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# SENSITIVITY ANALYSIS OF THE HETEROGENEOUS ARRAY

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## ABSTRACT

This paper investigates the robustness of direction finding techniques relatively to modelling errors for two structures of arrays (homogeneous or heterogeneous). The heterogeneous structure is based on the set up of non identical sensors and has been proposed in order to make an array polarization sensitive.

Regarding the MUSIC algorithm as a reference, the calculation of angular errors is limited to first order terms of the perturbed vectors or matrices. For the numerical simulations, examples involve trans horizon applications in the HF band (3-30 MHz). Computations have been carried out with a standard scheme of perturbations and the results indicate a better robustness of the heterogeneous array for a given level of uncertainty.

## 1. INTRODUCTION

Direction finding techniques operate with synchronous acquisitions at the output of an array of sensors and the associated covariance matrix is the relevant information for the most popular high resolution algorithms (Capon, Music, Weighted Subspace Fitting). This paper investigates the estimation of angular errors resulting from a perturbation on the steering-vector matrix. These uncertainties on the array response are due, for example, to imprecise positions of the sensors or to a default in the calibration of the electronic circuits connected to each of them. The expressions of the errors (limited to first order terms) are derived for two different structures of array. The first one is the classical homogeneous array set up with identical sensors. The second one is the heterogeneous structure, set up with different sensors, that we proposed for HF applications in order to make the array sensitive to the incoming polarization. Statistics of the angular errors are computed for the two solutions and indicate a greater robustness of the second structure.

## 2.EXPRESSIONS FOR ARRAY PROCESSING

### 2.1. Homogeneous (classical) array

A homogeneous array is set up with NC identical sensors associated with a reference point for the geometrical phase. In presence of NS incident waves with direction of arrival  $\theta_k$ ,

the NC output signals on the array are collected in the acquisition column vector  $\underline{\mathbf{X}}(t)$  as:

$$\underline{\mathbf{X}}(t) = \sum_{k=1}^{NS} \underline{\mathbf{a}}(\theta_k) s_k(t) + \underline{\mathbf{N}}(t) \quad (1)$$

where  $\underline{\mathbf{a}}(\theta_k)$  is the steering-vector for the D.O.A.  $\theta_k$ ,  $s_k(t)$  is the corresponding signal and  $\underline{\mathbf{N}}(t)$  is the noise vector. Associating the NS steering vectors in the matrix  $\underline{\mathbf{A}}$  provides the classical linear model of acquisitions:

$$\underline{\mathbf{X}}(t) = \underline{\mathbf{A}} \underline{\mathbf{S}}(t) + \underline{\mathbf{N}}(t) \quad (2)$$

where  $\underline{\mathbf{S}}(t)$  is the signal vector.

The covariance matrix of the acquisitions, defined as:

$$\underline{\mathbf{R}}_{\mathbf{xx}} = E[\underline{\mathbf{X}}(t) \underline{\mathbf{X}}(t)^H] \quad (3)$$

is then expressed as:

$$\underline{\mathbf{R}}_{\mathbf{xx}} = \underline{\mathbf{A}} \underline{\mathbf{R}}_{\mathbf{ss}} \underline{\mathbf{A}}^H + \sigma^2 \underline{\mathbf{Id}} \quad (4)$$

where  $\underline{\mathbf{R}}_{\mathbf{ss}}$  is the covariance of the incident signals:

$$\underline{\mathbf{R}}_{\mathbf{ss}} = E[\underline{\mathbf{S}}(t) \underline{\mathbf{S}}(t)^H] \quad (5)$$

### 2.2 Heterogeneous array

A heterogeneous array is made up of sensors which are different from one another. For each of them, the directional gain relatively to the angle  $\theta$ , called spatial response and denoted by  $F_n(\theta)$ ,  $n=1, \dots, NC$  is supposed to be known. Examples of spatial responses for HF antennas with a simple geometry are calculated in reference [1]. The computation refers to a deterministic model of the polarization at the exit point of the ionosphere.

