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Do river profiles record along-stream variations of low uplift rate?

S. Carretier,1 B. Nivière,2 M. Giamboni,3,4 and T. Winter5

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Spatial variations of gradients in landscapes may be used to identify and quantify recent deformation. The problem with doing this is to determine whether tectonic or climatic forcing is responsible for these variations, especially for low uplift rate environments (\(\leq 1 \text{ mm yr}^{-1}\)) where climate changes may have erased tectonic features. We evaluate the respective contribution of low uplift rate (\(\sim 0.1 \text{ mm yr}^{-1}\)) and Pleistocene climate oscillations on gradient variations of two comparable river profiles crossing different uplift zones in the southern Upper Rhine Graben. We compare the observed points of discontinuity in river profile (knickpoints) and convex portions (knickzones) with those predicted by a detachment-limited model that includes stochastic short-term and cyclic long-term variations in climate, a bedrock detachment threshold and rock uplift. The detachment-limited model is chosen as it predicts the development of persistent knickpoints. Differing values of the shear stress exponent, erosion threshold, climate variability and uplift pattern have been checked. Our modeling suggests that climate changes had no significant effects on profiles and that anomalies are more likely due to anticline growth. This surprising result arises from the combination of a very low regional uplift rate and the detachment-limited assumption. The detachment-limited model implies an upstream propagation of knickpoints and knickzones generated by uplift at the outlet during dry climate periods of low erosion. The greater the uplift rate, the larger the variations in river bed elevation. Thus, for high uplift rate, knickpoints and knickzones generated by climate oscillations are more likely to hide tectonic features. This result seems counterintuitive because it suggests that tectonic knickzones will be better preserved in low uplift rate environments, provided that the lithology is homogeneous.


1. Introduction

In this paper, we evaluate whether rock uplift or climate variations are responsible for knickpoints observed in the profiles of two rivers, the Largue and Ill Rivers, located in a low uplift rate (\(\sim 0.1 \text{ mm yr}^{-1}\)) region, the Sundgau area in the Upper Rhine Graben (France). The question of the origin of geomorphic features is fundamental in the field of active tectonics because topographic features may be used to infer cumulative displacements on faults and folds [e.g., Molnar et al., 1994; Lavé and Avouac, 2000; Carretier et al., 2002]. The answer is usually difficult because geomorphic features result from various tectonic-erosion interactions which may lead to similar forms. One may suppose that in low uplift rate settings semi-arid to humid climates, erosion will hamper the geomorphic response to tectonics. For example, if we assume an uplift rate of 0.1 mm yr\(^{-1}\), 1 m uplift will require 10 ky, a time span during which erosion and deposition may destroy such a small tectonic relief. Furthermore, assessing strain rates and seismic hazard in areas of low tectonic activity requires geomorphological markers that can record several seismic cycles. Because the mean recurrence time of events on slow structures may reach several thousand years to more than 10 ky [e.g., Camelbeeck and Meghraoui, 1998], the geomorphic record of uplift on such structures requires the preservation of markers on time spans of several 100 ky. Over such durations, climate variations may erase tectonic markers and produce ambiguous features such as knickpoints and rapid incision in rivers [Whipple, 2001; Zaprowski et al., 2005].

Tectonic geomorphology methods have been used in numerous studies to analyze active structures (see Table 1). Dating deformed markers is the most efficient method, but deformed markers are not easy to identify in slowly uplifting humid areas. Morphometry based on conceptual or analogue models provides useful insight but lacks a theoretical basis permitting the quantification of uplift rates. Finally, morphometry based on mechanistic modeling of erosion has
Table 1. Geomorphological Methods Used to Analyze Active Structures

<table>
<thead>
<tr>
<th>Description</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dating of deformed linear or planar markers (alluvial fans, fluvial terraces)</td>
<td>[e.g., Ritz et al., 2003; Rockwell et al., 1984; Molnar et al., 1994; Siani et al., 1997]</td>
</tr>
<tr>
<td>Variations of erosion or incision rates</td>
<td>[e.g., Pazzaglia and Brandon, 2001; Lavé and Avouac, 2000; Wobus et al., 2005; Krzyzowski et al., 2000; Maddy et al., 2000]</td>
</tr>
<tr>
<td>Morphology based on conceptual models or experiments on fluvial systems disrupted by tectonics</td>
<td>stream gradient index [e.g., Huch, 1973; Seebur and Gornitz, 1983]</td>
</tr>
<tr>
<td>Morphology based on a mechanical modeling of erosion</td>
<td>local relief [e.g., Hurtrez et al., 1999]</td>
</tr>
<tr>
<td></td>
<td>stream channel deflections [e.g., McGill and Sieh, 1991; Gaudemer et al., 1989; Jackson et al., 1996]</td>
</tr>
<tr>
<td></td>
<td>change in river sinuosity [e.g., Ouchi, 1985; Giamboni et al., 2005]</td>
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<tr>
<td></td>
<td>valley asymmetry [e.g., Cox, 1994]</td>
</tr>
<tr>
<td></td>
<td>terrace long profile geometry [e.g., Merritts et al., 1994]</td>
</tr>
<tr>
<td></td>
<td>marine and fluvial terrace morphology [e.g., Rosenbloom and Anderson, 1994; Arrowsmith et al., 1996]</td>
</tr>
<tr>
<td></td>
<td>river incision and width [e.g., Lavé and Avouac, 2001]</td>
</tr>
<tr>
<td></td>
<td>ratio of alluvial fan and drainage basin areas [e.g., Whipple and Trayler, 1996; Allen and Deniscoe, 2000]</td>
</tr>
<tr>
<td></td>
<td>steepness index [e.g., Lague et al., 2000; Kirby and Whipple, 2001; Hodges et al., 2004]</td>
</tr>
<tr>
<td></td>
<td>river deviation [e.g., Tomkin and Braun, 1999]</td>
</tr>
</tbody>
</table>

provided significant constraints on tectonic and erosion rates at the fault scarp scale [e.g., Hanks, 1999] but at the scale of drainage basin, this approach has suffered from uncertainties in erosion laws [e.g., van der Beek and Bishop, 2003; Tomkin et al., 2003]. Moreover, many studies at this scale assume a steady state topography that is difficult to demonstrate and may not be the case in many instances [Whipple, 2001]. Therefore quantitative methods capable of linking nonequilibrium morphologies and changes in tectonic uplift or climate are badly needed.

