface (middle) and substructuring of a tensegrity structure (right).

that satisfies:

- the kinematic admissibility: $U|_{\Omega_1} = U_d$ and $\epsilon_j = \frac{W_j}{l_j}$,

- the equilibrium written in a weak form: $\forall U^*, U^*|_{\Omega_1} = 0$ et $\sum_{j \in \Omega} F_j . W_j^* = \sum_{i \in \Omega_2} F_{d_i} . U_i^*$ with $W^* = BU^*$, where F_j is the internal effort in the link j,

- cables and bars behavior, see figure 1 (left), where $\bar{F}$ is the initial stress in the cables, and $\bar{W} = k^{-1}\bar{F}$ where k is the cable stiffness. Of course, $\bar{F}$ must be self balanced: $B^T \bar{F} = 0$.

In order to solve this problem, the strategy used here is an extension to the case of discrete problems of the micro macro approach developped for continuous medium in [5].

2.1   Substructuring

The first step of the problem reformulation consists of a decomposition of the structure into substructures and interfaces. We can proceed in two ways: either nodes are distributed among substructures, and interfaces are links joining a substructure to another, or links are distributed among substructures, and interfaces are nodes joining a substructure to another. Only this last case is considered herein, figure 1 (middle). Interfaces therefore exhibit a perfect behavior (equilibrium of efforts and continuity of displacements in interface nodes) and non smoothness is localized within the substructures. This modelling choice is identical to [1] and somehow the dual of the one proposed in [9] where the non linearity (contact in cracks) is isolated in the interfaces.

2.2   Two scale modelling

The definition of the micro and macro fields involves the interface fields of the substructured problem. We follow herein the approach developed in [2]. For a local interface $\Gamma_{EE'}$ between two substructures Ω_E and $\Omega_{E'}$, with a given discrete forces field on nodes of the interface $F_{EE'}$ acting on Ω_E, we define the macro part $F_{EE'}^M$ as the resultant and the moment of $F_{EE'}$. Using a force-oriented approach, we choose to write $F_{EE'} = F_{EE'}^M + F_{EE'}^m$ where $F_{EE'}^m$ is the micro complement to the initial field $F_{EE'}$. Micro and macro quantities of a displacement field $V_{EE'}$ on the interface nodes are obtained when expressing duality on the interface: $F_{EE'} . V_{EE'} = F_{EE'}^M . V_{EE'}^M + F_{EE'}^m . V_{EE'}^m$.

When compared to a continuous medium substructuring [5, 2], the discrete medium case leads to a higher connectivity of the decomposition (a large ratio number of interfaces to the number of substructures), as well as some interfaces with a small number of nodes. Such interfaces, designated as "weak", may have a too small number of nodes to split their discretization space into micro and macro subspaces (for instance less than four nodes in 3D, while the size of the macro space of resultant and moment is 9). For such weak interfaces, no macro part is defined.

3   Computational strategy

The solution strategy is close to the LATIN method as designed in [2]. The strategy is iterative, and each iteration includes a linear stage and a local stage. The linear stage leads to a problem similar to linear

elasticity, independent on each substructure, as well as a generalized global problem involving only the macro quantities (resultants and moments on interfaces). This strategy is not detailed herein, and the reader is suggested to refer to [2]. At the local stage, we obtain:

- on each interface, an explicit problem on each node of the interface. Due to the definition of interfaces used herein, this problem is still linear: the node behavior is to transfer displacement and effort through the interface.

- on each substructure, we are interested in cable and bar behaviors: we search for internal forces $\widehat{F}$ and extensions $\widehat{W}$ in elements.

For the bars, the unknowns $(\widehat{F}, \widehat{W})$ have to satisfy:

  - bar behavior (figure 1, left): $\widehat{F} = k\widehat{W}$ (k is the bar stiffnesses diagonal matrix),
  - a search direction from the estimated solution (F, W) obtained at the previous linear stage: $(\widehat{F} - F) + l(\widehat{W} - W) = 0$. l is a parameter of the method, homogeneous to k. It has no effect on the solution after convergence, but can change the convergence rate of the algorithm. The resulting problem is linear and explicit in $(\widehat{F}, \widehat{W})$.

For cables, the unknowns $(\widehat{F}, \widehat{W})$ have to satisfy:

  - cable behavior (figure 1, left),
  - a search direction, similar to the one used for the bars with the following change in variable:

$$\left\{ \begin{array}{l} \widehat{F}_T = \widehat{F} + \bar{F} \\ \widehat{R}_T = -(\widehat{W} + \bar{W}) + k^{-1}(\widehat{F} + \bar{F}) \end{array} \right. \tag{1}$$

where $\bar{F}$ is the previously mentioned initial selfstressed in cables and $\bar{W} = k^{-1}\bar{F}$.