In this context, the linear model for the output signals of the heterogeneous array is expressed as:

$$\underline{\mathbf{X}}_{\mathbf{h}}(t) = \sum_{k=1}^{NS} \underline{\mathbf{a}}_{\mathbf{h}}(\theta_k) s_k(t) + \underline{\mathbf{N}}_{\mathbf{h}}(t) \quad (6)$$

The components of the steering-vectors  $\underline{\mathbf{a}}_{\mathbf{h}}(\theta_k)$  combine the spatial responses and the exponentials which represent the phases  $\varphi_n(\theta_k)$  calculated with respect to the array geometry:

$$\underline{\mathbf{a}}_{\mathbf{h}}(\theta_k) = (F_1(\theta_k) e^{j\varphi_1(\theta_k)}, \dots, F_{NC}(\theta_k) e^{j\varphi_{NC}(\theta_k)})^T \quad (7)$$

It can be noticed that  $\underline{\mathbf{a}}_{\mathbf{h}}(\theta)$  does not have a constant norm ;

this remark will be taken into account when applying the MUSIC algorithm on this particular type of array.

Gathering the NS steering-vectors in matrix  $\mathbf{A}_h$  gives the linear model for the heterogeneous array:

$$\mathbf{X}_h(t) = \mathbf{A}_h \mathbf{S}(t) + \mathbf{N}_h(t) \quad (8)$$

and, assuming a spatially white additive noise, the corresponding covariance matrix is

$$\mathbf{R}_{xxh} = \mathbf{A}_h \mathbf{R}_{ss} \mathbf{A}_h^H + \sigma^2 \mathbf{Id} \quad (9)$$

### 3. SENSITIVITY ANALYSIS : PERTURBATION METHOD

Perturbations are supposed to affect the array response: random displacements of the sensors or difference in gain and phase for the different acquisition channels connected to the sensors. However, the assumption of spatially white noise is maintained keeping the noise covariance matrix proportional to the identity matrix.

Consequently, the modified covariance matrix can be written as:

$$\hat{\mathbf{R}}_{xx} = (\mathbf{A} + \Delta \mathbf{A}) \mathbf{R}_{ss} (\mathbf{A} + \Delta \mathbf{A})^H + \sigma^2 \mathbf{Id} \quad (10)$$

where  $\Delta \mathbf{A}$  is the perturbation of the array matrix due to errors on the manifold.

Several algorithms are based on the eigen decomposition of the covariance matrix  $\mathbf{R}_{xx}$ [2]. The eigen vectors of the noise subspace are the columns of matrix  $\mathbf{V}_n$ . The perturbation of the array manifold induces a variation  $\Delta \mathbf{V}_n$  of matrix  $\mathbf{V}_n$ :

$$\hat{\mathbf{V}}_n = \mathbf{V}_n + \Delta \mathbf{V}_n \quad (11)$$

The noise subspace of the perturbed covariance is characterized by the relation:

$$[(\mathbf{A} + \Delta \mathbf{A}) \mathbf{R}_{ss} (\mathbf{A} + \Delta \mathbf{A})^H + \sigma^2 \mathbf{Id}] (\mathbf{V}_n + \Delta \mathbf{V}_n) = (\sigma^2 \mathbf{Id} + \Delta \mathbf{A} \mathbf{A}) (\mathbf{V}_n + \Delta \mathbf{V}_n) \quad (12)$$

where  $\Delta \mathbf{A} \mathbf{A}$  is the perturbation affecting the diagonal of the noise eigen values.

Developing this expression and assuming that the second order terms can be neglected, we obtain the result [3]:

$$\Delta \mathbf{V}_n^H \mathbf{A} = -\mathbf{V}_n^H \Delta \mathbf{A} \quad (13)$$

Thanks to relation (13), the dependence of the angular error with the perturbation of the array manifold is established in the following section. The direction finding technique considered as the reference in this work is the MUSIC algorithm [4] which estimates the angle of arrival by minimizing relatively to the angular parameter  $\theta$  the quadratic form:

$$f(\theta) = \mathbf{a}^H(\theta) \mathbf{V}_n \mathbf{V}_n^H \mathbf{a}(\theta) \quad (14)$$