This paper is an attempt to give a physically based interpretation to knickpoints and knickzones observed in two rivers in a low uplift rate area. We define a knickpoint as a point of discontinuity in a river profile, that corresponds to either an abrupt change in slope or in bed elevation. We use the term knickzone to describe a portion of a profile which is broadly convex over several kilometers above knickpoints [e.g., Zaporowski et al., 2001]. We focus on river profiles because rivers transmit tectonic uplift to drainage basin slopes with a time delay. Thus current river profile anomalies may be caused by uplift rate variations which occurred several tens of thousand years ago [e.g., Whipple and Tucker, 1999]. This is the relevant time span to analyze slow deformation, which makes river profiles good candidates to record past and current deformation. In order to evaluate whether the knickpoints have a climatic or tectonic origin, we analyze the fit between observed profiles and synthetic ones obtained with a fluvial incision model subjected to climate oscillations (100 ky) and along-profile uplift rate variations. The river evolution model was proposed by Tucker and Bras [2000] and Tucker [2004] to model detachment-limited incision, including stochastic rainfall variability and a detachment threshold.

In the following we present the tectonic environment and the rivers, justify the choice of the model and describe the fitting procedure. The fit for various model parameters and climatic scenarios is analyzed and we discuss some possible generalization of our results.

2. Tectonic and Geomorphological Settings

The 50 × 50 km Sundgau region is located between the more external northern emergent front of the Miocene Western Alps (the Jura fold and thrust belt) and the southern end of the late Eocene Upper Rhine Graben (URG) (Figure 1a). The geological evolution of this area is well known and has been synthesized by, for example, Sissingh [1998], Schumacher [2002] and Dèzes et al. [2004] for the URG and by [Laubscher, 2001] for the Jura Mountains. We focus here on the relation of the Sundgau drainage network with the Plio-Pleistocene folding of the area based on the works of Nivière and Winter [2000] and Giamboni et al. [2004a, 2004b, 2005].

The area is located above a WSW-ENE Variscan transform zone that was potentially reactivated during the late Eocene rifting of Western Europe [Schumacher, 2002]. The Oligocene structural inheritance mainly consists of half-grabens controlled by roughly N10E-to N30E-trending west-dipping normal faults, like the Illfurth fault (Figure 1a).

During Middle Miocene time the Alpine thrust front started to propagate farther northward and influenced the area of the Jura Mountains. At the end of Miocene time, the Alpine front corresponded to EW folds including the Ferrette fold [Giamboni et al., 2004a]. Continued Alpine compression during the Pleistocene was responsible for propagating the deformation northward. The current frontal thrust is located near Mulhouse, on a N70E-trending branch of the Illfurth fault (Figure 1b) [Nivière and Winter, 2000]. Between the Ferrette fold and this thrust, several other ~10 km spaced WSW-ENE blind reverse faults formed, rooted ~1000 m deep in Triassic evaporite marls. These faults are post-Pliocene because they have deformed the Pliocene Sundgau Gravels and the Pleistocene terraces of the Rhine river [Nivière et al., 2006]. The Sundgau Gravels correspond to a paleo-Rhine deposit. This marker is assigned a late Zanclean to early Pliensarian age (4.2 to 2.9 My) by Fejar et al. [1998] whereas Petit et al. [1996] proposed an age of 3.4–2.6 My. These dates are based on assemblages of rodents and molluscs. The blind faults are associated with WSW-ENE anticlines revealed by the mapping of the base of the Sundgau Gravels [Giamboni et al., 2004a, 2004b] (Figure 1b). These anticlines are themselves associated with topographic ridges, whose elevations decrease from the south (650 m in the Ferrette fold) to the north (300 m) [Meyer et al., 1994; Nivière...
and Winter, 2000]. The anticlines add to a regional northwards tilting of the base of the Sundgau Gravels [Giamboni et al., 2004a]. The elevation also decreases regularly (slope \( \sim 1^\circ \)) from the \( \sim 650 \)-m-high southern crest of the Jura topographic front to \( \sim 290 \) m at Mulhouse in the north, the edge of the uplifted domain. This tilting is interpreted to be associated with the development of a tectonic wedge in front of the Jura Belt [Meyer et al., 1994; Nivière and Winter, 2000; Nivière et al., 2006]. The tilting of the Sundgau topography and of the base of the Sundgau Gravels allows an estimate of a mean regional post-Pliocene uplift rate relative to the city of Mulhouse of between 0.06 to 0.11 mm yr\(^{-1} \) [Nivière et al., 2006].

Giamboni et al. [2004a, 2005] analyzed the geomorphology of several rivers flowing northwards across the uplifted area, in particular the Ill and Largue Rivers which are reexamined in this study. Both rivers flow across the Sundgau from the Ferrette fold in the south and join each other 10 km upstream of Mulhouse (Figure 1b). This study focuses on the long profiles of the rivers to Mulhouse, the northern boundary of the uplifted domain (Figure 1b). These profiles have roughly the same length to Mulhouse (50 km), the same relief from the upper part of the Sundgau Gravels to Mulhouse (300 m) and the same drainage areas at their confluence (circa 350 km\(^2 \)). The drainage area accumulation with distances along the river courses are also very similar (Figure 2a). Their beds are covered by gravel and they incise into Oxfordian and Tertiary units.

Giamboni et al. [2005] analyzed the relationship between the stream-gradient index [Hack, 1973] along river profiles, lithological transitions, river junctions and the location of anticlines. High values of the stream-gradient index were found at river junctions and at lithological transitions, but the highest values were found above anticlines. High values of the stream-gradient index correspond to knickpoints and knickzones along river profiles. Knickpoints correspond here to discontinuities of river gradient, and there are no waterfalls associated with knickpoints. Figure 2a shows that some knickpoints along the Ill and Largue Rivers correspond to river junctions with large tributaries (large increase of drainage area). Moreover, Figure 2b shows that some of the lithological transitions are associated with knickpoints. Along profiles, the lithology varies between marls and sandstone, clay and marls, and limestones (Figure 2b). Knickpoints at lithological transitions are likely to propagate upstream because stratigraphic contacts are not vertical. The largest variation in river bed gradient and altitude occurs on the Ill profile above the Altkirch fold (Figures 1b and 2a). Here a knickzone (convex portion of the profile) is visible as a \( \sim 15 \) m offset of the river bed over 5 km (Figure 2a). The knickpoint at the foot of the knickzone does not correlate with any major change in drainage area nor lithological transition. This led Giamboni et al. [2005] to the conclusion that this knickzone is a geomorphic response to the uplift of the Altkirch fold. If true, the fact that the knickzone has not migrated upstream above the fold implies that the uplift is still persistent. A reach of the Largue River above the Dannemarie fold has a high stream-gradient index [Giamboni et al., 2005] (Figure 2a). It is probably mainly influenced by the lithological transitions above and at the edge of the Dannemarie fold. However, a local uplift may also steepen the river gradient. Current uplift above the Altkirch and Dannemarie folds is also supported by the narrowing of the valley above them [Giamboni et al., 2004a]. Remnants of fluvial terraces were also mapped by Giamboni et al. [2004b] along the Ill and Largue Rivers, but their continuity is not clear, so that it is not clear whether they are more numerous or folded above the anticlines.