For each cable, the resulting problem is a scalar complementary problem (LCP) [3]:

$$\left\{ \begin{array}{l} \widehat{R}_T - (l^{-1} + k^{-1})\widehat{F}_T = -[(W + \bar{W}) + l^{-1}(F + \bar{F})] \\ 0 \le \widehat{R}_T \perp \widehat{F}_T \ge 0 \end{array} \right. \tag{2}$$

4   Numerical tests

The proposed example concerns a tensegrity grid designed with the duplication of a selfstressed elementary module [7, 10]. Such a module is composed of 8 nodes, 12 cables and 4 bars. As boundary conditions on the whole grid, the nodes on two opposite lower edges are clamped and a uniform vertical force field is exerted on all of the nodes. The whole grid is splitted into 16 substructures (each of them contains 16 modules) and 42 interfaces (with 18 weak interfaces). As a comparison point of view, a similar substructuring of plate for continuum media, would lead to 16 substructures and only 24 interfaces, none of them beeing "weak".

For the considered test case, the loading can release a significant part of cables, up to a critical value for which there is a loss of stiffness, (similar to the buckling in continuous medium). Figure 2 presents the deformed configuration, and the number of slack cables as a function of the loading amplitude (in the present case, up to 46% of slack cables). We still stay within the stable domain of the structure behavior, for which it still possesses a stiffness reserve.

5   Conclusions

When the applied loadings are such that a large number of cables slacken, the simulation of large scale tensegrity structures requires the resolution of large nonsmooth systems. A multiscale approach, together with a domain decomposition, was used for solving these discrete systems when nonsmoothness lies inside substructures. In a short range, it will be interesting to have an automatic selection of an optimal search direction for the convergence rate of the algorithm and for the robustness of the termination criterion.
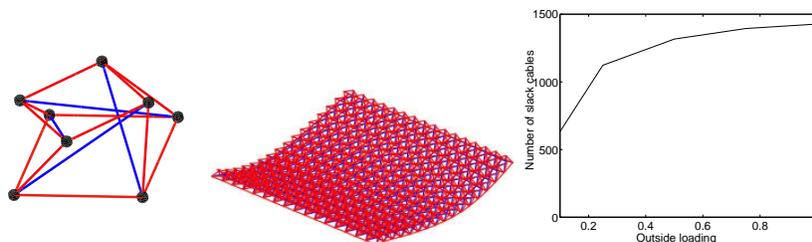
Figure 2: Module used (left) and results on a tensegrity structure.

The search direction chosen herein is local per element (bar or cable) but could also be chosen global per substructure; in such case, the LCP problem becomes vectorial and requires a suited resolution [8, 11]. The method also involves an homogenization at the substructure level (i.e. a group of selfstressed modules). An interpretation of the homogenized behavior will be studied. In a long range, the use of this approach will be extended to the dynamic behavior of granular media, for which an additional difficulty arises due to large changes in the configuration with the grain flow.

References

[1] M. Barboteu, P. Alart, and M. Vidrascu. A domain decomposition strategy for nonclassical frictional multi-contact problems. *Computer Methods in Applied Mechanics and Engineering*, 190:4785–4803, 2001.

[2] D. Dureisseix, P. Ladevèze, and O. Loiseau. A micro/macro and parallel computational strategy for highly heterogeneous structures. *International Journal for Numerical Methods in Engineering*, 52:121–138, 2001.

[3] Murty Katta. *Principal pivoting methos for LCP*. Department of Industrial and Operations Engineering, University of Michigan, 1997.

[4] P. Ladevèze. *Mécanique non-linéaire des structures — Nouvelle approche et méthodes de calcul non incrémentales*. Hermès, Paris, 1996.

[5] P. Ladevèze and D. Dureisseix. A micro/macro approach for parallel computing of heterogeneous structures. *Int. J. for Comput. Civil and Struct. Engng.*, 1:18–28, 2000.

[6] J.J. Moreau. Numerical aspects of sweeping process. *Comput. Methods Appl. Mech. Engrg.*, 177:329–349, 1999.

[7] R. Motro. *Tensegrity*. Hermes Science Publishing, London, 2003.

[8] S. Nineb, P. Alart, F. Dubois, and D. Dureisseix. Solveurs de systèmes de complémentarité et application à la tenségrité. In *Actes du 17e Congrès Français de Mécanique*, Troyes, 2005.

[9] A. Nouy, P. Ladevèze, and O. Loiseau. A multiscale computational approach for contact problems. *Computer Methods in Applied Mechanics and Engineering*, 191:4869–4891, 2002.

[10] Kazi-Aoual M.N., Quirant J. and Motro R. Designing tensegrity systems: the case of a double layer grid. *Engineering Structures*, 25(9):1121–1130, 2003.

[11] M. Renouf and P. Alart. Conjugate gradient type algorithms for frictional multi-contact problems: applications to granular materials. *Computer Methods in Applied Mechanics and Engineering*, 2004.

[12] M. Renouf, F. Dubois, and P. Alart. A parallel version of non smooth contact dynamics algorithm applied to simulation of granular medium. *Journal of Computational and Applied Mathematics*, 168:375–382, 2004.