#### 3.1 Homogeneous array

In absence of perturbations, the quadratic form is minimum for the exact angles of arrival:

$$f'(\theta_k) = \frac{df(\theta)}{d\theta} \bigg|_{\theta=\theta_k} = 0 \text{ and } \mathbf{V}_n^H \mathbf{a}(\theta_k) = 0 \text{ for } k=1, \dots, NS \quad (15)$$

With perturbations, the quadratic form is modified in:

$$\hat{f}(\theta) = \mathbf{a}^H(\theta) (\mathbf{V}_n + \Delta \mathbf{V}_n) (\mathbf{V}_n + \Delta \mathbf{V}_n)^H \mathbf{a}(\theta) \quad (16)$$

and reaches its minimums for angles equal to:

$$\hat{\theta}_k = \theta_k + \Delta \theta_k \quad (17)$$

where  $\Delta \theta_k$  is the angular error.

For these values of angle, we can express that:

$$\hat{f}'(\hat{\theta}_k) = 0 = \hat{f}'(\theta_k) + \Delta \theta_k \hat{f}''(\theta_k) + O(\Delta \theta_k^2) \quad (18)$$

if the terms of second order are neglected. The angular error is obtained by dividing the first derivative of the perturbed quadratic form by the second derivative. This calculation is addressed in ref [3], chap.11 and gives :

$$\Delta \theta_k \approx \frac{-\Re[\mathbf{a}'^H(\theta_k) \mathbf{V}_n \Delta \mathbf{V}_n^H \mathbf{a}(\theta_k)]}{\mathbf{a}'^H(\theta_k) \mathbf{V}_n \mathbf{V}_n^H \mathbf{a}'(\theta_k)} \quad (19)$$

and, using relation (15), we finally obtain [3]:

$$\Delta \theta_k \approx \frac{\Re[\mathbf{a}'^H(\theta_k) \mathbf{V}_n \mathbf{V}_n^H \Delta \mathbf{a}(\theta_k)]}{\mathbf{a}'^H(\theta_k) \mathbf{V}_n \mathbf{V}_n^H \mathbf{a}'(\theta_k)} \quad (20)$$

In this relation, the angular error  $\Delta \theta_k$  depends on the uncertainty affecting the steering-vector  $\mathbf{a}(\theta_k)$ .

Since the perturbation  $\Delta \mathbf{a}(\theta_k)$  is a random vector, the angular error is quantified with its statistics. Therefore, we calculate now the mean square error of the angle of arrival.

Denoting  $\mathbf{f}_k = \mathbf{V}_n^H \mathbf{a}'(\theta_k)$ , the angular error  $\Delta \theta_k$  is written as :

$$\Delta \theta_k = \frac{\mathbf{f}_k^H \mathbf{V}_n^H \Delta \mathbf{a}(\theta_k) + \Delta \mathbf{a}(\theta_k)^H \mathbf{V}_n \mathbf{f}_k}{2 \mathbf{f}_k^H \mathbf{f}_k} \quad (21)$$

Then, the related mean square error is calculated as :

$$E[\Delta \theta_k^2] = \frac{\Re\{\mathbf{f}_k^H \mathbf{V}_n^H (\mathbf{C}_{kk} \mathbf{V}_n \mathbf{f}_k + \mathbf{D}_{kk} \mathbf{V}_n^* \mathbf{f}_k^*)\}}{2 \mathbf{f}_k^H \mathbf{f}_k^2} \quad (22)$$

where matrices  $\mathbf{C}_{kk} = E[\Delta \mathbf{a}(\theta_k) \Delta \mathbf{a}(\theta_k)^H]$  and  $\mathbf{D}_{kk} = E[\Delta \mathbf{a}(\theta_k) \Delta \mathbf{a}(\theta_k)^T]$  contain statistics of the perturbation  $\Delta \mathbf{a}(\theta_k)$ .

#### 3.2 Heterogeneous array

In this section, we derive the calculation of the angular error resulting from a perturbed array response  $\Delta \mathbf{A}_h$  of a heterogeneous structure.