The present-day climate of the Sundgau area is humid, with annual mean precipitation over the last 30 years of 500 mm. Considering the low relief of the Sundgau and the proximity of both rivers, the climate can be considered as homogeneous over this area.

On the whole, these data suggest that at least the knickzone above the Altkirch fold has a tectonic origin. Nevertheless, it is striking that such low relief features generated by a low uplift rate (in the order of 0.1 mm yr\(^{-1} \)) have survived long-term climate variations during the Pleistocene. One could imagine that incision (and deposition) should have smoothed the knickpoints, or that cyclic incision due to climate variations may have generated knickzones with comparable amplitudes. In order to provide some possible explanations of such an apparent paradox, we explore the responses of a river incision model to climate and uplift variations.

3. Modeling Approach

3.1. Long River Profile Model

Different mechanical models of long-term fluvial incision have been proposed, but there is no clear agreement on the broader applicability of one compared to the others [Stock and Montgomery, 1999; van der Beek and Bishop, 2003; Tomkin et al., 2003]. Actually, very few studies have checked the consistency of the models with natural examples regarding their ability to correctly predict incision, response time and river profile shape [see Whipple, 2004]. In parallel, some studies have proposed guidelines to choose...
an appropriate model from the river long profile [see Whipple and Tucker, 1999; Tucker and Whipple, 2002; Carretier and Lucazeau, 2005; Brocard and van der Beek, 2006]. We use such guidelines to justify the selected model. The consistency of model predictions is discussed to justify a posteriori the selected model, but it is necessary to keep in mind that the proposed answers to the question addressed in this paper are strongly related to the assumptions of the
following model. We use a detachment-limited model, which links the erosion rate to a power law of the excess shear stress applied to the river bed (usually called "shear stress model with threshold") (see Appendix A). This model (with negligible or significant threshold) allows the formation and the long-time preservation of well-marked knickpoints [Howard et al., 1994; Whipple and Tucker, 1999] associated with localized uplift or elevation changes at base level. This is not possible with a pure transport-limited model (erosion limited by the ability of the river to transport sediment coming from upstream), which predicts the rapid smoothing of knickpoints [e.g., Ouchi, 1985; Howard et al., 1994; Poisson and Avouac, 2004]. The sensitivity of gradient to some lithological variations, even
small, is also an argument in favor of detachment-limited behavior, as transport-limited gradients do not depend on the bedrock strength.

[14] Recent studies have explored how erosion models account for the variability of water discharges and their capacity to exceed erosion thresholds [Tucker and Bras, 2000; Crave and Davy, 2001; Molnar, 2001; Snyder et al., 2003a; Tucker, 2004; Lague et al., 2005]. The combination of climate variability and erosion thresholds seems to be a necessary condition to predict relevant response times and scaling relationships between uplift and fluvial relief. Thus we use the Stochastic Threshold Model proposed by Tucker and Bras [2000] and Tucker [2004]. This model combines the shear stress model with a threshold for a storm event with a rectangular Poisson model of discharge distribution. Tucker [2004] provided an approximate analytical solution of the long-term mean incision rate of the form (as reformulated by Snyder et al. [2003a, 2003b]; see Appendix A),

$$(E) = K_R K_C K_{\tau_c} A^a S^b,$$  \hspace{1cm} (1)

where $K_R$ encompasses the physical parameters, $K_C$ encompasses the climatic parameters, $K_{\tau_c}$ is a threshold term equal to 1 for negligible threshold ($\tau_c = 0$ Pa) and tending asymptotically toward zero for increasing critical runoff (the minimum runoff necessary for particle detachment $K_{\tau_c}$). $A$ is the drainage area and $S$ the river gradient. The climate is described via two parameters in $K_C$ (see equation (A14)): The variability factor $F_{\text{var}}$ is the inverse of the proportion of time with rain [Tucker and Bras, 2000], and $P$ is the mean storm rainfall intensity. The mean precipitation rate $\langle P \rangle$ is the ratio of $P$ over $F_{\text{var}}$. For example, an arid climate corresponds to high $F_{\text{var}}$ and low $\langle P \rangle$. Although the Poisson model is not a good fit for classical discharge events distribution for small and very large discharge events [Lague et al., 2005], we will see that our main conclusions do not depend critically on the Poisson model.

[15] The long-term height variations of river profiles are modeled using the following equation:

$$\frac{\partial h(x, t)}{\partial t} = -\langle E \rangle + U(x, t),$$  \hspace{1cm} (2)

where $h(x, t)$ is the profile evolution function of location along profile $x$ and time $t$ and $U(x, t)$ is an uplift rate function relative to the city of Mulhouse which is located out of the uplifted domain. The time scale over which the mean erosion rate $\langle E \rangle$ converges toward a nearly constant value should be on the order of $10^2-10^3$ years (return time of the most rare rainfall events [Lague et al., 2005]). Longer-period variations in $\langle E \rangle$ (e.g., 100 ky) can be obtained by varying the mean and variability of precipitation rates in order to model oscillations between dry and wet periods.

[16] Our approach is a perturbation analysis. It consists of modeling equilibrium profiles with constant parameters, then modeling the impact of climatic changes, and finally modeling the impact of localized uplift. For each scenario, we test the fit between observed and synthetic profiles. These fits help us evaluate the relative contribution of uplift and climate variations on knickzones development. The synthetic profiles are obtained by downstream finite difference integration of equation 2 (~50 years time step, 40 m spatial resolution) with fixed elevation at the outlet. Profiles are modeled from the source to Mulhouse without taking into account lithological variations. Drainage area $A$ along profile is approximated by a power law function of the downstream distance $x$ (Hack’s law $A = 11.5 \times x^{1.6}$). The fit is evaluated by calculating the standard deviation (STD) between observed and synthetic profiles.

$$\text{STD} = \sqrt{\frac{1}{p-1} \sum_{j=1}^{p} \left(h_{\text{model}}(x_j) - h_{\text{data}}(x_j)\right)^2}, \quad (3)$$

where the $j$ [1, p] refers to the points of the profiles.

