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Derivation of a frequency independent criterion to determine the limp model validity for highly porous sound absorbing materials

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Abstract

A frequency independent criterion to determine the limp model validity for highly porous sound absorbing materials is investigated. Although limp model is derived from the Biot theory assuming that the porous frame has no bulk stiffness, it is shown that this model can be used for highly porous materials since the criterion is respected. Abacus giving the criterion limit as function of the porous layer thickness are given for two configurations: sound absorption of a porous layer backed by a rigid backing and sound radiation of a plate covered by a porous layer. Its is shown that the derived criterion matches Beranek criterion but it is less restrictive.

PACS numbers:

I. INTRODUCTION

The present paper proposes a straightforward criterion to determine if an absorbing porous material can be modeled with the "equivalent fluid" limp model[1–7] instead of the poroelastic Biot model[8,9]. The use of the limp model is advantageous in finite element models for example, because it allows a great decrease of the calculation cost.

In a recent paper Doutres *et al.*[7] propose a criterion to identify the frequency bands for which a porous material behavior can be simulated with the limp model. The criterion was based on a frequency dependent parameter FSI (Frame Stiffness Influence) derived from the properties of the material. This parameter characterizes the influence of the frameborne wave on the fluid phase displacement related to the frame stiffness influence. It is shown that as the value of FSI increases, the difference between the limp and the Biot simulations increases. A critical value of FSI, below which the porous behavior can be simulated with the limp model was determined for two sets of boundary conditions and three frequency bands characteristic of the frame behavior.

In this paper, the identification process is more straightforward and easy to carry out to give an estimation on the accuracy of using the limp model in the whole frequency range. It is based on a frequency independent parameter derived from FSI, called FSI_r, which only requires the knowledge of the frame bulk modulus and the isothermal fluid pressure.

II. FREQUENCY INDEPENDENT PARAMETER FSI_r

This section presents the limp model and the how the frequency independent parameter FSI_r is determined from the frequency dependent one FSI developed in reference[7].

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A. Porous material modeling

The Biot theory[8,9] is commonly used to model the deformation of a poroelastic solid saturated by a compressible fluid. This theory states that three waves can propagate in a porous media: two compressional waves and a shear wave. This work deals with one dimensional applications and only the two compressional waves are considered. These waves are characterized by a complex wave number δ_i $(i = 1, 2)$ and a displacement ratio μ_i. This ratio indicates in which medium the waves mainly propagate. Here, the wave with the subscript $i = 1$ propagates mainly in the fluid phase and is referred to as the "airborne" wave. The wave with the subscript $i = 2$ propagates mainly in the solid phase and is referred to as the "frameborne" wave.

The limp model is derived from the Biot theory, assuming that the frame has no bulk stiffness[1–7]. This model describes the propagation of one compressional wave in an "equivalent fluid" medium that has the bulk modulus of the air in the pores and the density of the air modified by the inertia effect of the solid phase and its interaction with the fluid phase. The compressional wave is characterized by the wave number $\delta_{limp} = \omega\sqrt{\tilde{\rho}_{limp}/\tilde{K}_f}$ with

$$\tilde{\rho}_{limp} = \phi \frac{\rho_t \tilde{\rho}_f/\phi - \rho_0^2}{\rho_t + \tilde{\rho}_f/\phi - 2\rho_0} \ . \tag{1}$$

Note that the formulation of $\tilde{\rho}_{limp}$ has been derived from the simplification of Eq. (33) of reference[7] and has the form of the one recently proposed by Panneton[6]. In this equation, ϕ is the porosity, $\rho_t = \rho_1 + \phi\rho_0$ is the total apparent mass of the equivalent fluid with ρ_1 the density of the porous material and ρ_0 the density of the interstitial fluid. $\tilde{\rho}_f$ and $\tilde{K}_f$ are the effective density and the bulk modulus of the fluid phase of the well known "rigid frame equivalent fluid" and account for the visco-inertial and elastic interactions between air and frame in the poroelastic aggregate model: the reader can refer to references[9–12] for a precise description of these parameters.

