Polynomial Estimation of Time-varying Multi-path Gains with ICI Mitigation in OFDM Systems

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Abstract

In this paper, we consider the case of a high speed mobile receiver operating in an orthogonal-frequency-division-multiplexing (OFDM) communication system. We present an iterative algorithm for estimating multi-path complex gains with inter-sub-carrier-interference (ICI) mitigation (using comb-type pilots). Each complex gain variation is approximated by a polynomial representation, within several OFDM symbols. Assuming the knowledge of delay-related information, polynomial coefficients are obtained from the time-averaged gain values, which are estimated using the LS criterion. The channel matrix is easily computed and the ICI is reduced by using successive interference suppression (SIS) during data symbol detection. The algorithm's performance is further enhanced by an iterative procedure, performing channel estimation and ICI mitigation at each iteration. Theoretical analysis and simulation results for a Rayleigh fading channel show that the proposed algorithm has low computational complexity and good performance in the presence of high normalised Doppler spread.

Index Terms

[0]Part of this work will be presented in IEEE ISCCSP, St. Julians, MALTA, March 2008 [1]

OFDM, ICI, SIS, channel estimation, time-varying channels.

I. INTRODUCTION

ORTHOGONAL frequency division multiplexing (OFDM) is an attractive technique for high-speed data transmission in mobile communications. Currently, OFDM has been adapted to the digital audio and video broadcasting (DAB/DVB) systems, to high-speed wireless local area networks (WLAN) such as IEEE802.11x, HIPERLAN/2, and to multimedia mobile access communications (MMAC), ADSL, digital multimedia broadcasting (DMB) and multi-band OFDM type ultra-wideband (MB-OFDM UWB) systems, etc. In OFDM systems, each sub-carrier has a narrow bandwidth, which makes the signal robust against frequency selectivity which can arise from multi-path delay spread. However, OFDM is relatively sensitive to the time-domain selectivity, which is induced by rapid temporal variations of a mobile channel. Such variations corrupt the orthogonality of the OFDM sub-carrier waveforms, leading to inter-sub-carrier-interference (ICI).

In the case of wideband mobile communication sytems, dynamic channel estimation is needed, because the radio channel is frequency selective and time-varying [5]. In practice, the channel may change significantly, even within one OFDM symbol. It is thus preferable to estimate channel by inserting pilot tones, called comb-type pilots, into each OFDM symbol [6]. Assuming such a strategy, conventional methods consist generally of estimating the channel at pilot frequencies and next interpolating [8] the channel frequency response. The estimation of the channel at the pilot frequencies can be based on Least Square (LS) or Linear Minimum Mean-Square-Error (LMMSE). LMMSE has been shown to have better performance than LS [6]. In [7], the complexity of LMMSE is reduced by deriving an optimal low-rank estimator with singular-value-decomposition.

In [9] the channel estimator is based on a parametric channel model, which directly estimates the time delays and complex attenuations of the multi-path channel. This estimator yields the best performance of all comb-type pilot channel estimators, as long as the channel remains invariant within one OFDM symbol.

Recently, the basis expansion model (BEM) was introduced to approximate OFDM channel variations. Firstly, for slow fading assumptions, [16] used a polynomial basis function model for the channel response

in a time-frequency window, whereas [17] modeled the correlated discrete-time fading channel using a Karhunen-Loeve(KL) orthogonal expansion.

For fast time-varying channels, many existing works resort to estimating the equivalent discrete-time channel taps, which are modeled by the BEM [18] [19]. The BEM methods [18] are Karhunen-Loeve BEM (KL-BEM), prolate spheroidal BEM (PS-BEM), complex-exponential BEM (CE-BEM) and polynomial BEM (P-BEM). The KL-BEM is optimal in terms of mean square error (MSE), but is not robust to statistical channel mismatches, whereas the PS-BEM is a general approximation for all kinds of channel statistics, although its band-limited orthogonal spheroidal functions have maximal time concentration within the considered interval. The CE-BEM is independent of channel statistics, but induces a large modeling error. Finally, a great deal of attention has been paid to the P-BEM [19], although its modeling performance is rather sensitive to the Doppler spread; nevertheless, it provides a better fit for low, than for high Doppler spreads. In [22], a piece-wise linear method is used to approximate the channel taps, and the channel tap slopes are estimated from the cyclic prefix or from both adjacent OFDM symbols.

For ICI mitigation, MMSE and successive interference cancellation (SIC) schemes, with optimal ordering, were developed in [23]. Since the number of sub-carriers is usually very large, this receiver is highly complex. In [24], a low-complexity MMSE and decision-feedback equalizer (DFE) were developed, based on the fact that most of a symbol's energy is distributed over just a few sub-carriers, and that ICI on a sub-carrier originates mainly from its neighbouring sub-carriers.

As channel delay spread increases, the number of channel taps also increases, thus leading to a large number of BEM coefficients [18]. In such a case, more pilot symbols are needed in order to estimate the BEM coefficients. In contrast to the research described in [18], we sought to directly estimate the physical channel, instead of the equivalent discrete-time channel taps. This means estimating the physical propagation parameters such as multi-path delays and multi-path complex gains. For a fast time-varying channel, the channel matrix in the OFDM system depends on the multi-path delays and time-variations of the multi-path complex gains within a single OFDM symbol. In [2], we proposed an algorithm for channel matrix estimation and inter-sub-carrier-interference (ICI) reduction, which is executed per block of OFDM symbols. Assuming the availability of delay information, the time-varying complex gains within a given OFDM symbol are obtained by interpolating the estimated time averaged values over each symbol

of the block. This algorithm is very demanding in terms of computing power.

In the present paper, we present a new low-complexity iterative algorithm for the estimation of complex gains with ICI mitigation in OFDM downlink mobile communication systems which use comb-type pilots. By exploiting the nature of the channel, the delays are assumed to be invariant and perfectly estimated as we have already done in OFDM [2] and CDMA [3] [4] contexts. It should be noted that an initial, and generally accurate estimation of the number of paths and time delays can be obtained by using the MDL (minimum description length) and ESPRIT (estimation of signal parameters by rotational invariance techniques) methods [9] [11]. Firstly, we compute the time average of the complex gains, over the effective duration of the OFDM symbol, by using LS criterion as was done in [2]. Then, we show that the time-variation of each complex gain can be approximated in a polynomial fashion within several OFDM symbols, where the coefficients of each polynomial are calculated from the estimated time-averaged values. Hence, thanks to the use of polynomial modeling, the channel matrix can be computed with low complexity from the estimated coefficients, and the ICI is reduced using SIS in data symbol detection. We provide theoretical and simulated Mean Square Error (MSE) multi-path channel complex gain estimation analysis, expressed in terms of the normalised (with respect to the OFDM symbol-time) Doppler spread. By taking advantage of an iterative procedure, at each step of which the ICI is estimated and then removed, the algorithm proposed here has demonstrated considerable improvements in performance, whilst reducing computational complexity, when compared to that described in [2].

The organisation of the present paper is as follows: Section II introduces the OFDM baseband model, whereas Section III describes the polynomial modeling. Section IV covers the algorithm used to estimate the polynomial coefficients, as well as the iterative algorithm. Section V presents the results of simulations which validate our technique. Finally, our conclusions are presented in Section VI.

$Notation$: The notations used in this paper are as follows. Upper (lower) bold face letters denote matrices (column vectors). $[\mathbf{x}]_k$ denotes the kth element of the vector $\mathbf{x}$, and $[\mathbf{X}]_{k,m}$ denotes the $[k,m]$th element of the matrix $\mathbf{X}$. $\mathbf{I}_N$ is a $N \times N$ identity matrix and diag$\{\mathbf{x}\}$ is a diagonal matrix with $\mathbf{x}$ on its main diagonal. The superscripts $(\cdot)^T$ and $(\cdot)^H$ stand respectively for transpose and Hermitian operators. $|\cdot|$, Tr$(\cdot)$ and E$[\cdot]$ are respectively the determinant, trace and expectation operations, and Re$(\cdot)$, $\|\cdot\|$ and $(\cdot)^*$ are respectively the real part, magnitude and conjugate of a complex number or matrix. $\|\mathbf{X}\|^2$ is the

Frobenius matrix norm, $J_0(\cdot)$ denotes the zeroth-order Bessel function of the first kind and $\delta_{k,m}$ is the Kronecker symbol.

II. System Model

If we consider an OFDM system with N sub-carriers, the duration of an OFDM symbol can be written as $T = vT_s$, with $v = N + N_g$ where N_g is the length of the cyclic prefix and T_s is the sampling time. On the transmitter side, an N-point IFFT is applied to a normalized QAM-symbols data block $\{x_{(n)}[b]\}$ ($i.e., \mathrm{E}\left[x_{(n)}[b]x_{(n)}[b]^*\right] = 1$), where n and b represent respectively the OFDM symbol index and the sub-carrier index. A cyclic prefix (CP), which is a copy of the last samples of the IFFT output, is added to avoid inter-symbol-interference (ISI) caused by multi-path fading channels. The output baseband signal of the transmitter can be represented as:

$$s(t) = \sum_{n=-\infty}^{\infty} \sum_{q=-N_g}^{N-1} s_{(n)}[q] g_e(t - qT_s - nT) \tag{1}$$

where $g_e(t)$ is the impulse response of the transmission analogue filter and $s_{(n)}[q]$, with $q \in [-N_g, N-1]$, are the $(N + N_g)$ samples of the IFFT output completed by the cyclic prefix of the nth OFDM symbol, given by:

$$s_{(n)}[q] \;=\; \frac{1}{N} \sum_{b=-\frac{N}{2}}^{\frac{N}{2}-1} x_{(n)}[b] e^{j2\pi \frac{bq}{N}} \tag{2}$$

It is assumed that the signal is transmitted over a multi-path Rayleigh fading channel characterized by:

$$h(t,\tau) \;=\; \sum_{l=1}^{L} \alpha_l(t) \delta(\tau - \tau_l T_s) \tag{3}$$

where L is the total number of propagation paths, α_l is the lth complex gain of variance $\sigma^2_{\alpha_l}$ and τ_l is the lth delay normalized by the sampling time (τ_l is not necessarily an integer). The L individual elements of $\{\alpha_l(t)\}$ are uncorrellated with respect to each other. They are wide-sense stationary (WSS), narrow-band complex Gaussian processes, with the so-called Jakes' power spectrum of maximum Doppler frequency f_d [10]. The average energy of the channel is normalized to one ($i.e., \sum_{l=1}^{L} \sigma^2_{\alpha_l} = 1$).