3.2. Parameter Values

[17] The values of physical parameters used for the two rivers are discussed in Appendix A. We do not have the value of the variability factor $F_{\text{var}}$ in the study area. We fixed $F_{\text{var}}$ at 10, which lies within the range of values in the United States [Tucker, 2004], but it is not constrained here. In our study, it is fundamental that the model predicts the largest variations of incision rate when the climate varies. If not, the model would not allow us to evaluate the maximum impact of climate change on river profiles and thus to compare the relative impact of climate and uplift. We vary $F_{\text{var}}$ in section 4 to evaluate the effects of climate variations. Moreover, adjustment of the unknown parameter $k_c$ (equation (A2)) depends on the choice of $F_{\text{var}}$. In order to evaluate whether our results can critically depend on the choice of these two parameters, we calculate variations in the mean erosion rate $\langle E \rangle$ (equation 1) associated with $F_{\text{var}}$ variations. Figure 3a shows these variations for differing values of the critical detachment stress $\tau_c$ and the shear stress exponent $a$ (equation (A2)). Each curve corresponds to differing values of $k_c$, adjusted so that the mean erosion rate equals the uplift rate and the fluvial relief corresponds to that of studied rivers. For the red curves, $k_c$ is adjusted with $F_{\text{var}} = 10$ and for the green curves, with $F_{\text{var}} = 30$. It appears in both cases that 90% variations around these values correspond to more than 50–70% variation in the mean erosion rate $\langle E \rangle$ (Figure 3a). Thus, even if the value $F_{\text{var}} = 10$ is not correct for the present days or as the mean value over geological time, variation around this value corresponds to the largest fluctuations of mean erosion rate, and thus to the largest possible river profile elevation variations due to climate change. We also plot in Figure 3b the dependence of $\langle E \rangle$ on the mean precipitation rate when it varies around the value $\langle P \rangle = 500$ mm yr$^{-1}$. This shows that the mean incision rate can increase much more than the tested range, in particular if $a = 5/2$. Nevertheless, we will see that a larger range of values would not change our main conclusion regarding the climatic impact on river profiles.

[18] Three parameters remain undefined: $k_c$, $\tau_c$ and $a$ (Equation (A1)). The exponent $a$ of the shear stress excess can take different values between 1 and 5/2 depending on the main detachment process [Whipple et al., 2000]. As this process can vary over time because of variations of sediment supply (abrasion tool), we test the two extreme values. We point out here that a simplification of the shear stress model with threshold has been used to make an analytical solution tractable (equation (A2)). This leads to a significant
overestimation of the erosion rate for the value $a = 5/2$ when the shear stress is close to the critical shear stress. For example, if $t = 2 \tau_c$, the erosion rate predicted by equation (A2) is 4.6 times greater than that predicted by equation (A1). This implies that the modeled incision will be a maximum, which is not a problem if we want to estimate the maximum possible profile variations due to climate change.

It is usually difficult to evaluate the critical shear stress $\tau_c$ (equation (A1)) [e.g., Snyder et al., 2003a]). Here we assume that the present day river gradients represent gradients below which no detachment occurs ($\epsilon = 0$) for mean discharge conditions. This assumption is partially supported by the presence of gravels covering the bed of the rivers. Using equation (A3) with the mean value $Q_{\text{m}}$, we estimate a value of $\tau_c$ of ~9.5 Pa. In the following, we will use the values of $\tau_c$ of 0 Pa, 47 Pa and 100 Pa, in order to evaluate the effect of negligible and significant critical shear stresses on model fits. These values are consistently lower than or equal to other estimates (e.g., ~100–200 Pa for mudstone and sandstone [Snyder et al., 2003a]). In the case where $\tau_c = 0$ Pa, the mean erosion rate (equation (1)) simplifies into the classical stream power law ($K_{\text{tc}} = 1$ in equation 1)). Therefore the comparison of model fits with no or large critical shear stresses will help us discuss the validity of the stream power law and stochastic threshold models in the studied example. The parameter $k_e$ (equation (A2)) remains undetermined. It is adjusted in the next section.

4. Results

4.1. Initial Equilibrium Profiles

An interesting feature of the Ill and Largue River profiles is their close similarity consistent with the fact that they flow over the same uplifted initial surface. We assume a common initial profile for the two rivers. This profile is determined such that a dynamic equilibrium profile fits the fluvial relief at 6 km from the source. We do not study the fit between 0 and 6 km because the rivers flow across significantly differing lithologies (and potentially uplift rates) in this portion compared to the Tertiary units exposed downstream. Varying $\tau_c$ along the profile would allow us to
adjust river profiles in this portion, but that is beyond the scope of this study. Moreover, this would not change significantly the profile further downstream because incision depends on the local gradient and drainage area. The best fit dynamic equilibrium profile is obtained by imposing a constant and homogeneous Quaternary regional uplift rate of 0.1 mm yr$^{-1}$ (relative to the pinned bottom boundary) to an initial horizontal profile and by adjusting $k_e$ to fit the fluvial relief. Several climatic scenarios are tested: a constant and homogeneous climate ($P = 500$ mm yr$^{-1}$ and $F_{var} = 10$) and variable climatic conditions.

4.1.1. Constant Climate

Figure 4 shows the observed and modeled profiles for $a = 1$ and 5/2, and for $\tau_c = 47$ Pa and 0 Pa. The response times to achieve equilibrium are between 2 and 3 My. The adjusted values of $k_e$ are three orders of magnitude lower for $a = 5/2$ than for $a = 1$. The STD is given on the graphs and in Table 2. It is around 6 m. In order to evaluate the sensitivity of the fluvial relief to $k_e$, profiles have been plotted for 10% variations around $k_e$. This leads to a relative change of the fluvial relief of between 2 and 12% and STD of 3.9 to 18.4 m. The change is smaller for $a = 5/2$ than for $a = 1$. The sensitivity significantly differs for $\tau_c = 47$ Pa and 0 Pa if $a = 1$, but is not significant if $a = 5/2$. This probably comes from the simplification of the model (equation (A2)) which decreases the effect of $\tau_c$ for $a = 5/2$, as noted in section 3. These results show that a significant threshold value decreases the sensitivity of the relief to $k_e$.

4.1.2. Varying Climate

[23] In order to evaluate the impact of varying climate on the evolution of river profiles, we vary $P$ and $F_{var}$ around the mean values of 500 mm yr$^{-1}$ and 10, respectively. Present day data in the United States show that $P$ and $F_{var}$ vary between an arid pole (low $P$, high $F_{var}$) and a humid pole (high $P$, low $F_{var}$) [Tucker, 2004]). Thus we impose out-of-phase sinusoidal variations of these two parameters to model long-term oscillations of climate during Quaternary times. The tested variations are: periods ($T$) of 100 ky and 200 ky, phase differences ($\phi$) of $T/2$ and $T/4$, amplitudes of $\pm 50\%$ and $\pm 90\%$ around the mean values.