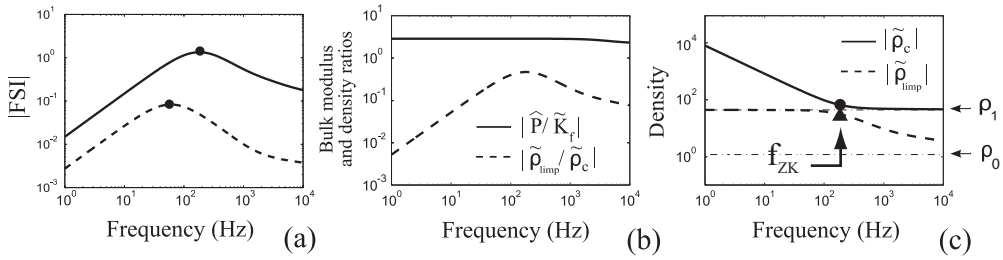

FIG. 1. Frequency behavior of the parameter FSI; (a) material (- -) B and (–) C: (•) are the approximated values at f_{ZK} (see Eq. (6)); (b) ratio of the characteristic Bulk modulus and densities for material C; (c) densities of the two characteristic waves for material C: (▲) and (•) are the approximated densities at f_{ZK} (see Eq. (4))

B. Frequency behavior of FSI

The FSI parameter is based on the assumption that the use of the limp model is possible when the frameborne wave contribution is negligible in the considered application. This condition is observed when the ratio μ_2/μ_1 is small compared to 1[7]. This can be written in terms of a frequency dependent parameter, FSI, expressed as a ratio of two characteristic wave numbers

$$FSI = \frac{\delta_{limp}^2}{\delta_c^2} = \frac{\tilde{\rho}_{limp}}{\tilde{\rho}_c}\frac{\widehat{P}}{\tilde{K}_f} \ . \tag{2}$$

$\delta_c = \omega\sqrt{\tilde{\rho}_c/\widehat{P}}$ is the wave number of a wave, called "c" wave, that propagates in a medium that has the bulk modulus of the frame in vacuum

$$\widehat{P} = \frac{E(1+j\eta)(1-\nu)}{(1-2\nu)(1+\nu)} \ , \tag{3}$$

and the density of the frame in fluid, $\tilde{\rho}_c = \rho_1 - \tilde{\rho}_{12}/\phi$, whith $j = \sqrt{-1}$, E the Young modulus, η the loss factor, ν the Poisson ratio of the frame and $\tilde{\rho}_{12}$ the inertial coefficient which accounts for the interaction between inertial forces of the solid and fluid phases together with viscous dissipation[9].

Figure 1(a) presents the FSI for two materials which can be found in automotive or building applications: material B is a high density fibrous material and material C is a

polymer foam with a stiff skeleton and a larger airflow resistivity. The properties of these materials are given in Table I. This figure shows that the amplitude of the FSI increases with the bulk modulus of the porous skeleton.

The frequency behavior of FSI is studied regarding the figures 1(b) and 1(c) in the case of foam C. Figure 1(b) shows that the bulk modulus ratio $|\widehat{P}/\tilde{K}_f|$ is almost constant. The slight decrease at high frequencies is due to the increase of the bulk modulus of the fluid phase which accounts for the thermal effects[10]. Note that the Young modulus of the skeleton is assumed to be constant in this work. This figure also shows that the ratio of the two characteristic waves $|\tilde{\rho}_{limp}/\tilde{\rho}_c|$ exhibits a bell shape that can be related to the inertial and viscous effects. At low frequencies, the viscous effects are predominant and the two phases are strongly coupled (see Fig. 1(c)): the density of the limp wave is equal to the density of the porous material ρ_1 and the density of the "c" wave is increased. At high frequencies, the viscous effects decrease and the inertial effects become important: the density of the limp wave is close to that of the fluid and the density of the "c" wave is equal to that of the porous material. In between these two asymptotic behaviors, $\tilde{\rho}_{limp}$ and $\tilde{\rho}_c$ are close and their ratio reaches a maximum. This occurs at the frequency defined by Zwikker and Kosten[13], $f_{ZK} = \phi^2\sigma/2\pi\rho_1$, that indicates the frequency above which the inertial forces per unit volume on the material are superior to the viscous ones.

C. Derivation of the frequency independent parameter $\mathbf{FSI}_r$

The criterion proposed in this paper consists in comparing a frequency independent ratio which characterizes the frame stiffness influence with critical values. Details on critical values are given in the next section. The frequency independent parameter is derived from FSI and is set as its maximum value to ensure the uniqueness of the solution in the whole frequency range.