On the receiver side, after passing to discrete time by means of low-pass filtering and A/D conversion, the CP is removed assuming that its length is no less than the maximum delay. Afterwards, an N-point FFT is applied to transform the time sequence into the frequency domain. If we consider that the N

transmission sub-carriers are within the flat region of the frequency response of each of the transmitter and receiver filters, then, omitting the time index n, the N received sub-carriers are given by [2] [9]:

$$\mathbf{y} \;=\; \mathbf{H}\,\mathbf{x} + \mathbf{w} \tag{4}$$

where **x**, **y**, **w** are $N \times 1$ vectors given by:

$$\begin{aligned}
\mathbf{x} &= \Big[x[-\frac{N}{2}], x[-\frac{N}{2}+1], ..., x[\frac{N}{2}-1]\Big]^T \\
\mathbf{y} &= \Big[y[-\frac{N}{2}], y[-\frac{N}{2}+1], ..., y[\frac{N}{2}-1]\Big]^T \\
\mathbf{w} &= \Big[w[-\frac{N}{2}], w[-\frac{N}{2}+1], ..., w[\frac{N}{2}-1]\Big]^T
\end{aligned}$$

and **H** is a $N \times N$ matrix with elements given by:

$$[\mathbf{H}]_{k,m} \;=\; \frac{1}{N}\sum_{l=1}^{L}\Big[e^{-j2\pi(\frac{m-1}{N}-\frac{1}{2})\tau_l}\sum_{q=0}^{N-1}\alpha_l(qT_s)e^{j2\pi\frac{m-k}{N}q}\Big] \tag{5}$$

where $\{\alpha_l(qT_s)\}$ is the T_s spaced sampling of the lth complex gain value, and $w[b]$ is white complex Gaussian noise with variance σ^2. The channel matrix contains the time average of the channel frequency response $[\mathbf{H}]_{k,k}$ on its diagonal and the coefficients of ICI $[\mathbf{H}]_{k,m}$ for $k \neq m$. It sould be noted that **H** would clearly be a diagonal matrix if the complex gains were time-invariant within one OFDM symbol.

III. Complex Gain Polynomial Modeling

In this section, we show that, for realistically high Doppler spread f_dT, each sampled complex gain $\boldsymbol{\alpha}_l = \big[\alpha_l(-N_gT_s), ..., \alpha_l\big((vN_c - N_g - 1)T_s\big)\big]^T$ within N_c OFDM symbols can be approximated by a polynomial model containing N_c coefficients ($i.e.$, a (N_c-1) degree polynomial). Thus, for $q \in \mathcal{D} = [-N_g, vN_c - N_g - 1]$, $\alpha_l(qT_s)$ can be expressed as:

$$\alpha_l(qT_s) \;=\; \sum_{d=0}^{N_c-1} c_{d,l}\; q^d + \xi_l[q] \tag{6}$$

where $\mathbf{c}_l = \Big[c_{0,l}, ..., c_{N_c-1,l}\Big]^T$ are the N_c polynomial coefficients and $\xi_l[q]$ is the model error. We will also show that a good approximation can be obtained by calculating the N_c coefficients from only $\overline{\boldsymbol{\alpha}}_l = \Big[\overline{\alpha}_{l,0}, ..., \overline{\alpha}_{l,N_c-1}\Big]^T$, where $\overline{\alpha}_{l,d} = \frac{1}{N}\sum_{q=dv}^{dv+N-1}\alpha_l(qT_s)$ is the time average computed over the effective duration of the $(d+1)$th OFDM symbol of the lth complex gain.

Optimal Polynomial: The optimal polynomial $\boldsymbol{\alpha}_{\mathbf{opt}_l}$, which is least-squares fitted (linear and polynomial regression) [15] to $\boldsymbol{\alpha}_l$, and its N_c coefficients $\mathbf{c}_{\mathbf{opt}_l}$ are given by:

$$\begin{aligned} \boldsymbol{\alpha}_{\mathbf{opt}_l} &= \mathbf{Q}^T \mathbf{c}_{\mathbf{opt}_l} = \mathbf{S}\boldsymbol{\alpha}_l \\ \mathbf{c}_{\mathbf{opt}_l} &= \left(\mathbf{Q}\mathbf{Q}^T\right)^{-1}\mathbf{Q}\boldsymbol{\alpha}_l \end{aligned} \tag{7}$$

where $\mathbf{Q}$ is a $N_c \times vN_c$ matrix of elements $[\mathbf{Q}]_{k,m} = (m - N_g - 1)^{(k-1)}$ and $\mathbf{S} = \mathbf{Q}^T\left(\mathbf{Q}\mathbf{Q}^T\right)^{-1}\mathbf{Q}$ is a $vN_c \times vN_c$ matrix. It provides the MMSE approximation for all polynomials containing Nc coefficients, given by:

$$\begin{aligned} \mathrm{MMSE}_l &= \frac{1}{vN_c}\mathrm{E}\left[(\boldsymbol{\alpha}_l - \boldsymbol{\alpha}_{\mathbf{opt}_l})^H(\boldsymbol{\alpha}_l - \boldsymbol{\alpha}_{\mathbf{opt}_l})\right] \\ &= \frac{1}{vN_c}\mathrm{Tr}\left((\mathbf{I}_{vN_c} - \mathbf{S})\mathbf{R}_{\boldsymbol{\alpha}_l}(\mathbf{I}_{vN_c} - \mathbf{S}^T)\right) \end{aligned} \tag{8}$$

where $\mathbf{R}_{\boldsymbol{\alpha}_l} = \mathrm{E}\left[\boldsymbol{\alpha}_l\boldsymbol{\alpha}_l^H\right]$ is the $vN_c \times vN_c$ correlation matrix of $\boldsymbol{\alpha}_l$. Since $\alpha_l(t)$ is wide-sense stationary (WSS) narrow-band complex Gaussian processes with the so-called Jakes' power spectrum [10] then:

$$[\mathbf{R}_{\boldsymbol{\alpha}_l}]_{k,m} = \sigma^2_{\alpha_l} J_0\Big(2\pi f_d T_s(k-m)\Big) \tag{9}$$

Desired Polynomial: Our aim is now to find the polynomial approximation of N_c coefficients, based solely on knowledge of $\overline{\boldsymbol{\alpha}}_l$. This polynomial $\boldsymbol{\alpha}_{\mathbf{des}_l}$ and its coefficients $\mathbf{c}_{\mathbf{des}_l}$ are given by:

$$\begin{aligned} \boldsymbol{\alpha}_{\mathbf{des}_l} &= \mathbf{Q}^T\mathbf{c}_{\mathbf{des}_l} = \mathbf{V}\,\overline{\boldsymbol{\alpha}}_l \\ \mathbf{c}_{\mathbf{des}_l} &= \mathbf{T}^{-1}\overline{\boldsymbol{\alpha}}_l \end{aligned} \tag{10}$$

where $\mathbf{T}$ is the $N_c \times N_c$ transfer matrix between $\mathbf{c}_{\mathbf{des}_l}$ and $\overline{\boldsymbol{\alpha}}_l$, and $\mathbf{V} = \mathbf{Q}^T\mathbf{T}^{-1}$. For $N_c = 3$, $\mathbf{T}$ is given by:

$$\mathbf{T} = \begin{bmatrix} 1 & \frac{N-1}{2} & \frac{(N-1)(2N-1)}{6} \\ 1 & \frac{N-1}{2} + v & \frac{(N-1)(2N-1)}{6} + (N-1)v + v^2 \\ 1 & \frac{N-1}{2} + 2v & \frac{(N-1)(2N-1)}{6} + 2(N-1)v + 4v^2 \end{bmatrix}$$

Notice that, for $N_c = 2$, the resulting transfer matrix will be the 2×2 upper block matrix in the top-left corner of the above $\mathbf{T}$ matrix (defined for $N_c = 3$). The MSE of this polynomial modeling is given by:

$$\begin{aligned} \mathrm{MSE}_{\mathrm{des}_l} &= \tfrac{1}{vN_c}\mathrm{E}\left[\mathbf{e}_{\mathbf{des}_l}\mathbf{e}^H_{\mathbf{des}_l}\right] = \\ \tfrac{1}{vN_c}\mathrm{Tr}&\left(\mathbf{R}_{\boldsymbol{\alpha}_l} + \mathbf{V}\,\mathbf{R}_{\overline{\boldsymbol{\alpha}}_l}\mathbf{V}^T - \mathbf{R}_{\boldsymbol{\alpha}_l\overline{\boldsymbol{\alpha}}_l}\mathbf{V}^T - \mathbf{V}\,\mathbf{R}^H_{\boldsymbol{\alpha}_l\overline{\boldsymbol{\alpha}}_l}\right) \end{aligned} \tag{11}$$

where $\mathbf{e}_{\mathbf{des}_l} = \boldsymbol{\alpha}_l - \boldsymbol{\alpha}_{\mathbf{des}_l}$ is the model error, $\mathbf{R}_{\overline{\boldsymbol{\alpha}}_l}$ is the $N_c \times N_c$ correlation matrix of $\overline{\boldsymbol{\alpha}}_l$ and $\mathbf{R}_{\boldsymbol{\alpha}_l\overline{\boldsymbol{\alpha}}_l}$ is the $vN_c \times N_c$ cross-correlation matrix between $\boldsymbol{\alpha}_l$ and $\overline{\boldsymbol{\alpha}}_l$ with elements given by:

$$\begin{aligned}
[\mathbf{R}_{\overline{\boldsymbol{\alpha}}_l}]_{k,m} &= \frac{\sigma^2_{\alpha_l}}{N^2} \sum_{q_1=kv-v}^{kv-N_g-1} \sum_{q_2=mv-v}^{mv-N_g-1} J_0\Big(2\pi f_d T_s(q_1-q_2)\Big) \\
[\mathbf{R}_{\boldsymbol{\alpha}_l\overline{\boldsymbol{\alpha}}_l}]_{k,m} &= \frac{\sigma^2_{\alpha_l}}{N} \sum_{q=mv-v}^{mv+N_g-1} J_0\Big(2\pi f_d T_s(k-q-N_g-1)\Big)
\end{aligned} \tag{12}$$