Figure 5 shows the resulting profiles after an approximately constant fluvial relief has been achieved (after $\sim 2.2$ My for $a = 1$ and $\tau_c > 0$ Pa and for $a = 5/2$ and after $\sim 3.5$ My for $a = 1$ and $\tau_c = 0$ Pa). The STD and the adjusted

Table 2. Adjusted $k_e$ of Dynamic Equilibrium Profiles Under Constant and Homogeneous Climate and Uplift

<table>
<thead>
<tr>
<th>$a$</th>
<th>$\tau_c$ (Pa)</th>
<th>best-fit $k_e$ (m yr$^{-1}$ Pa$^{-1}$)</th>
<th>STD (m)</th>
<th>STD $(k_e + 10%k_e)$</th>
<th>STD $(k_e - 10%k_e)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>47</td>
<td>$3.64 \times 10^{-5}$</td>
<td>6.2(L)</td>
<td>12.6(L)</td>
<td>5.5(L)</td>
</tr>
<tr>
<td>5/2</td>
<td>47</td>
<td>$1.62 \times 10^{-8}$</td>
<td>6.2(L)</td>
<td>10.4(L)</td>
<td>3.9(L)</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>$1.85 \times 10^{-5}$</td>
<td>6.2(L)</td>
<td>18.3(L)</td>
<td>5.5(L)</td>
</tr>
<tr>
<td>5/2</td>
<td>0</td>
<td>$1.44 \times 10^{-8}$</td>
<td>6.2(L)</td>
<td>11.2(L)</td>
<td>4.1(L)</td>
</tr>
</tbody>
</table>

*See Figure 4. (L) refers to the Ill River, and (L) refers to the Largue River. Uplift rates are relative to the pinned bottom condition of river profiles.
values of $k_e$ are given in Table 3. The profiles consist of a system of knickzones which propagate and vanish upstream. The knickzones disappear upstream partly because the finite difference solution tends to smooth any advecting step. A second reason is that deviation from the linear dependence of erosion rate on river gradient ($n$) implies progressive rounding and widening of knickzones [Whipple, 2001; Tucker and Whipple, 2002]. This is the case in our experiments ($n = 0.67$ for $a = 1$ and $n = 1.67$ for $a = 5/2$). The third reason is that knickzones retreat at a speed that decreases upstream because of the decreasing drainage area and the smoothing of the knickzones [Gardiner, 1983; Whipple and Tucker, 1999]. Thus, as the most upstream knickzones widen and slow, they are overtaken by the following ones and they coalesce. It is interesting to note that the angular or rounded shape of the knickzones is directly linked to how the incision rate evolves with increasing climate variability. Figure 5 and the upper diagrams of Figure 3 show that rounded knickzones correspond to a decreasing incision rate with increasing variability factor $F_{\text{var}}$. This does not depend on the value of $a$ (and thus on the gradient exponent $n$ of the stream power law; see Appendix A), contrary to what is predicted with the classical stream power law without stochastic climate [Tucker and Whipple, 2002]. Consequently, the shape of knickzones generated by sinusoidal climate oscillations seems not to be useful for inferring the parameter $n$ of the incision law if the threshold $\tau_c$ is significant compared to shear stress at mean climatic conditions.

Knickzones develop at river outlets because the profiles are tectonically advected upwards during low erosion efficiency periods and relax during times of increased erosion efficiency, similarly to river responses to successive base-level changes [Snyder et al., 2002; Bishop et al., 2005]. The relaxation begins at the outlet and propagates upstream as a wave, which is characteristic of the detachment-limited model (equation (1)). Consequently, the change of river bed elevation is mainly controlled by the time period of climatic

![Figure 5](https://example.com/figure5.png)

**Figure 5.** Effect of variable climate. The Ill and Largue profiles are in green. The model profiles are computed by imposing sinusoidal variations to the mean precipitation rate ($P$) and to the variability factor $F_{\text{var}}$. The reference mean values are $P = 500$ mm yr\(^{-1}\) and $F_{\text{var}} = 10$ and the constant uplift rate relative to the pinned bottom condition is $U = 0.1$ mm yr\(^{-1}\). The fitting results are given in Table 3. Inset gray diagrams show an enlargement of the last 20 km of the profiles. The profiles have been slightly shifted vertically for ease of viewing. A 5-m ruler is displayed to measure the amplitude of elevation changes because of climate oscillations. These figures show that elevation changes along a single profile are smaller than 5 m whatever the tested scenario and erosion parameters. This suggests that climate variations cannot explain the main knickzone along the Ill River.
Table 3. Adjusted $k_c$ of Models With Variable Climate

<table>
<thead>
<tr>
<th>a</th>
<th>$\tau_c$ Pa</th>
<th>$b$ T years</th>
<th>$d_0$, years</th>
<th>$k_c$, m yr$^{-1}$ Pa$^{-1}$</th>
<th>STD, m</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>47 50% 10^5</td>
<td>5 10^4</td>
<td>3.64 10^{-5}</td>
<td>5.1(1) 6.5(L)</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>47 90% 10^5</td>
<td>5 10^5</td>
<td>4.44 10^{-5}</td>
<td>6.1(1) 6.3(L)</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>47 90% 2 10^5</td>
<td>10^3</td>
<td>4.44 10^{-5}</td>
<td>5.7(1) 6.9(L)</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>47 90% 10^5</td>
<td>5 10^5</td>
<td>4.79 10^{-5}</td>
<td>6.6(1) 6.7(L)</td>
<td></td>
</tr>
<tr>
<td>5/2</td>
<td>47 90% 10^5</td>
<td>5 10^5</td>
<td>1.04 10^{-5}</td>
<td>5.3(1) 6.3(L)</td>
<td></td>
</tr>
<tr>
<td>5/2</td>
<td>47 90% 10^5</td>
<td>5 10^5</td>
<td>2.3 10^{-5}</td>
<td>5.7(1) 6.1(L)</td>
<td></td>
</tr>
<tr>
<td>5/2</td>
<td>47 90% 10^5</td>
<td>5 10^5</td>
<td>2.24 10^{-5}</td>
<td>6.2(1) 5.3(L)</td>
<td></td>
</tr>
<tr>
<td>5/2</td>
<td>47 90% 10^5</td>
<td>5 10^5</td>
<td>2.15 10^{-5}</td>
<td>6.0(1) 5.2(L)</td>
<td></td>
</tr>
<tr>
<td>5/2</td>
<td>47 90% 10^5</td>
<td>5 10^5</td>
<td>3.85 10^{-5}</td>
<td>6.5(1) 6.2(L)</td>
<td></td>
</tr>
<tr>
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<td>47 90% 10^5</td>
<td>5 10^5</td>
<td>4.79 10^{-5}</td>
<td>6.6(1) 6.7(L)</td>
<td></td>
</tr>
<tr>
<td>5/2</td>
<td>47 90% 10^5</td>
<td>5 10^5</td>
<td>1.04 10^{-5}</td>
<td>5.3(1) 6.3(L)</td>
<td></td>
</tr>
<tr>
<td>5/2</td>
<td>47 90% 10^5</td>
<td>5 10^5</td>
<td>1.04 10^{-5}</td>
<td>5.3(1) 6.3(L)</td>
<td></td>
</tr>
</tbody>
</table>

*See Figure 5. T is the period of climate oscillation, and $d_0$ is the difference of phase between variations of the mean precipitation rate ($P$) and of the variability factor $F_{var}$ (see Figure 5). (I) refers to the Ill River, and (L) refers to the Largue River.