According to Fig. 1, this maximum can be approximated from the densities of both the limp and the "c" waves expressed at f_{ZK}. Assuming that the density of air ρ_0 is negligible

TABLE I. Measured properties of materials B and C[14].

Porous	B	C
Airflow resistivity: σ (kN s/m^4)	23	57
Porosity: ϕ	0.95	0.97
Tortuosity: α_∞	1	1.54
Viscous length: Λ (μm)	54.1	24.6
Thermal length: Λ' (μm)	162.3	73.8
Frame density: ρ_1 (kg/m^3)	58	46
Young's Modulus at 5 Hz: E (kPa)	17	214
Structural loss factor at 5 Hz: η	0.1	0.115
Poisson's ratio: ν	0	0.3

compared with the one of the porous material ρ_1, these densities are given by

$$\tilde{\rho}_c(f_{ZK}) \approx \rho_1(1 - j\frac{1}{\phi}) \ , \ \tilde{\rho}_{limp}(f_{ZK}) \approx \rho_1 \frac{(1 - j\phi)}{(1 + \phi^2)}. \tag{4}$$

These values are presented on Fig. 1(c) for material C: (▲) for $\tilde{\rho}_{limp}(f_{ZK})$ and (•) for $\tilde{\rho}_c(f_{ZK})$. At f_{ZK}, the bulk modulus of the air in the pores is close to its isotherm value: $\tilde{K}_f(f_{ZK}) \approx P_0$(101.3 kPa). Hence, the modulus of the maximum FSI at f_{ZK} is given by

$$|\mathrm{FSI}(f_{ZK})| \approx \frac{|\widehat{P}|}{P_0} \frac{\phi}{1 + \phi^2} \ , \tag{5}$$

which can be simplified for highly porous sound materials (ϕ close to 1) as

$$\mathrm{FSI}_r = \frac{|\widehat{P}|}{2P_0} \ . \tag{6}$$

FSI_r is then easy to calculate and requires only the knowledge of the bulk modulus of the frame ($\widehat{P}$). The two parameters FSI_r and f_{ZK}, given in Table II for materials B and C are presented with black points in Fig. 1(a).

TABLE II. Criterion parameters of materials B and C.

Material	B	C
f_{ZK} (Hz)	57	186
FSI_r at f_{ZK}	0.08	1.43

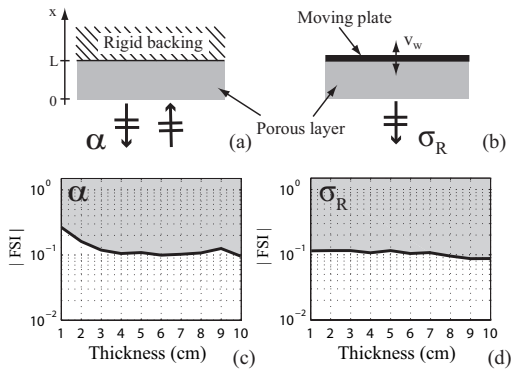

FIG. 2. Design of the two configurations and evolution of the FSI critical values (bold line) as function of the thickness: (a,c) absorption coefficient, (b,d) radiation efficiency.

III. FREQUENCY INDEPENDENT CRITICAL VALUES AND APPLICATION

In the previous section, the frequency independent parameter FSI_r which characterizes the intrinsic frame influence has been introduced. The next step is to identify the critical values under which the limp model can be used instead of the Biot model. The reader should refer to ref.[7] for a detailed description of the procedure.

The influence of the frame vibration on the porous material behavior depends on the stiffness of the material but also on frequency and on the boundary conditions applied to the porous layer[1–7]. The critical values of FSI are derived for two specific configurations: sound absorption of a porous layer backed by a rigid wall (Fig. 2(a)) and sound radiation of a plate covered by a porous layer (Fig. 2(b)).

For a given configuration and porous layer thickness, the critical values of FSI are de-

termined from the difference between the limp and the Biot simulations carried out for a wide range of simulated porous materials in the frequency range $[1 - 10\ 000$ Hz$]$. 256 simulated materials are built with random values of the main porous properties (see TAB. III) to have a set representative of usual absorbing materials. The porosity is set to 0.97 and corresponds to highly porous materials. The viscous characteristic length Λ is derived from the shape factor M^{12} set to unity (cylindrical pores) and the thermal length Λ' is three times the viscous characteristic length. For all materials, the difference between the two models is sorted in terms of their FSI (see Eq. (2)) in the whole frequency range. The FSI increases with the difference between the two models and the critical values of FSI are set when the difference between the two models exceeds the acceptable limit of 0.1 for the absorption coefficient and 3 dB for the radiation efficiency.