As can be seen in Fig 1, even with just $N_c = 2$ coefficients, we have $\text{MSE}_{\text{des}} \approx$ MMSE and for $f_dT \leq 0.1$, $\text{MSE}_{\text{des}} \leq 10^{-4}$. This proves that, for high realistic values of f_dT, we can approximate $\boldsymbol{\alpha}_l$ by a polynomial model with N_c coefficients and can calculate the polynomial approximation using only the time average values $\overline{\boldsymbol{\alpha}}_l$. More explanation about polynomial modeling for Jakes' process can be found in [1].

Under this polynomial approximation, the channel matrix (see equation (5)) for the nth of N_c OFDM symbols can be defined simply as:

$$\begin{aligned}
\mathbf{H}_{(n)} &= \frac{1}{N} \sum_{d=0}^{N_c-1} \mathbf{B}_{(n,d)} \\
\text{with } \mathbf{B}_{(n,d)} &= \mathbf{M}_{(n,d)}\ \text{diag}\{\mathbf{F}\boldsymbol{\chi}_d\}
\end{aligned} \tag{13}$$

where $\boldsymbol{\chi}_d = \Big[c_{d,1}, ..., c_{d,L}\Big]^T$, $\mathbf{F}$ is the $N \times L$ Fourier matrix and $\mathbf{M}_{(n,d)}$ is a $N \times N$ matrix given by:

$$\begin{aligned}
[\mathbf{F}]_{k,m} &= e^{-j2\pi(\frac{k-1}{N}-\frac{1}{2})\tau_m} \\
\big[\mathbf{M}_{(n,d)}\big]_{k,m} &= \sum_{q=0}^{N-1} \big(q+(n-1)v\big)^d\ e^{-j2\pi\frac{m-k}{N}q}
\end{aligned} \tag{14}$$

where $n \in [1, N_c]$. Notice that the terms of the matrix $\mathbf{M}_{(n,d)}$ can easily be computed and stored, using the properties of power series. This simplified representation of the channel matrix will be used throughout the algorithm as we present in the next section.

IV. Estimation of Polynomial Coefficients and The Iterative Algorithm

In this section, we propose a method based on comb-type pilots and multi-path time delay information. This method consists in estimating the N_c coefficients of the polynomial fitted to the time averaged complex gains, over the effective duration of the N_c OFDM symbols.

A. Pilot Pattern and Received Pilot Sub-carriers

The N_p pilot sub-carriers are fixed during transmission and are inserted evenly into the N sub-carriers. As opposed to the methods described in [9] [8], the distance L_f (in frequency domain) between two adjacent pilots can be selected without the need to respect the sampling theorem. However, as will be seen in equation (20), N_p must fulfill the following requirement: $N_p \geq L$.

Let $\mathcal{P}$ denote the set containing the index positions of the N_p pilot sub-carriers defined by:

$$\mathcal{P} = \{p_s \mid p_s = (s-1)L_f + 1,\ s = 1, ..., N_p\} \tag{15}$$

The received pilot sub-carriers can be written as the sum of three components:

$$\mathbf{y_p} = \text{diag}\{\mathbf{x_p}\}\mathbf{h_p} + \mathbf{H_p x} + \mathbf{w_p} \tag{16}$$

where the $N_p \times 1$ vectors $\mathbf{x_p}$, $\mathbf{y_p}$ and $\mathbf{w_p}$ are given by:

$$\begin{aligned} \mathbf{x_p} &= \left[x[p_1], x[p_2], ..., x[p_{N_p}]\right]^T \\ \mathbf{y_p} &= \left[y[p_1], y[p_2], ..., y[p_{N_p}]\right]^T \\ \mathbf{w_p} &= \left[w[p_1], w[p_2], ..., w[p_{N_p}]\right]^T \end{aligned}$$

In the above, $\mathbf{h_p}$ is a $N_p \times 1$ vector and $\mathbf{H_p}$ is a $N_p \times N$ matrix with elements given by:

$$\begin{aligned} [\mathbf{h_p}]_k &= [\mathbf{H}]_{p_k, p_k} \\ [\mathbf{H_p}]_{k,m} &= \begin{cases} [\mathbf{H}]_{p_k, m} & \text{if } m \neq p_k \\ 0 & \text{if } m = p_k \end{cases} \end{aligned} \tag{17}$$

The first component is the desired term without ICI and the second component is the ICI term. $\mathbf{h_p}$ can be writen as the Fourier transform for the different complex gains time average $\boldsymbol{a} = \left[\overline{\alpha}_1, ..., \overline{\alpha}_L\right]^T$:

$$\mathbf{h_p} = \mathbf{F_p}\boldsymbol{a} \tag{18}$$

where $\overline{\alpha}_l = \dfrac{1}{N}\displaystyle\sum_{q=0}^{N-1} \alpha_l(qT_s)$ and $\mathbf{F_p}$ is the $N_p \times L$ Fourier transform matrix given by:

$$[\mathbf{F_p}]_{k,m} = [\mathbf{F}]_{p_k, m} \tag{19}$$

B. Estimation of Polynomial Coefficients

The complex gain time averages, taken over the effective duration of each OFDM symbol for the different paths, are estimated using the LS criterion. By neglecting the ICI contribution, the LS-estimator of $\boldsymbol{a}$, which minimizes $\left(\mathbf{y_p} - \text{diag}\{\mathbf{x_p}\}\mathbf{F_p}\boldsymbol{a}\right)^H \left(\mathbf{y_p} - \text{diag}\{\mathbf{x_p}\}\mathbf{F_p}\boldsymbol{a}\right)$, is represented by:

$$\begin{aligned} \boldsymbol{a}_{\mathbf{LS}} &= \mathbf{G}\mathbf{y_p} \\ \text{with} \quad \mathbf{G} &= \left(\mathbf{F_p}^H \text{diag}\{\mathbf{x_p}\}^H \text{diag}\{\mathbf{x_p}\}\mathbf{F_p}\right)^{-1} \mathbf{F_p}^H \text{diag}\{\mathbf{x_p}\}^H \end{aligned} \tag{20}$$

where $\mathbf{G}$ is a $L \times N_p$ matrix. By estimating $\boldsymbol{a}$ for N_c consecutive OFDM symbols, the N_c polynomial coefficients of each complex gains are obtained (as shown in section III) by:

$$\hat{\mathbf{C}}_{\mathbf{des}} = \mathbf{T}^{-1}\mathbf{A_{LS}} \tag{21}$$

where $\hat{\mathbf{C}}_{\mathbf{des}} = [\hat{\mathbf{c}}_{\mathbf{des}_1}, ..., \hat{\mathbf{c}}_{\mathbf{des}_L}]$ and $\mathbf{A_{LS}} = [\overline{\boldsymbol{\alpha}}_{\mathbf{LS}_1}, ..., \overline{\boldsymbol{\alpha}}_{\mathbf{LS}_L}]$ are $N_c \times L$ matrices.

C. Iterative Algorithm

In the iterative algorithm for channel estimation and ICI suppression, the OFDM symbols are grouped into blocks of N_c OFDM symbols each. The iterative algorithm is shown in Fig. 2, where $\{r_{(n)}[q]\}$ is the received sampled signal without CP. The complete algorithm is divided into two modes: channel matrix estimation mode and detection mode, as shown in Fig. 2(a). The first of these involves estimation of the N_c polynomial coefficients, $\mathbf{C_{des}}$, by means of an LS-estimator and computation of the channel matrix as shown in Fig. 2(b). The second mode involves the detection of data symbols using a successive data interference suppression (SIS) scheme with one tap frequency equalizer (see Appendix C). A feedback technique is used between these two modes, performing iteratively ICI suppression and channel matrix estimation. The algorithm is executed in two stages: an initialization stage and a sliding stage. The initialization stage is applicable to the first received block of N_c OFDM symbols only ($i.e.\ n = 1, ..., N_c$), whereas the sliding stage applies to each of the following OFDM symbols ($i.e.\ n > N_c$), whilst making use of the $(N_c - 1)$ previously estimated (using reduced ICI), time averaged complex gains. The initialization and sliding stages proceed as follows:

$initialization$:

$i \leftarrow 1$

if (initialization stage);

$\mathbf{Y}_{\mathbf{p}_{(i)}} = [\mathbf{y}_{\mathbf{p}_{(1,i)}}, ..., \mathbf{y}_{\mathbf{p}_{(N_c,i)}}]$ $where$ $\mathbf{y}_{\mathbf{p}_{(n,i)}} = \mathbf{y}_{\mathbf{p}_{(n)}}$ $n = 1, ..., N_c$

$elseif$ (sliding stage);

$n \leftarrow n + 1$

$\left\{[\mathbf{A_{LS}}]_{k,m},\ k = 1, .., N_c - 1\right\} = \left\{[\mathbf{A_{LS}}]_{k,m},\ k = 2, .., N_c\right\}$