The uplift rate between the Dannemarie and Illfurth anticlines, a reach where the regional rock uplift rate relative to Mulhouse is probably lower [Nivière and Winter, 2000]. We perform a parameter search procedure to calculate best fit values of the added or subtracted uplift. The best fit value corresponds to the minimum of the STD (root mean square inversion). The fitting procedure includes also the time span after the onset of anticline growth (the age of the anticline).

[27] Figure 6 displays model results for the two rivers and for the case $\tau_c = 47$ Pa and $a = 1$. In the case of the Ill River, a new equilibrium profile develops from the anticline (U1) and propagates upstream. A knickzone appears above the anticline and remains fixed. The profile is progressively uplifted vertically above it and upstream. In the case of the Largue River, the profile is uplifted above the Dannemarie fold (U2) and then subsides because of the lower uplift rate downstream (U3). The best fit profiles account for the knickzone in the Ill River near km 33 and for the steeper reach of the Largue profile around km 25. The fit between profiles is better than fits obtained in previous sections (STD is between 1 and 2 m, compared to 6 m for initial equilibrium profiles). Moreover, the product of uplift rates and best fit age predict cumulative uplifts consistent with the fold amplitude of the base of the Sundgau Gravels (~50 m, Figure 1b). The inset diagram of Figure 6a shows that the curves of STD versus anticline age are roughly flat after ~1 My for the Ill profile. This is because the transient wave has propagated to the upper edge of the model space. This suggests that the Ill River has reached a new equilibrium, at least in the studied reach. For the Largue River, the curve of STD versus anticline age shows a minimum. The goodness of fit decreases with increasing ages. This suggests that the Largue profile corresponds to a transient profile adjusting to local uplift rate perturbation. The predicted ages (minimum age for the Ill River) for the onset of anticline growth are similar for the two rivers (~1 My).

[28] The behavior of these transient rivers and the resulting fits are very similar for different values of $\tau_c$ and $a$ (Table 4). The only differences are (1) the more or less rounded shape of the knickzones which is a second-order observation given the simplified form of the imposed tectonic uplift, (2) the response time, and (3) the adjusted uplift rates. Only the best fit response times vary significantly (between 0.65 and 2.1 My), but even added to the time required to achieve a first dynamic equilibrium (~2 My), these remain consistently lower than the upper bound age of the Sundgau Gravels (between 2.6 and 4.2 My). Without further constraints on the age of anticline growth, it is not possible to decide between the tested models, which all appear equally good.

5. Discussion

5.1. Effect of Uplift Rate

[29] The main conclusion of our study is that climate change over the Quaternary may have had minor effects on the evolution of the two river profiles compared to those of anticline growth, even though the rock uplift rate is low. This arises from two common outcomes of the tested detachment-limited models: (1) Elevation change cannot exceed the rock uplift during low erosion efficiency periods, and (2) the incision during high erosion efficiency periods
cannot propagate far upstream because the response time of the river is too long compared to the timescale of climatic oscillations. The first limitation is set up by the low regional uplift rate $U$. Higher rock uplift rates should allow larger climate-induced profile variations. To illustrate this, we plot in Figure 7 different profiles obtained by changing $k_c$ or $U$ by a factor of 10. For a given fluvial relief, greater profile variations occur for greater uplift rate. In these cases, climate-driven variations of river bed elevation are more likely to obscure spatial variations of the uplift rate, although this should depend obviously on the magnitude of these variations and on the distance of local uplift relative to the base level.

[30] The impact of $U$ on climate-induced elevation change can be shown analytically in the case of non-stochastic rainfall and negligible $\tau_c$ (stream power law model). The maximum possible elevation change can be defined as the difference of fluvial relief between two equilibrium profiles, corresponding to two differing climate or tectonic conditions. Whipple and Tucker [1999] have shown that the stream power law predicts that the dynamic equilibrium relief $R$ scales as

$$R \propto P^{-a} U^{1-c},$$

where $P$ represents a mean precipitation rate. Taking the log derivative, we can express the relationship between fractional changes of $R$, $P$ and $U$ [Roe et al., 2006].

$$\frac{dR}{R} = -\frac{m}{n} \frac{dP}{P} + \frac{1}{n} \frac{dU}{U}.$$  

This shows that relief change is greater for an uplift rate change than for a precipitation rate for an identical amount of relative change of both parameters, and for classical values of $\frac{m}{n} = 0.5$ and $n < 2$. If $U$ is constant, then

$$dR = -U^{1-c-m} \frac{dP}{P} P^{-a}.$$  

This means that the climate-induced relief variations are greater for larger uplift rates. Snyder et al. [2003a] and Tucker [2004] showed that the relationship between $R$ and $U$ is weaker for large uplift rates than for low uplift rates if $\tau_c > 0$ Pa and for a Poisson distribution of floods. Lague et al. [2005] demonstrated also a weaker relationship between relief and increasing high uplift rates by using a frequency-magnitude distribution of discharge characterized by a “heavy” tail, with power-law scaling of the probability density function of floods. This reinforces our conclusion because increase of high uplift rate will produce a relatively smaller relief increase than that produced by increase of a low uplift rate. Thus tectonic nickpoints and knickzones are even more likely to dominate those generated during climatic variations for low uplift rate than for high uplift rate. We note, however, that this is valid only for a pinned lower boundary condition, as imposed in our experiments. If the boundary condition corresponds to a strong contrast between the detachment-limited uplifted domain and a transport-limited alluvial plain, differential erosion at the transition can produce a local base-level fall which propagates upstream as a knickzone [Carretier and Lucazeau, 2005], possibly removing uplift signatures above faults and folds. Similarly, climatic
sea-level fluctuations at river outlets can generate migrating knickzones [Snyder et al., 2002; Bishop et al., 2005]. Moreover, the limited climatic incision is strongly controlled by the regional uplift relative to the pinned outlet boundary condition. Without regional uplift, climatic incision could be greater during high erosion periods, and it could affect the whole river profile [e.g., Zaprowski et al., 2005], possibly removing local uplift signatures.