The modified acquisition covariance matrix is expressed as:

$$\hat{\mathbf{R}}_{\mathbf{x}\mathbf{x}\mathbf{h}} = (\mathbf{A}_{\mathbf{h}} + \Delta\mathbf{A}_{\mathbf{h}})\mathbf{R}_{\mathbf{ss}}(\mathbf{A}_{\mathbf{h}} + \Delta\mathbf{A}_{\mathbf{h}})^H + \sigma^2\mathbf{Id} \quad (23)$$

Similarly to relation (13), it can be demonstrated that the corresponding perturbation  $\Delta\mathbf{V}_{\mathbf{n}}$  of the noise subspace verifies:

$$\Delta\mathbf{V}_{\mathbf{n}}^H\mathbf{A}_{\mathbf{h}} = -\mathbf{V}_{\mathbf{n}}^H\Delta\mathbf{A}_{\mathbf{h}} \quad (24)$$

The MUSIC algorithm is based on the orthogonality between the steering-vector of an incident signal and the noise subspace [4]. The implementation operating with an estimation of the covariance matrix, the corresponding vectors are only approximately orthogonal. For this reason, the variable vector to be projected in the noise subspace should have a constant norm for all directions of arrival under test. This condition is obviously fulfilled for the homogeneous array with a norm of  $\underline{\mathbf{a}}(\theta)$  been equal to  $\sqrt{NC}$  whatever the angle of arrival.

On the contrary, the steering-vector of the heterogeneous array has not this property as indicated in section 2.2. Therefore, the quadratic form to be minimized in this case is written as [1]:

$$f(\theta) = \underline{\mathbf{b}}_{\mathbf{h}}^H(\theta)\mathbf{V}_{\mathbf{n}}\mathbf{V}_{\mathbf{n}}^H\underline{\mathbf{b}}_{\mathbf{h}}(\theta) \quad (25)$$

where

$$\underline{\mathbf{b}}_{\mathbf{h}}(\theta) = \underline{\mathbf{a}}_{\mathbf{h}}(\theta) / \|\underline{\mathbf{a}}_{\mathbf{h}}(\theta)\| \quad (26)$$

is the normalized steering-vector. Actually, the measure of orthogonality between a variable vector and a given subspace implies that the vector has a constant norm.

The angular error for the heterogeneous array is expressed as an equivalent of relation (27):

$$\Delta\theta_k \equiv \frac{-\Re[\underline{\mathbf{b}}_{\mathbf{h}}^H(\theta_k)\mathbf{V}_{\mathbf{n}}\Delta\mathbf{V}_{\mathbf{n}}^H\underline{\mathbf{b}}_{\mathbf{h}}(\theta_k)]}{\underline{\mathbf{b}}_{\mathbf{h}}^H(\theta_k)\mathbf{V}_{\mathbf{n}}\mathbf{V}_{\mathbf{n}}^H\underline{\mathbf{b}}_{\mathbf{h}}(\theta_k)} \quad (27)$$

where  $\underline{\mathbf{b}}_{\mathbf{h}}'(\theta) = \frac{d\underline{\mathbf{b}}_{\mathbf{h}}(\theta)}{d\theta}$  is the derivative of the normalized steering-vector relatively to the angle of arrival.

To take benefit of relation (32), we express that:

$$\Delta\mathbf{V}_{\mathbf{n}}^H\underline{\mathbf{b}}_{\mathbf{h}}(\theta_k) = \Delta\mathbf{V}_{\mathbf{n}}^H \frac{\underline{\mathbf{a}}_{\mathbf{h}}(\theta_k)}{\|\underline{\mathbf{a}}_{\mathbf{h}}(\theta_k)\|} = -\mathbf{V}_{\mathbf{n}}^H \frac{\Delta\underline{\mathbf{a}}_{\mathbf{h}}(\theta_k)}{\|\underline{\mathbf{a}}_{\mathbf{h}}(\theta_k)\|} \quad (28)$$

to finally obtain the expression of the angular error:

$$\Delta\theta_k \equiv \frac{\|\underline{\mathbf{a}}_{\mathbf{h}}(\theta_k)\|^{-1}\Re[\underline{\mathbf{b}}_{\mathbf{h}}^H(\theta_k)\mathbf{V}_{\mathbf{n}}\mathbf{V}_{\mathbf{n}}^H\Delta\underline{\mathbf{a}}_{\mathbf{h}}(\theta_k)]}{\underline{\mathbf{b}}_{\mathbf{h}}^H(\theta_k)\mathbf{V}_{\mathbf{n}}\mathbf{V}_{\mathbf{n}}^H\underline{\mathbf{b}}_{\mathbf{h}}(\theta_k)} \quad (29)$$

Statistics of this error are calculated relatively to the characteristics of the perturbations affecting the array responses quantified by the matrices

$$\mathbf{C}_{\mathbf{k}\mathbf{k}\mathbf{h}} = E[\Delta\underline{\mathbf{a}}_{\mathbf{h}}(\theta_k)\Delta\underline{\mathbf{a}}_{\mathbf{h}}(\theta_k)^H] \text{ and}$$

$$\mathbf{D}_{\mathbf{k}\mathbf{k}\mathbf{h}} = E[\Delta\underline{\mathbf{a}}_{\mathbf{h}}(\theta_k)\Delta\underline{\mathbf{a}}_{\mathbf{h}}(\theta_k)^T].$$

Denoting  $\underline{\mathbf{g}}_{\mathbf{k}} = \mathbf{V}_{\mathbf{n}}^H\underline{\mathbf{b}}'(\theta_k)$ , we can finally express the mean square angular error for the heterogeneous array as:

$$E[\Delta\theta_k^2] = \frac{\Re\{\underline{\mathbf{g}}_{\mathbf{k}}^H\mathbf{V}_{\mathbf{n}}^H(\mathbf{C}_{\mathbf{k}\mathbf{k}\mathbf{h}}\mathbf{V}_{\mathbf{n}}\underline{\mathbf{g}}_{\mathbf{k}} + \mathbf{D}_{\mathbf{k}\mathbf{k}\mathbf{h}}\mathbf{V}_{\mathbf{n}}^*\underline{\mathbf{g}}_{\mathbf{k}}^*)\}}{2\|\underline{\mathbf{a}}_{\mathbf{h}}(\theta_k)\|^2(\underline{\mathbf{g}}_{\mathbf{k}}^H\underline{\mathbf{g}}_{\mathbf{k}})^2} \quad (30)$$

## 4. NUMERICAL SIMULATIONS

### 4.1 Antenna arrays for HF direction finding

The active loop antenna is the standard sensor for HF direction finding system. Several loop antennas of the same type are classically associated in a circular uniform array for HF direction finding. The sensors are then equi-spaced along a circle and set up with the same orientation on an horizontal ground. This structure is considered in this section as the reference for an homogeneous array.

In the second array which is considered, the antennas are subject to a rotation of  $N_d$  degrees around a vertical axis every two positions within the array so that the structure becomes heterogeneous. The spatial responses of the antennas ( $F_n(\theta)$ ,  $n=1, \dots, NC$ ) are computed with an electromagnetic simulation software (NEC2D) coupled with a predictive model of the polarization emerging from the ionosphere [1]. The ground reflection is taken into account with an estimation of parameters conductivity and permittivity.

### 4.2 Perturbation model

In the expression of the perturbed steering-vector, errors in modulus and phase are separated. For a given angle of arrival (AOA)  $\theta_k$ , the component of index  $n$  in this vector is expressed as:

$$\hat{\mathbf{a}}(\theta_k)_n = (1 + \Delta m_{nk})e^{j\Delta\phi_{nk}}\mathbf{a}(\theta_k)_n \quad (31)$$

where  $\Delta m_{nk}$  and  $\Delta\phi_{nk}$  are respectively the error son modulus and phase of component  $n$ .