$\mathbf{y}_{\mathbf{p}_{(n,i)}} = \mathbf{y}_{\mathbf{p}_{(n)}}$

$recursion$:

1) if (initialization stage); $\mathbf{A}_{\mathbf{LS}}^{T} = \mathbf{G}\mathbf{Y}_{\mathbf{p}_{(i)}}$

$elseif$ (sliding stage); $\boldsymbol{a}_{\mathbf{LS}} = \mathbf{G}\mathbf{y}_{\mathbf{p}_{(n,i)}}$

$\left\{[\mathbf{A_{LS}}]_{N_c,m},\ m = 1, .., L\right\} = \left\{[\boldsymbol{a}_{\mathbf{LS}}]_{m},\ m = 1, .., L\right\}$

2) $\hat{\mathbf{C}}_{\mathbf{des}} = \mathbf{T}^{-1}\mathbf{A_{LS}}$

3) compute the channel matrix using (13)

if (initialization stage); $\hat{\mathbf{H}}_{(n,i)}$ $n = 1, ..., N_c$

$elseif$ (sliding stage); $\hat{\mathbf{H}}_{(N_c,i)}$

4) remove the pilot ICI from the received data sub-carriers $\mathbf{y}_{\mathbf{d}_{(n)}}$

5) detection of data symbols $\hat{\mathbf{x}}_{\mathbf{d}_{(n,i)}}$

6) $\mathbf{y}_{\mathbf{p}_{(n,i+1)}} = \mathbf{y}_{\mathbf{p}_{(n)}} - \hat{\mathbf{H}}_{\mathbf{p}_{(n,i)}}\hat{\mathbf{x}}_{(n,i)}$

7) $i \leftarrow i + 1$

where i represents the iteration number. Notice that at the end of the initialization stage, $n = N_c$.

D. Computational Complexity

The purpose of this section is to determine the implementation complexity in terms of the number of the multiplications needed for the sliding stage. The matrices $\mathbf{F}$, $\mathbf{F_p}$, $\mathbf{G}$, $\mathbf{T}^{-1}$ and $\mathbf{M}_{(n,d)}$ are pre-computed

and stored if the pilot sub-carriers are fixed and the delays are invariant for a great number of OFDM symbols. The complexity of the LS-estimator of $\boldsymbol{a}$ in step 1 is $L \times N_p$ and for the estimation of N_c polynomial coefficients in step 2 it is $L \times N_c^2$. The computational cost of computing the channel matrix $\mathbf{H}_{(n)}$ in step 3 is $NN_c(N+L)$, which is less than that in [2] which is $LN^2(N+1)$. The complexity of removing the ICI in step 4, 5 and 6 is $N_p(N-N_p) + \frac{(N-N_p)(N-N_p+1)}{2} + N_p(N-1)$. In conclusion, the significant reduction in computational complexity, in comparison with that found in [2], is mainly due to the fact that the calculation of the channel matrix is based on the polynomial coefficients, with no need to construct complex gain time variations using low-pass interpolation.

E. Mean Square Error (MSE) Analysis

The MSE between the lth exact complex gain and the lth estimated polynomial (characterised by N_c coefficients and fitted to the time average values within N_c OFDM symbols) is defined by:

$$\text{MSE}_l \;=\; \frac{1}{vN_c}\text{E}\big[(\boldsymbol{\alpha}_l - \hat{\boldsymbol{\alpha}}_{\mathbf{des}_l})^H(\boldsymbol{\alpha}_l - \hat{\boldsymbol{\alpha}}_{\mathbf{des}_l})\big] \tag{22}$$

where $\hat{\boldsymbol{\alpha}}_{\mathbf{des}_l} = \mathbf{V}\overline{\boldsymbol{\alpha}}_{\mathbf{LS}_l}$ is the lth estimated polynomial, which gives (see Appendix B):

$$\begin{aligned}\text{MSE}_l \;&=\; \text{MSE}_{\text{des}_l} + \frac{1}{vN_c}\mathbf{g}_l^H\Big(\mathbf{R_1} + \mathbf{R_2}\Big)\mathbf{g}_l \\ &\quad -\frac{2}{vN_c}\text{Re}\Big(\mathbf{r_3}\mathbf{g}_l\Big)\end{aligned} \tag{23}$$

where $\mathbf{g}_l^T$ is the lth row of the matrix $\mathbf{G}$, and $\mathbf{R_1}$, $\mathbf{R_2}$ and $\mathbf{r_3}$ are computed in Appendix B. The first component on the right hand side is the MSE of the polynomial approximation, the second component is the MSE of the lth estimated polynomial and the third component is the cross-covariance term. It should be noted that if the ICI are completely eliminated, then, $\mathbf{R_2}$ and $\mathbf{r_3}$ are respectively a matrix/vector of zeros. Expression (23) thus becomes:

$$\text{MSE}_l \text{ (without ICI)} \;=\; \text{MSE}_{\text{des}_l} + \frac{1}{vN_c}\mathbf{g}_l^H\mathbf{R_1}\mathbf{g}_l \tag{24}$$

where the second component on the right hand side is the MSE of the lth estimated polynomial without ICI. This component is due to the error in the estimator of $\boldsymbol{a}$ without ICI (see (32) in Appendix B), which in our algorithm is the error of the LS-estimator without ICI (see (33) in Appendix B). The lower bound

(LB) of the estimator of $\boldsymbol{a}$ (without ICI) thus leads to the LB of the MSE between the exact complex gain and the estimated polynomial MSE_l (without ICI).

It is clear that our LS-estimator is unbiased. So, the CRAMER-RAO BOUND (CRB) [14] is an important criterion for evaluating the quality of our LS-estimator, since it provides the MMSE bound among all unbiased estimators. The Standard CRB (SCRB) [14] for the estimator of $\boldsymbol{a}$ with known ICI is given by (see Appendix A):

$$\mathbf{SCRB}_{\boldsymbol{a}} \quad = \quad \frac{1}{\text{SNR}} \left(\mathbf{F}_{\mathbf{p}}^{H} \text{diag}\{\mathbf{x_p}\}^{H} \text{diag}\{\mathbf{x_p}\} \mathbf{F_p} \right)^{-1} \tag{25}$$

where $\text{SNR} = \frac{1}{\sigma^2}$ is the normalized signal to noise ratio. Hence, from (32) in Appendix B, the LB of the MSE between the lth exact complex gain and the lth estimated polynomial is given by:

$$\text{LB}_l \quad = \quad \text{MSE}_{\text{des}_l} + \mathcal{G} \times [\mathbf{SCRB}_{\boldsymbol{a}}]_{l,l} \tag{26}$$

where $\mathcal{G} = \frac{\|\mathbf{V}\|^2}{vN_c}$ is a noise amplification gain. Interpreting the right hand side of (26), the first component is the model error MSE_{des} which depends on f_dT and N_c whereas the second component is the LB of the MSE of the lth estimated polynomial which depends on SNR and N_c. Consequently, the number of coefficients N_c needs to be chosen such that an acceptable tradeoff can be found between model error and noise reduction. It can easily be shown that:

$$\begin{cases} \text{MSE}_l \text{ (with ICI)} & > \quad \text{LB}_l \\ \text{MSE}_l \text{ (without ICI)} & = \quad \text{LB}_l \end{cases} \tag{27}$$

Thus, by iteratively estimating and removing the ICI, the MSE_l will converge towards LB_l.

V. Simulation Results

In this section, the theory described above is demonstrated by simulation, and the performance of the iterative algorithm is tested. The mean square error (MSE) and the bit error rate (BER) performances are examined in terms of the average signal-to-noise ratio (SNR) [9] [8], and the maximum Doppler spread f_dT (normalized by $1/T$) for the Rayleigh channel. The normalized channel model is Rayleigh as recommended by GSM Recommendations 05.05 [12] [13], using the parameters shown in Table I. A 4QAM-OFDM system is used with normalized symbols: $N = 128$ sub-carriers, $N_g = \frac{N}{8}$ sub-carriers, $N_p = 16$ pilots ($i.e., L_f = 8$) and $\frac{1}{T_s} = 2MHz$. The BER performance is evaluated under a relatively

rapid time-varying channel, using the values $f_dT = 0.05$ and $f_dT = 0.1$, corresponding to a vehicle driven at speeds $V_m = 140km/h$ and $V_m = 280km/h$, respectively, for $f_c = 5GHz$.

Fig. 3 provides a comparison between the MSE of the exact complex gain and of the estimated polynomial, in terms of f_dT for $N_c = 2$ and 3, at SNR $= 20$dB and 40dB. It is observed that for moderate values of SNR, the approximation achieved with $N_c = 2$ coefficients is better than that found using $N_c = 3$ coefficients. However, for high values of SNR, the opposite tendency is observed. This is due to the noise component in equation (23), and to the third coefficient which is poorly estimated, especially in the case of low SNR, because it is negligible compared to the noise level [1]. However, this difference between the MSE does not have a strong influence on the BER, as can be seen in Fig. 4.

Fig. 5 illustrates the evolution of MSE as the number of iterations progresses, as a function of SNR, for $f_dT = 0.1$. It is found that, with all ICI, the MSE obtained by simulation agrees with the theoretical value given in (11). After only one iteration, a great improvement is realized and the MSE is very close to the LB of our algorithm, especially in regions of low and moderate SNR. This is because at low SNR, the noise is dominant with respect to the ICI level, whereas for high SNR, the ICI is not completely removed due to data symbol detection errors. Fig. 5 also shows that, for $f_dT = 0.1$ and SNR ≤ 30dB, the MSE of the polynomial approximation MSE_{des} is negligible, and the main contribution to the MSE is that produced by the LS-estimator. In this case, from (26) we indeed have $\text{LB}_l \approx \mathcal{G} \times [\mathbf{SCRB_a}]_{l,l}$, since MSE_{des} is negligible when compared to SCRB, as can be seen by comparing Fig. 1 with Fig. 11. To find the smallest possible LB, we thus have to choose $N_c = 2$, since $\mathcal{G}$ increases as a function of N_c as shown in Table II. However, for high SNR levels, LB tends asymptotically towards MSE_{des}, meaning that the smallest possible LB will be achieved when $N_c > 2$.