5.2. Limitations of the Model

The first limitation of the tested detachment-limited models is that they can only predict incision, and not deposition. This is a strong limitation for predicting variations of river bed elevation when climate varies. In particular, the model can not account for back limb sedimentation if the gradient is reversed behind folds (which does not occur in these experiments). It is very likely that deposition and transport-limited conditions occurred during Quaternary times, and it is possible that erosion shifted toward detachment-limited conditions only because of anticline uplift [Whipple and Tucker, 2002]. For example, the Sundgau Gravels are overlain by a discontinuous Wurmian loess cover [e.g., Vogt, 1992] whose thickness is less than 20 m. These deposits should have covered the entire drainage network and their removal may have been transport-limited. However, neglecting transport-limited periods should not affect our conclusion that climatic induced knickzones are less important than those generated by differential rock uplift. Indeed, deposition usually smoothes variations of river gradients. Therefore neglecting deposition with detachment-limited models should predict the highest elevation changes. On the other hand, assuming constant detachment limited conditions underestimates the response times because periods of transport-limited conditions are not taken into account. However, even underestimated, response times associated with climate induced knickzones migration are still greater than imposed climatic variations, which explains why knickzones are observed in synthetic profiles. Thus underestimation of response time should not affect our conclusions.

Model response times are likely to be modified by other time-dependent processes that have been neglected here, such as time variations of $\tau_e$ due to variation of the vegetation cover, variation of at-a-station river width geometry ($k_w$ and $s$ in equation A7) during the incision of different rock types and because of local uplift and variation of runoff because of infiltration variations. Variation of abrasion efficiency because of variable rate of gravel supply may also have had a significant impact on river response times. We tested two end-member models which may be correct for bedrock abrasion: a model with an excess shear stress dependence more than linear ($\alpha = 5/2$, [Whipple et al., 2000]), and a model with $\alpha = 1$, which may be consistent, as suggested by Lague et al. [2005], with a recent model of abrasion by saltation proposed by Sklar and Dietrich [2004]. However, Sklar and Dietrich [2004] have clearly shown that their model should imply variations of the erosional efficiency coefficient $k_e$ (equation (A1)) along the river profile and through time, which has not been taken into account in our study. The different values of $\alpha$ led to similar river responses, but with different response times to local increased uplift. This difference may be greater with temporal variations of $k_e$. The evaluation of consequences of variable $k_e$ and the difference in predicted response times may be used to test the values of $\alpha$. In our study, it was not possible to do this because of the uncertainty on the ages of anticline growth.

6. Conclusion

Our study suggests that long-term climate variations have not significantly affected the studied river profiles when compared with differential rock uplift across growing anticlines. This results from two assumptions: (1) the long-
Table A1. Model Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \epsilon )</td>
<td>( [L \cdot T^{-1}] )</td>
<td>rate of channel incision (equation (A1))</td>
</tr>
<tr>
<td>( \tau )</td>
<td>( [M \cdot L^{-1} \cdot T^{-2}] )</td>
<td>average bed shear stress (equation (A1))</td>
</tr>
<tr>
<td>( \tau_c )</td>
<td>( [M \cdot L^{-1} \cdot T^{-2}] )</td>
<td>critical detachment shear stress (equation (A1))</td>
</tr>
<tr>
<td>( k_e )</td>
<td>( [M^{-1} \cdot L^{-1} \cdot T^{-3}] )</td>
<td>incision efficiency factor (equation (A1))</td>
</tr>
<tr>
<td>( a )</td>
<td>( [L^3 \cdot T^{-1}] )</td>
<td>exponent in the incision rate relationship (equation (A1))</td>
</tr>
<tr>
<td>( Q )</td>
<td>( [L^3 \cdot T^{-1}] )</td>
<td>water discharge (equation (A3))</td>
</tr>
<tr>
<td>( S )</td>
<td>( [L] )</td>
<td>river gradient (equation (A3))</td>
</tr>
<tr>
<td>( w )</td>
<td>( [L] )</td>
<td>river width (equation (A3))</td>
</tr>
<tr>
<td>( k_t )</td>
<td>( [L^{-2} \cdot \tau^{-2}] )</td>
<td>factor in the friction law (equations (A3), (A4), and (A5))</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>( [L^3 \cdot T^{-2}] )</td>
<td>exponent of water discharge per unit width in the friction law (equation (A3))</td>
</tr>
<tr>
<td>( \beta )</td>
<td>( [M \cdot L^{-3}] )</td>
<td>exponent of river gradient in the friction law (equation (A3))</td>
</tr>
<tr>
<td>( f )</td>
<td>( [L \cdot T^{-2}] )</td>
<td>flow density (equation (A4))</td>
</tr>
<tr>
<td>( g )</td>
<td>( [L^3 \cdot T^{-1}] )</td>
<td>Darcy friction factor (equation (A4))</td>
</tr>
<tr>
<td>( N )</td>
<td>( [L \cdot T^{-1}] )</td>
<td>gravitational acceleration (equation (A4))</td>
</tr>
<tr>
<td>( R )</td>
<td>( [L \cdot T^{-1}] )</td>
<td>Manning friction factor (equation (A5))</td>
</tr>
<tr>
<td>( A )</td>
<td>( [L^3] )</td>
<td>run off (equation (A6))</td>
</tr>
<tr>
<td>( k_a )</td>
<td>( [L^{-3} \cdot T^{-1} \cdot R^0] )</td>
<td>drainage area (equation (A6))</td>
</tr>
<tr>
<td>( b )</td>
<td>( [L^3 \cdot T^{-1}] )</td>
<td>exponent in the at-station hydraulic geometry power law (equation (A7))</td>
</tr>
<tr>
<td>( Q_m )</td>
<td>( [L^3 \cdot T^{-1}] )</td>
<td>mean water discharge (equation (A8))</td>
</tr>
<tr>
<td>( w_m )</td>
<td>( [L] )</td>
<td>river width for mean discharge condition (equation (A8))</td>
</tr>
<tr>
<td>( s )</td>
<td>( [L \cdot T^{-1}] )</td>
<td>exponent in the at-station hydraulic geometry relationship (equation (A8))</td>
</tr>
<tr>
<td>( R_a )</td>
<td>( [L \cdot T^{-3}] )</td>
<td>mean runoff (equation (A9))</td>
</tr>
<tr>
<td>( K_r )</td>
<td>( [L^{-1} \cdot \tau^{-1} \cdot S^{-1} \cdot \rho] )</td>
<td>mean rate of channel incision (equation (A10))</td>
</tr>
<tr>
<td>( K_c )</td>
<td>( [L^{-1} \cdot \tau^{-1} \cdot S^{-1} \cdot \rho] )</td>
<td>threshold factor in the stochastic threshold model equation (A10))</td>
</tr>
<tr>
<td>( I )</td>
<td>( [L \cdot T^{-1}] )</td>
<td>climate factor in the stochastic threshold model (equation (A10))</td>
</tr>
<tr>
<td>( P )</td>
<td>( [L \cdot T^{-1}] )</td>
<td>infiltration rate (equation (A15))</td>
</tr>
<tr>
<td>( F_p )</td>
<td>( [L \cdot T^{-1}] )</td>
<td>precipitation rate of a rainfall event (equation (A15))</td>
</tr>
<tr>
<td>( m )</td>
<td>( [L^3 \cdot T^{-1}] )</td>
<td>mean precipitation rate (equation (A15))</td>
</tr>
<tr>
<td>( n )</td>
<td>( [L] )</td>
<td>drainage area exponent in the stochastic threshold model (equation (A10))</td>
</tr>
<tr>
<td>( F_{sur} )</td>
<td></td>
<td>gradient exponent in the stochastic threshold model (equation (A10))</td>
</tr>
</tbody>
</table>