The two abacus given in Figs. 2(c) and 2(d) present the FSI critical values determined in the frequency range $[1 - 10\ 000$ Hz$]$ for the two configurations and thicknesses from 1 to 10 cm. For a given material, the limp model can be used if the FSI_r is situated below the critical value (white area of the abacus) and the Biot model should be preferred if FSI_r exceeds the critical value (gray area of the abacus). According to Fig. 2, the critical FSI values are similar for the two considered configurations and slightly depend on the porous thickness: critical FSI≈ 0.1.

The procedure for determining the porous materials for which the limp model can be used in the considered frequency range $[1 - 10\ 000$ Hz$]$ is as follows: (i) the bulk modulus of the frame $\widehat{P}$ has to be determined; (ii) the parameter FSI_r is evaluated using Eq. (6); (iii) the critical values of FSI are chosen in Fig. 2 according to the studied configuration and the thickness of the porous layer; (iv) FSI_r is finally compared to the critical values: the limp model can be used in the whole frequency range if FSI_r is below the FSI critical value.

In the case of material B, FSI_r is equal to 0.08 (see TAB. II) which is below the FSI critical values of the absorption and radiation configurations and for all thicknesses: the limp model can be used for all porous thicknesses. The FSI_r of material C is equal to 1.43 (see

TABLE III. Range of values for the properties of the simulated materials (μ' is the viscosity of air).

Air flow resistivity: σ (kN s/m^4)	1-100
Porosity: ϕ	0.97
Tortuosity: α_∞	1-2
Shape factor: M	1
Viscous length: Λ (μm)	$\sqrt{8\alpha_\infty\mu'/\sigma\phi}$
Thermal length: Λ' (μm)	3Λ
Frame density: ρ_1 (kg/m^3)	10-90
Young's Modulus at 5 Hz: E (kPa)	3-200
Structural loss factor at 5 Hz: η	0.01-0.2
Poisson's ratio: ν	0.3

TAB. II) which is above the FSI critical values of the absorption and radiation configurations and for all thicknesses: the Biot model should be preferred for all layer thicknesses. These predictions agree with the simulations presented in Fig. 4 of reference[7].

IV. COMPARISON WITH THE BERANEK CRITERION

Beranek[1] first proposed the limp approximation from the poroelastic formulation of Zwikker and Kosten[13] using the assumption that, for soft materials, the two waves which propagate in the porous medium are decoupled. He also gave a criterion to identify the porous materials for which this assumption can be applied, that is $|\widehat{P}/K_f| < 0.05$. Its criterion limit has been obtained from the study of materials having a frame bulk modulus from 28 to 100 kPa. Note that, for highly porous materials (ϕ close to 1), the FSI$_r$ given in Eq. (6) is half the Beranek parameter. In the case of the absorption and radiation configurations, the critical value of FSI is found around 0.1 [see Figs. 2(c) and 2(d)] from the study of materials having a frame bulk modulus from 3 to 200 kPa. The proposed

criterion can thus be written as $|\widehat{P}/K_f| < 0.2$. Hence, the main difference with the Beranek criterion is the critical limit: the FSI_r criterion being much less restrictive. Indeed, according to TAB. II, the Beranek criterion rejects the use of the limp model for both materials B and C.

V. CONCLUSION

In this paper, a straightforward criterion to identify the absorbing porous materials which can be modeled with the limp model has been derived from the poroelastic Biot model. The identification process relies on a frequency independent parameter function of the bulk modulus of the frame $\widehat{P}$ and the porosity. In the case of highly porous material, it can be reduced to $\mathrm{FSI}_r = |\widehat{P}|/2P_0$ which is half the parameter proposed by Beranek[1]. It is shown that for the majority of porous thicknesses (between 1 and 10 cm), the absorption coefficient and the radiation efficiency could be simulated with the limp model instead of the complete Biot model if $\mathrm{FSI}_r < 0.1$, i.e. the bulk modulus of the frame should be lower than 20 kPa, which is less restrictive than Beranek[1] criterion.

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