Fig. 7 gives the BER performance of our proposed iterative algorithm, for $N_c = 2$, when compared with that achieved using the conventional methods (LS and LMMSE criteria with low-pass interpolation (LPI) in the frequency domain) [6] [8], our previously proposed algorithm [2], and the SIS algorithm with perfect channel knowledge for $f_dT = 0.05$ and $f_dT = 0.1$. As a reference, we also plot the performance obtained with perfect channel and ICI knowledge. This result shows that our algorithm has better performance than the conventional methods and our previously published algorithm [2]. Moreover, the approach presented here enables an improvement in BER to be achieved after each iterative step,

because each iteration necessarily results in an improvement in the estimation of ICI. After two iterations, a significant improvement occurs; the performance of our algorithm comes very close to that found with the SIS algorithm, using perfect channel knowledge. For high values of SNR, our algorithm does not achieve the same performance as with perfect channel and ICI knowledge, because an error floor remains, due to the data symbol detection error.

Fig. 6 gives the BER in terms of N_p for $f_dT = 0.1$, $N_c = 2$ and SNR $= 20$dB. It is obvious that when the number of pilots is increased, the performance will improve. It is interesting to note that the results presented here demonstrate that with a lesser number of pilots, our algorithm has better performance than conventional methods.

Fig. 8 shows the BER performance of our proposed iterative algorithm, for $N_c = 2$ and $f_dT = 0.1$ with IEEE802.11a standard channel coding [21]. The convolutional encoder has a rate of 1/2, and its polynomials are $P_0 = 133_8$ and $P_1 = 171_8$ and the interleaver is a bit-wise block interleaver with 16 rows and 14 columns. It can clearly be seen that a significant improvement in BER occurs with channel coding, and that for high SNR there is always an error floor due to data symbol detection errors.

Fig. 9 gives the BER performance after three iterations of our proposed iterative algorithm, for $N_c = 2$ and $f_dT = 0.1$, with imperfect delay knowledge. SD denotes the standard deviation of the time delay errors (modeled as zero mean Gaussian variables). It can be noticed that the algorithm is not very sensitive to a delay error of SD$< 0.1T_s$. By using the ESPRIT method [9] to estimate the delays, we have a SD$< 0.05T_s$, for all SNR as shown in Fig. 10. When combined with the ESPRIT method, our algorithm thus has negligible sensitivity to delay errors.

VI. Conclusion

In this paper, we have presented an iterative algorithm of low-complexity, for the estimation of polynomial coefficients for multi-path complex gains, thereby mitigating the inter-sub-carrier-interference (ICI) of OFDM systems. The rapid time-variation complex gains are tracked by exploiting the fact that the delays can be assumed to be invariant (over several symbols) and perfectly estimated. Theoretical analysis and simulations show that by estimating and removing the ICI at each iteration, multi-path complex gain estimation and coherent demodulation can be significantly improved, especially after the

first iteration in the case of high Doppler spread. Moreover, our algorithm has better performance than conventional methods, and its BER performance is very close to the performance of an SIS algorithm in the case of perfect channel knowledge.

APPENDIX A
CRB FOR THE ESTIMATOR OF $\boldsymbol{a}$

If it is assumed that $\mathbf{ICI_p} = \mathbf{H_p x}$ in (16) are known, the vector $\mathbf{y_p}$ for a given $\boldsymbol{a}$ is a complex Gaussian with mean vector $\mathbf{m} = \text{diag}\{\mathbf{x_p}\}\mathbf{F_p}\boldsymbol{a} + \mathbf{ICI_p}$ and covariance matrix $\mathbf{\Omega_1} = \sigma^2\mathbf{I}_{N_p}$. Thus, the probability density function $p(\mathbf{y_p}|\boldsymbol{a})$ is defined as:

$$p(\mathbf{y_p}|\boldsymbol{a}) = \frac{1}{|2\pi\mathbf{\Omega_1}|} e^{-\frac{1}{2}(\mathbf{y_p}-\mathbf{m})^H \mathbf{\Omega_1}^{-1}(\mathbf{y_p}-\mathbf{m})}$$

Since $\boldsymbol{a}$ is a complex Gaussian vector with zero mean and covariance matrix $\mathbf{\Omega_2}$, the probability density function of $\boldsymbol{a}$ can be defined as:

$$p(\boldsymbol{a}) = \frac{1}{|2\pi\mathbf{\Omega_2}|} e^{-\frac{1}{2}\boldsymbol{a}^H \mathbf{\Omega_2}^{-1}\boldsymbol{a}}$$

where $\mathbf{\Omega_2}$ is a $L \times L$ diagonal matrix of elements given by:

$$[\mathbf{\Omega_2}]_{l,l} = \text{E}\Big[[\boldsymbol{a}]_l [\boldsymbol{a}]_l^*\Big] = \frac{\sigma_{\alpha_l}^2}{N^2} \sum_{q_1=0}^{N-1}\sum_{q_2=0}^{N-1} J_0\Big(2\pi f_d T_s (q_1 - q_2)\Big)$$

The Standard CRB (SCRB) and the Bayesian CRB (BCRB) for the estimator of $\boldsymbol{a}$ are defined as [14]:

$$\begin{aligned} \mathbf{SCRB}_{\boldsymbol{a}} &= \Big(-\text{E}\Big[\tfrac{\partial^2}{\partial \boldsymbol{a}^2}\, ln\big(p(\mathbf{y_p}|\boldsymbol{a})\big)\Big]\Big)^{-1} \\ \mathbf{BCRB}_{\boldsymbol{a}} &= \Big(-\text{E}\Big[\tfrac{\partial^2}{\partial \boldsymbol{a}^2}\, ln\big(p(\mathbf{y_p}, \boldsymbol{a})\big)\Big]\Big)^{-1} \end{aligned} \tag{28}$$

where $p(\mathbf{y_p}, \boldsymbol{a}) = p(\mathbf{y_p}|\boldsymbol{a})p(\boldsymbol{a})$ is the joint probability density function of $\mathbf{y_p}$ and $\boldsymbol{a}$ and, the expectation is computed over $\mathbf{y_p}$ and $\boldsymbol{a}$. Notice that SCRB and BCRB are used for the estimation of deterministic and random variables, respectively.

The results of the second derivatives of $ln\big(p(\mathbf{y_p}|\boldsymbol{a})\big)$ and $ln\big(p(\mathbf{y_p}, \boldsymbol{a})\big)$ with respect to $\boldsymbol{a}$ are given by:

$$\begin{aligned} \frac{\partial^2}{\partial \boldsymbol{a}^2}\, ln\big(p(\mathbf{y_p}|\boldsymbol{a})\big) &= -\mathbf{F_p}^H \text{diag}\{\mathbf{x_p}\}^H \mathbf{\Omega_1}^{-1} \text{diag}\{\mathbf{x_p}\}\mathbf{F_p} \\ \frac{\partial^2}{\partial \boldsymbol{a}^2}\, ln\big(p(\mathbf{y_p}, \boldsymbol{a})\big) &= -\mathbf{F_p}^H \text{diag}\{\mathbf{x_p}\}^H \mathbf{\Omega_1}^{-1} \text{diag}\{\mathbf{x_p}\}\mathbf{F_p} - \mathbf{\Omega_2}^{-1} \end{aligned} \tag{29}$$

Hence, substituting (29) into (28) yields:

$$\begin{aligned}\mathbf{SCRB}_{\boldsymbol{a}} &= \sigma^2 \left(\mathbf{F}_{\mathbf{p}}^H \text{diag}\{\mathbf{x_p}\}^H \text{diag}\{\mathbf{x_p}\}\mathbf{F_p}\right)^{-1} \\ \mathbf{BCRB}_{\boldsymbol{a}} &= \left(\frac{1}{\sigma^2}\mathbf{F}_{\mathbf{p}}^H \text{diag}\{\mathbf{x_p}\}^H \text{diag}\{\mathbf{x_p}\}\mathbf{F_p} + \mathbf{\Omega}_{\mathbf{2}}^{-1}\right)^{-1}\end{aligned}$$

It should be noticed that in our specific problem, SCRB is independant of $\boldsymbol{a}$. SCRB thus defines the lower bound, if the a priori distribution of $\boldsymbol{a}$ is not used in the estimation method, whereas BRCB takes this information into account. This is illustrated in Fig. 11, which plots the SCRB $=$ Tr($\mathbf{SCRB}_{\boldsymbol{a}}$) and BCRB $=$ Tr($\mathbf{BCRB}_{\boldsymbol{a}}$) as a function of SNR for the channel defined in Table I, with $N = 128$, $N_p = 16$ and $f_d T = 0.1$. It can be observed that there is a small difference between SCRB and BCRB at low values of SNR only. We can thus compare the MSE of our LS-estimator of $\boldsymbol{a}$ with SCRB instead of BCRB. Moreover, for a known ICI, the optimal estimators of deterministic $\boldsymbol{a}$ and random (Gaussian) $\boldsymbol{a}$ are the LS and maximum likelihood (ML) estimators, respectively. The LS-estimator was used (for deterministic $\boldsymbol{a}$) because it requires less information than the ML-estimator.