where \( \tau \) and \( \tau_c \) \( [M \cdot L^{-1} \cdot T^{-2}] \) are shear stress and critical shear stress for detachment, respectively. Equation (A1) is simplified into the following form to make an analytical solution tractable for stochastic climate:

\[
\epsilon = k_e (\tau - \tau_a)^a. \tag{A2}
\]

[36] The shear stress is expressed as

\[
\tau = k_t \left( \frac{Q}{w} \right) \alpha S^b, \tag{A3}
\]

where \( Q \) \( [L^3 \cdot T^{-1}] \) is the water discharge, \( w \) \( [L] \) the river width and \( S \) the river gradient.

[37] Empirical relations are as follows. If the Darcy-Weisbach law is used,

\[
k_t = 1/2 \rho g^{3/2} \left( V^2 \right)^{1/3}, \tag{A4}
\]

and if the Manning law is used,

\[
k_t = \rho g \left( V^2 \right)^{1/3}, \tag{A5}
\]

where \( g \) \( [L \cdot T^{-2}] \) is the gravitational acceleration, \( \rho \) \( [M \cdot L^{-3}] \) the density and \( f \) and \( N \) are friction coefficients.

Discharge-drainage area (\( R \) \( [L \cdot T^{-1}] \)) is the runoff, \( A \) \( [L^2] \) is the drainage area,

\[
Q = R A. \tag{A6}
\]

Width-discharge is

\[
w = k_w Q^b. \tag{A7}
\]
At-station width-discharge (the mean discharge $Q_m$ [L^3T^{-1}]) and the related river width $w_m$ [L] are taken as reference is

$$w/w_m = (Q/Q_m)^{0.389}.$$  \(\text{(A8)}\)

“Mean” discharge-runoff ($R_m$ [L T^{-1}] is runoff for mean discharge) is

$$Q_m = R_m A.$$  \(\text{(A9)}\)

The stochastic threshold model for mean incision rate $\langle E \rangle$ [L T^{-1}] is

$$\langle E \rangle = K_R K_c K_{rc} A^{4/5} S^\alpha,$$  \(\text{(A10)}\)

where

$$m = \alpha a (1 - b),$$  \(\text{(A11)}\)

$$n = a b^3,$$  \(\text{(A12)}\)

$$K_R = k_k k_w^{-\omega a} k_s^a,$$  \(\text{(A13)}\)

$$K_c = F_{var}^{-1} (P)^{\gamma_b} R_m^{b+1} \exp \left( - \frac{I}{P} \right) \Gamma (\gamma_b + 1),$$  \(\text{(A14)}\)

$$K_{rc} = \frac{\Gamma (\gamma_b + 1, \frac{P}{I}) - \left( \frac{P}{I} \right)^{\gamma_b} \exp \left( - \frac{P}{I} \right) \Gamma (\gamma_b + 1)}{\Gamma \left( \gamma_b + 1, \frac{P}{I} \right)},$$  \(\text{(A15)}\)

$$R_c = \left( \tau_a I \right)^{-\rho} \tau_a^{-\omega \alpha (1 - b) \sigma_s} A^\gamma \left( \frac{\gamma_b}{\gamma_b + 1} \right)^{\beta_s} S^{-\beta_s},$$  \(\text{(A16)}\)

$$F_{var} = P / \langle P \rangle,$$  \(\text{(A17)}\)

$$\gamma_b = \alpha a (1 - s),$$  \(\text{(A18)}\)

$$\epsilon_b = \alpha a (b - s).$$  \(\text{(A19)}\)

$\langle P \rangle$ [L T^{-1}] is the mean precipitation rate. $P$ [L T^{-1}] is the instantaneous precipitation rate of a rainfall event. $I$ [L T^{-1}] is the infiltration rate. $F_{var}$ is called the variability factor. $\Gamma(\cdot)$ is the incomplete Gamma function. Note that $K_{rc}$ depends on the gradient $S$ and drainage area $A$.

### A2. Parameter Values

[40] The parameter $k_i$ (equations (A3), (A4) and (A5)) is estimated to be $1.33 \times 10^{-2}$ Pa m^{-1.3} yr^{0.67} with the Darcy-Weisbach relationship (water with suspended load density $\rho = 1400$ kg m^{-3}, gravitational acceleration $g = 9.81$ m s^{-2}, roughness parameter $f = 0.08$ for gravel-cobble bed) and $1.4 \times 10^{-2}$ Pa m^{-1.3} yr^{0.6} with the Manning relationship (roughness $N = 0.03$ for gravel bed). Both values do not change our results significantly when combined with corresponding values of $\alpha$ (2/3 with Darcy-Weisbach and 3/5 with Manning) and $\beta$ (2/3 with Darcy-Weisbach and 7/10 with Manning). The Darcy-Weisbach parameters are used for the calculations.

[41] The width-discharge relation (equation (A7)) is calibrated using the classical exponent $b = 0.5$ [Leopold and Maddock, 1953]; [Montgomery and Gran, 2001] and taking $k_0 = 0.4 \times 10^{-3}$ m^{-0.5} yr^{-0.5} to obtain a river width of about 4 m at 35 km for mean discharge conditions.

[42] Mean at-station discharge $Q_m$ in the at-station width-discharge relation (equation (A8)) is evaluated using a mean precipitation rate $\langle P \rangle = 500$ mm yr^{-1} and a mean infiltration rate $\langle I \rangle = 250$ mm yr^{-1} averaged over 30 years and over an area of 50 km x 50 km centered on the Sundgau area. The geometrical exponent $s$ is taken as 0.25, which corresponds to a concave channel section.

[43] Consequently, if $a = 1$ then $\gamma_b = 0.5$, $\epsilon_b = 0.167$, $n = 0.67$, $m = 0.335$ and if $a = 5/2$, $\gamma_b = 1.25$, $\epsilon_b = 0.42$, $n = 1.67$, and $m = 0.84$. The remaining parameters ($F_{var}$, $k_c$, $\tau_b$) are discussed in section 3.

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