APPENDIX B
MEAN SQUARE ERROR OF THE COMPLEX GAINS ESTIMATOR

Let $\mathbf{\Delta_p} = \left[\mathbf{ICI}_{\mathbf{p}_{(n-N_c+1)}}, ..., \mathbf{ICI}_{\mathbf{p}_{(n)}}\right]$ with $\mathbf{ICI}_{\mathbf{p}_{(n)}} = \mathbf{H}_{\mathbf{p}_{(n)}}\mathbf{x}_{(n)}$ and $\mathbf{W_p} = \left[\mathbf{w}_{\mathbf{p}_{(n-N_c+1)}}, ..., \mathbf{w}_{\mathbf{p}_{(n)}}\right]$. The error matrix of the LS-estimator over N_c OFDM symbols is given by:

$$\boldsymbol{\mathcal{E}} = \mathbf{A}_{\mathbf{LS}}^T - \mathbf{A}^T = \mathbf{G}\left(\mathbf{\Delta_p} + \mathbf{W_p}\right)$$

The error between the lth exact complex gain and the lth estimated polynomial is given by:

$$\mathbf{e}_l = \boldsymbol{\alpha}_l - \mathbf{V}\overline{\boldsymbol{\alpha}}_{\mathbf{LS}_l} = \mathbf{e}_{\mathbf{des}_l} - \mathbf{V}\boldsymbol{\epsilon}_l \quad (30)$$

$$= \mathbf{e}_{\mathbf{des}_l} - \mathbf{V}\left(\mathbf{\Delta_p} + \mathbf{W_p}\right)^T \mathbf{g}_l \quad (31)$$

where $\boldsymbol{\epsilon}_l^T$ and $\mathbf{g}_l^T$ are the lth rows of the matrices $\boldsymbol{\mathcal{E}}$ and $\mathbf{G}$, respectively. Since the noise and the ICI are uncorrelated, the MSE between the lth exact complex gain and the lth estimated polynomial is given by (23), where $\mathbf{R_1}$, $\mathbf{R_2}$ and $\mathbf{r_3}$ are defined by:

$$\mathbf{R_1} = \text{E}\left[\mathbf{W}_{\mathbf{p}}^* \mathbf{V}^H \mathbf{V} \mathbf{W}_{\mathbf{p}}^T\right] = \sigma^2 \|\mathbf{V}\|^2 \mathbf{I}_{N_p}, \quad \mathbf{R_2} = \text{E}\left[\mathbf{\Delta}_{\mathbf{p}}^* \mathbf{V}^H \mathbf{V} \mathbf{\Delta}_{\mathbf{p}}^T\right] \quad \text{and} \quad \mathbf{r_3} = \text{E}\left[\mathbf{e}_{\mathbf{des}_l}^H \mathbf{V} \mathbf{\Delta}_{\mathbf{p}}^T\right]$$

$\mathbf{ICI}_{\mathbf{p}_{(n)}}$ can be written as the sum of two components:

$$\mathbf{ICI}_{\mathbf{p}_{(n)}} \;=\; \mathbf{ICI}_{\mathbf{pp}_{(n)}} + \mathbf{ICI}_{\mathbf{dd}_{(n)}}$$

where $\mathbf{ICI}_{\mathbf{pp}_{(n)}} = \mathbf{H}_{\mathbf{pp}_{(n)}}\mathbf{x}_{\mathbf{p}_{(n)}}$ and $\mathbf{ICI}_{\mathbf{dd}_{(n)}} = \mathbf{H}_{\mathbf{dd}_{(n)}}\mathbf{x}_{\mathbf{d}_{(n)}}$, with $\mathbf{H}_{\mathbf{pp}_{(n)}}$ and $\mathbf{H}_{\mathbf{dd}_{(n)}}$ are a $N_p \times N_p$ and a $N_p \times (N - N_p)$ matrices, respectively, whose elements are given by:

$$\left[\mathbf{H}_{\mathbf{pp}_{(n)}}\right]_{k,m} = \begin{cases} \left[\mathbf{H}_{(n)}\right]_{p_k,p_m} & \text{if } k \neq m \\ 0 & \text{if } k = m \end{cases}$$

$$\left[\mathbf{H}_{\mathbf{dd}_{(n)}}\right]_{k,m} = \left[\mathbf{H}_{(n)}\right]_{p_k,t_m} \quad \text{with } t_m \in [1,N] - \mathcal{P} \quad \text{for } m \in [1, N - N_p]$$

where $\{p_k\}$ are defined in (15). Hence, the matrix $\mathbf{R_2}$ becomes: $\mathbf{R_2} = \mathbf{R_{pp}} + \mathbf{R_{dd}}$ where $\mathbf{R_{pp}} = \mathrm{E}\left[\mathbf{\Delta}^*_{\mathbf{pp}}\mathbf{V}^H\mathbf{V}\mathbf{\Delta}^T_{\mathbf{pp}}\right]$ and $\mathbf{R_{dd}} = \mathrm{E}\left[\mathbf{\Delta}^*_{\mathbf{dd}}\mathbf{V}^H\mathbf{V}\mathbf{\Delta}^T_{\mathbf{dd}}\right]$, since the data symbols and the coefficients $\left[\mathbf{H}_{(n)}\right]_{k,m}$ are uncorrelated. The data symbols are normalized (*i.e.* $\mathrm{E}\left[x_{(u_1)}[d_1]x^*_{(u_2)}[d_2]\right] = \delta_{d_1,d_2}\delta_{u_1,u_2}$), such that the elements $[\mathbf{R_{pp}}]_{k,m}$, $[\mathbf{R_{dd}}]_{k,m}$ and $[\mathbf{r_3}]_k$, with $k, m \in [1, N_p]$, can be calculated as:

$$\begin{aligned}
[\mathbf{R_{pp}}]_{k,m} &= \sum_{u=1}^{vN_c}\sum_{u_1=1}^{N_c}\sum_{u_2=1}^{N_c}[\mathbf{V}]_{u,u_1}[\mathbf{V}]_{u,u_2}\left[\mathbf{Z}_{\mathbf{p}_{(k,m)}}\right]_{u_1,u_2} \\
[\mathbf{R_{dd}}]_{k,m} &= \sum_{u=1}^{vN_c}\sum_{u_1=1}^{N_c}\sum_{u_2=1}^{N_c}[\mathbf{V}]_{u,u_1}[\mathbf{V}]_{u,u_2}\left[\mathbf{Z}_{\mathbf{d}_{(k,m)}}\right]_{u_1,u_2} \\
[\mathbf{r_3}]_k &= \mathrm{E}\left[\sum_{u=1}^{vN_c}\sum_{u_1=1}^{N_c}[\mathbf{V}]_{u,u_1}\left[\mathbf{Z}_{\mathbf{1}_{(k)}}\right]_{u,u_1}\right] - \mathrm{E}\left[\sum_{u=1}^{vN_c}\sum_{u_1=1}^{N_c}\sum_{u_2=1}^{N_c}[\mathbf{V}]_{u,u_1}[\mathbf{V}]_{u,u_2}\left[\mathbf{Z}_{\mathbf{2}_{(k)}}\right]_{u_1,u_2}\right]
\end{aligned}$$

where $\left[\mathbf{Z}_{\mathbf{p}_{(k,m)}}\right]_{u_1,u_2}$, $\left[\mathbf{Z}_{\mathbf{d}_{(k,m)}}\right]_{u_1,u_2}$, $\left[\mathbf{Z}_{\mathbf{1}_{(k)}}\right]_{u,u_1}$ and $\left[\mathbf{Z}_{\mathbf{2}_{(k)}}\right]_{u_1,u_2}$ are given by:

$$\begin{aligned}
\left[\mathbf{Z}_{\mathbf{p}_{(k,m)}}\right]_{u_1,u_2} &= \mathrm{E}\left[\left[\mathbf{\Delta_{pp}}\right]_{m,u_1}\left[\mathbf{\Delta_{pp}}\right]^*_{k,u_2}\right] = \mathrm{E}\left[\sum_{\substack{d_1=p_1\\ d_1\neq p_m}}^{p_{N_p}}\sum_{\substack{d_2=p_1\\ d_2\neq p_k}}^{p_{N_p}}\left[\mathbf{x}_{(u_1)}\right]_{d_1}\left[\mathbf{x}_{(u_2)}\right]^*_{d_2}\left[\mathbf{H}_{(u_1)}\right]_{p_m,d_1}\left[\mathbf{H}_{(u_2)}\right]^*_{p_k,d_2}\right] \\
&= \frac{1}{N^2}\sum_{\substack{d_1=p_1\\ d_1\neq p_m}}^{p_{N_p}}\sum_{\substack{d_2=p_1\\ d_2\neq p_k}}^{p_{N_p}}\sum_{l=1}^{L}\sigma^2_{\alpha_l}\left[\mathbf{x}_{(u_1)}\right]_{d_1}\left[\mathbf{x}_{(u_2)}\right]^*_{d_2}e^{-j2\pi\frac{d_1-d_2}{N}\tau_l}\sum_{q_1=0}^{N-1}\sum_{q_2=0}^{N-1}e^{j2\pi\frac{(d_1-p_m)q_1-(d_2-p_k)q_2}{N}}J_0\Big(2\pi f_d T_s\big((q_1-q_2)+(u_1-u2)v\big)\Big)
\end{aligned}$$

$$\begin{aligned}
\left[\mathbf{Z}_{\mathbf{d}_{(k,m)}}\right]_{u_1,u_2} &= \mathrm{E}\left[\left[\mathbf{\Delta_{dd}}\right]_{m,u_1}\left[\mathbf{\Delta_{dd}}\right]^*_{k,u_2}\right] = \mathrm{E}\left[\sum_{\substack{d_1=1\\ d_1\neq p_s}}^{N}\sum_{\substack{d_2=1\\ d_2\neq p_s}}^{N}\left[\mathbf{x}_{(u_1)}\right]_{d_1}\left[\mathbf{x}_{(u_2)}\right]^*_{d_2}\left[\mathbf{H}_{(u_1)}\right]_{p_m,d_1}\left[\mathbf{H}_{(u_2)}\right]^*_{p_k,d_2}\right] \\
&= \frac{\delta_{u_1,u_2}}{N^2}\sum_{l=1}^{L}\sigma^2_{\alpha_l}\sum_{\substack{d=1\\ d\neq p_s}}^{N}\sum_{q_1=0}^{N-1}\sum_{q_2=0}^{N-1}e^{j2\pi\frac{(d-p_m)q_1-(d-p_k)q_2}{N}}J_0\Big(2\pi f_d T_s(q_1-q_2)\Big)
\end{aligned}$$

$$\left[\mathbf{Z_{1}}_{(k)}\right]_{u,u_1} = \mathrm{E}\left[\alpha_l^*\big((u-1)T_s\big)\left[\mathbf{\Delta_{pp}}\right]_{k,u_1}\right] = \mathrm{E}\left[\sum_{\substack{d=p_1\\ d\neq p_k}}^{p_{N_p}} \alpha_l^*\big((u-1)T_s\big)\left[\mathbf{x}_{(u_1)}\right]_d \left[\mathbf{H}_{(u_1)}\right]_{p_k,d}\right]$$

$$= \frac{\sigma_{\alpha_l}^2}{N}\sum_{\substack{d=p_1\\ d\neq p_k}}^{p_{N_p}} \left[\mathbf{x}_{(u_1)}\right]_d e^{-j2\pi(\frac{d-1}{N}-\frac{1}{2})\tau_l} \sum_{q=0}^{N-1} e^{j2\pi\frac{(d-p_k)q}{N}} J_0\Big(2\pi f_d T_s\big((q-u+1)+(u_1-1)v\big)\Big)$$

$$\left[\mathbf{Z_{2}}_{(k)}\right]_{u_1,u_2} = \mathrm{E}\left[\overline{\boldsymbol{\alpha}}_{l,u_2-1}^*\left[\mathbf{\Delta_{pp}}\right]_{k,u_1}\right] = \mathrm{E}\left[\sum_{\substack{d=p_1\\ d\neq p_k}}^{p_{N_p}} \overline{\boldsymbol{\alpha}}_{l,u_2-1}^* \left[\mathbf{x}_{(u_1)}\right]_d \left[\mathbf{H}_{(u_1)}\right]_{k,d}\right]$$

$$= \frac{\sigma_{\alpha_l}^2}{N^2}\sum_{\substack{d=p_1\\ d\neq p_k}}^{p_{N_p}} \left[\mathbf{x}_{(u_1)}\right]_d e^{-j2\pi(\frac{d-1}{N}-\frac{1}{2})\tau_l} \sum_{q_1=0}^{N-1} e^{j2\pi\frac{(d-k)q_1}{N}} \sum_{q_2=0}^{N-1} J_0\Big(2\pi f_d T_s\big((q_1-q_2)+(u_1-u_2)v\big)\Big)$$

Notice that the elements of the matrix $\mathbf{R_2}$ and $\mathbf{r_3}$ depend on known pilot symbols.

If the ICI are completely eliminated , the elements of $\boldsymbol{\mathcal{E}}$ are uncorrelated with respect to each other and the elements of $\mathbf{e}_{\mathbf{des}_l}$. Thus, from (30) we can write:

$$\mathrm{MSE}_l \text{ (without ICI)} = \mathrm{MSE}_{\mathrm{des}_l} + \frac{\|\mathbf{V}\|^2}{vN_c} E\Big[[\boldsymbol{\mathcal{E}}]_{l,1}[\boldsymbol{\mathcal{E}}]_{l,1}^*\Big] \tag{32}$$

Combining (32) and (31), for the case of the LS-estimator, thus leads to:

$$\mathrm{MSE}_l \text{ (without ICI)} = \mathrm{MSE}_{\mathrm{des}_l} + \frac{\|\mathbf{V}\|^2\|\mathbf{g}_l\|^2}{vN_c\ \mathrm{SNR}} \tag{33}$$

APPENDIX C
SUCCESSIVE INTERFERENCE SUPPRESSION METHOD

The received data sub-carriers, without contributions from pilot sub-carriers, are given by:

$$\mathbf{y_d} = \mathbf{H_d x_d} + \mathbf{w_d}$$

where $\mathbf{x_d}$ is the transmitted data, $\mathbf{y_d}$ is the received data and $\mathbf{w_d}$ is the noise at the data sub-carrier positions, given by $(N-N_p)\times 1$ vectors, and $\mathbf{H_d}$ is a $(N-N_p)\times(N-N_p)$ data channel matrix obtained by eliminating rows and columns at the $\mathcal{P}$ position in the channel matrix $\mathbf{H}$.

Through the implementation of a successive interference suppression (SIS) scheme, with optimal ordering and one tap frequency equalizer, the data can be estimated. Optimal ordering of the data channel

matrix $\mathbf{H_d}$, computed from the largest to the smallest magnitude of the diagonal elements, is given by:

$$\begin{aligned}\mathcal{O} \;=\; &\Big\{O_1,\ O_2,\ ...,\ O_{N-N_p}\ |\\ &i\ <\ j\ \text{ if }\ \|\,[\mathbf{H_d}]_{O_i,O_i}\,\|\ >\ \|\,[\mathbf{H_d}]_{O_j,O_j}\,\|\ \Big\}\end{aligned}$$

The detection algorithm can now be described as follows:

$$\begin{aligned}&initialization:\\ &i \leftarrow 1\\ &\mathcal{O} = \Big\{O_1,\ O_2,\ ...,\ O_{N-N_p}\Big\}\\ &\mathbf{y_{d}}_{(i)} = \mathbf{y_d}\\ &recursion:\\ &[\mathbf{x_{ed}}]_{O_i} = [\mathbf{y_{d}}_{(i)}]_{O_i} / \,[\mathbf{H_d}]_{O_i,O_i}\\ &[\hat{\mathbf{x}}_\mathbf{d}]_{O_i} = Q\big(\,[\mathbf{x_{ed}}]_{O_i}\,\big)\\ &\mathbf{y_{d}}_{(i+1)} = \mathbf{y_{d}}_{(i)} - [\hat{\mathbf{x}}_\mathbf{d}]_{O_i}\,\mathbf{h_{d}}_{O_i}\\ &i \leftarrow i+1\end{aligned}$$

where $Q(.)$ denotes the quantization operation appropriate to the constellation in use, and $\mathbf{h_{d}}_{O_i}$ is the O_ith column of the data channel matrix $\mathbf{H_d}$. Notice that the complexity could be reduced, with very little loss in performance, if SIS were processed on a small number of adjacent sub-carriers only [20].

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TABLE I

CHANNEL PARAMETERS

Rayleigh Channel		
Path Number	*Average Power* (dB)	*Normalized Delay*
1	-7.219	0
2	-4.219	0.4
3	-6.219	1
4	-10.219	3.2
5	-12.219	4.6
6	-14.219	10

TABLE II

THE GAIN $\mathcal{G}$ IN EXPRESSION (26), FOR $N = 128$ AND $N_g = 16$

N_c	2	3	4
$\mathcal{G} = \frac{\|\mathbf{v}\|^2}{vN_c}$	1.17	1.39	1.73

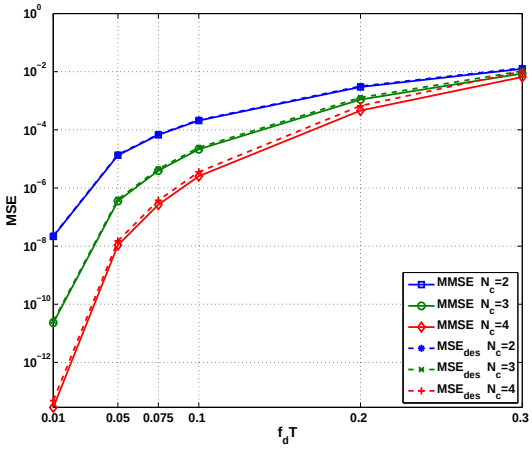


Fig. 1. Comparison between MMSE and MSE_{des} for a normalized channel with $L = 6$ paths

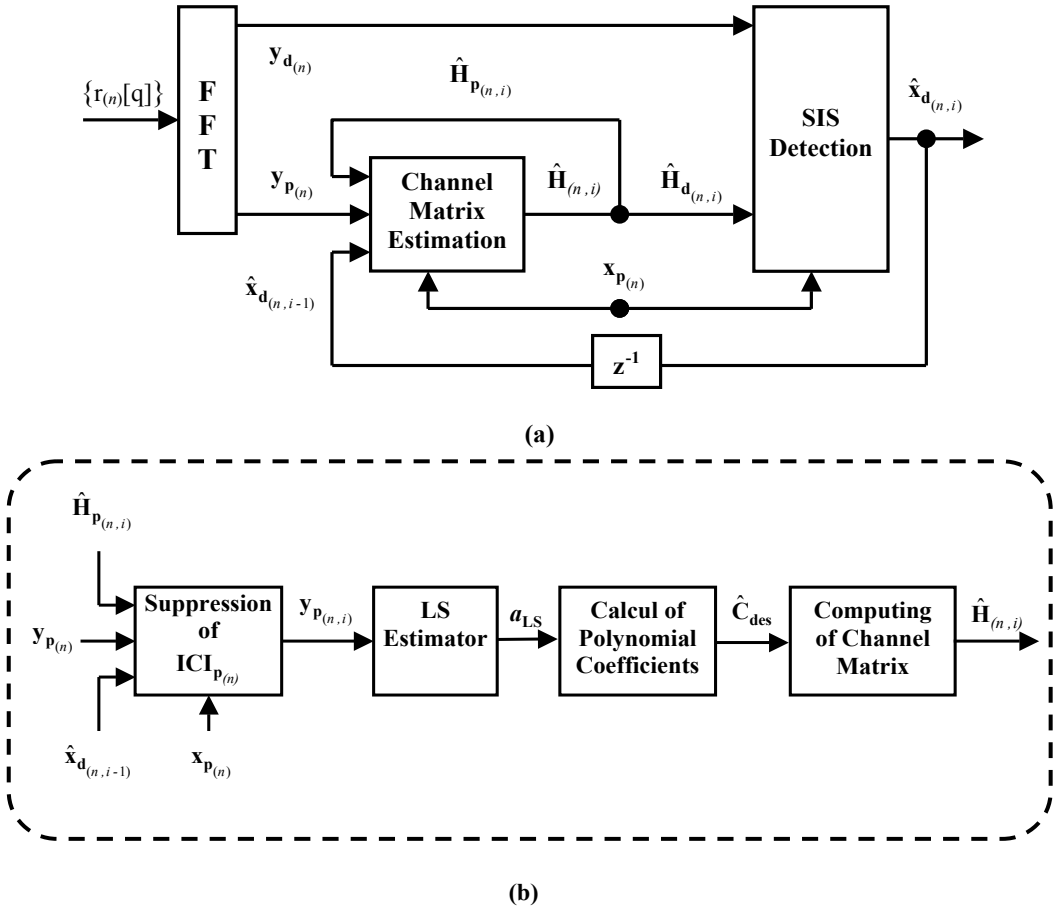


Fig. 2. Block diagrams of the iterative algorithm: (a) overall channel estimator and ICI suppression block diagram; and (b) channel matrix estimation block diagram

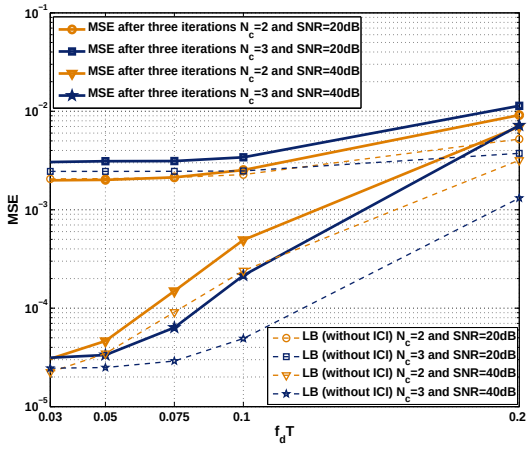


Fig. 3. Comparison of MSE, for $N_c = 2$ and 3, at $SNR = 20$dB and 40dB

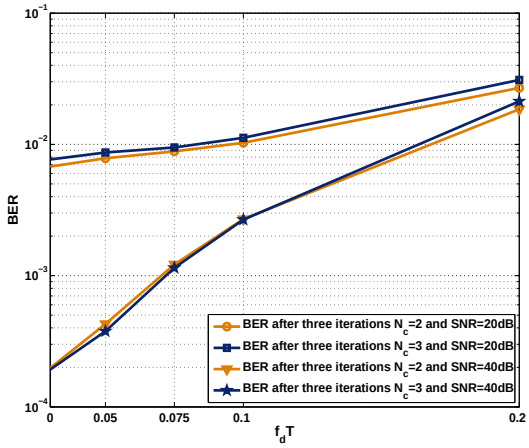


Fig. 4. Comparison of BER, for $N_c = 2$ and 3, at $SNR = 20$dB and 40dB

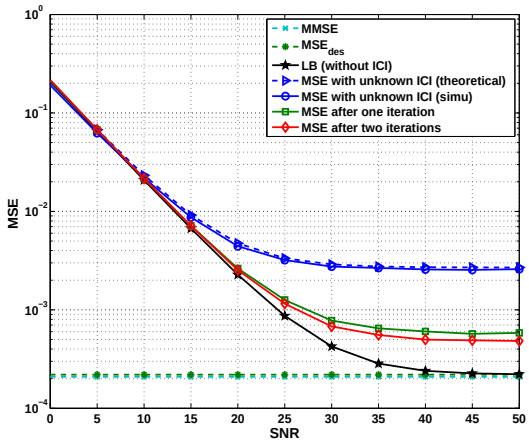


Fig. 5. Mean square error of the polynomial approximation, for $f_dT = 0.1$ and $N_c = 2$

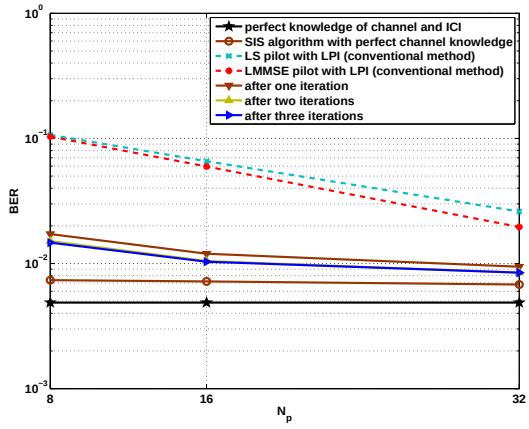
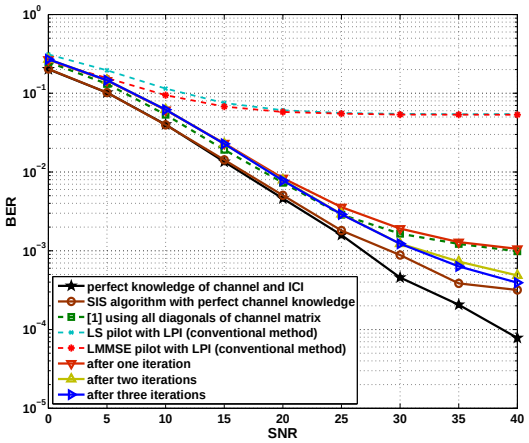
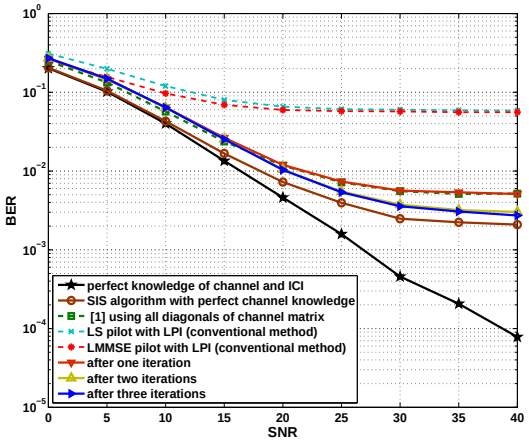


Fig. 6. Comparison of BER, for $f_dT = 0.1$, $N_c = 2$ and $SNR = 20$dB



(a)



(b)

Fig. 7. Comparison of BER vs SNR, for $N_c = 2$: (a) $f_dT = 0.05$; (b) $f_dT = 0.1$

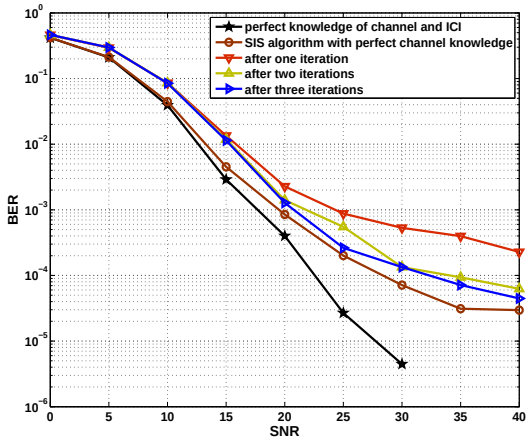


Fig. 8. Comparison of BER, in the case of the IEEE802.11a convolutional code, for $N_c = 2$ and $f_dT = 0.1$

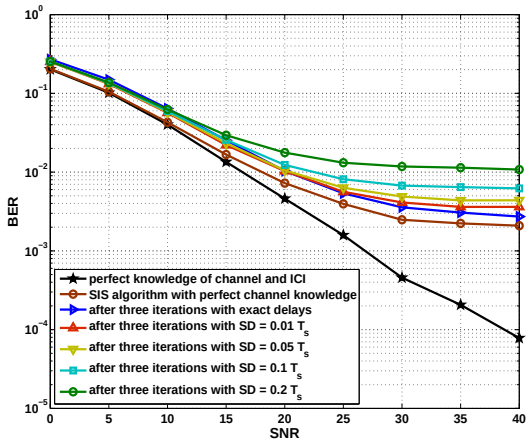


Fig. 9. Comparison of BER, for the case of imperfect knowledge of delays, for $N_c = 2$ and $f_dT = 0.1$

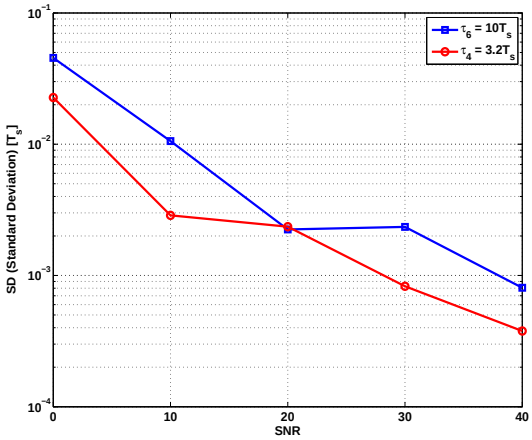


Fig. 10. Delay estimation errors for the fourth and sixth paths, using the ESPRIT method [9] (estimated correlation matrix, averaged over 1000 OFDM symbols, $i.e$ 0.072sec), for $f_dT = 0.1$

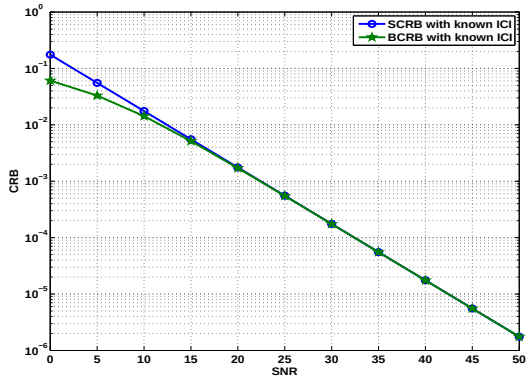


Fig. 11. SCBR and BCRC, with $N = 128$, $N_p = 16$ and $f_dT = 0.1$