The perturbation of the steering-vector can be written as:

$$\Delta\underline{\mathbf{a}}(\theta_k) = \hat{\mathbf{a}}(\theta_k) - \mathbf{a}(\theta_k) = \underline{\mathbf{d}\mathbf{G}_{\mathbf{k}}} \otimes \mathbf{a}(\theta_k) \quad (32)$$

with

$$\underline{\mathbf{d}\mathbf{G}_{\mathbf{k}}} = [(1 + \Delta m_{1k})e^{j\Delta\phi_{1k}} \dots (1 + \Delta m_{Nck})e^{j\Delta\phi_{Nck}}]^T$$

and  $\otimes$  is the Schur-Hadamard product.

In the calculation of angular mean square errors for AOA  $\theta_k$ , matrices  $\mathbf{C}_{\mathbf{k}\mathbf{k}}$ ,  $\mathbf{D}_{\mathbf{k}\mathbf{k}}$ ,  $\mathbf{C}_{\mathbf{k}\mathbf{k}\mathbf{h}}$  and  $\mathbf{D}_{\mathbf{k}\mathbf{k}\mathbf{h}}$  are then expressed as:

$$\mathbf{C}_{\mathbf{k}\mathbf{k}} = \mathbf{P}_1 \otimes \underline{\mathbf{a}}(\theta_k)\underline{\mathbf{a}}(\theta_k)^H \quad (33)$$

$$\mathbf{D}_{\mathbf{k}\mathbf{k}} = \mathbf{P}_2 \otimes \underline{\mathbf{a}}(\theta_k)\underline{\mathbf{a}}(\theta_k)^T$$

$$\mathbf{C}_{\mathbf{k}\mathbf{k}\mathbf{h}} = \mathbf{P}_1 \otimes \underline{\mathbf{a}}_{\mathbf{h}}(\theta_k)\underline{\mathbf{a}}_{\mathbf{h}}(\theta_k)^H$$

$$\mathbf{D}_{\mathbf{k}\mathbf{k}\mathbf{h}} = \mathbf{P}_2 \otimes \underline{\mathbf{a}}_{\mathbf{h}}(\theta_k)\underline{\mathbf{a}}_{\mathbf{h}}(\theta_k)^T$$

with

$$P_1(n,l) = E[(1 + \Delta m_{nk})e^{j\Delta\phi_{nk}} - 1][(1 + \Delta m_{lk})e^{j\Delta\phi_{lk}} - 1]^*$$

$$P_2(n,l) = E[(1 + \Delta m_{nk})e^{j\Delta\phi_{nk}} - 1][(1 + \Delta m_{lk})e^{j\Delta\phi_{lk}} - 1]$$

For the numerical simulations which are presented in the next section, the assumptions concerning the perturbations are:

- independent perturbations of two different sensors
- $\Delta m_n$  and  $\Delta g_n$  mutually independent for a given sensor
- $\Delta m_n$  and  $\Delta g_n$  have zero mean values

### 4.3 Comparison of robustness for two structures

The two circular arrays are characterized by geometrical parameters: number of sensors  $NC=10$ , diameter of each loop  $d=1.3m$ , array radius  $R=20m$ , inter-element rotation  $Nd=20^\circ$  (heterogeneous case).

The scenario of the reception involves  $NS=2$  signals with a carrier frequency  $f_0=10$  MHz, impinging on the arrays with azimuth of arrival  $Az_1=40^\circ$  and  $Az_2=50^\circ$ . The common elevation of arrival is supposed to be known. The signal to noise ratio SNR is equal to 12 dB and the B.T product (bandwidth by difference of group delays) equal to 3.

Uncertainties with the same magnitude are supposed to affect the two array manifolds. For each sensor, the phase variation is uniformly distributed on the interval  $[-\delta\phi_{\max}; +\delta\phi_{\max}]$  with  $\delta\phi_{\max} = 15^\circ$ . The modulus error is also uniformly distributed in an interval  $[-\delta m_{\max}; +\delta m_{\max}]$ ,

$\delta m_{\max}$  being a variable parameter in the numerical simulation adjustable from 0 up to 40%.

The corresponding angular rms error affecting the azimuth estimation is computed for the 2 types of array according to relations (30) and (38). The results are plotted on figure 3 (homogeneous) and figure 4 (heterogeneous).

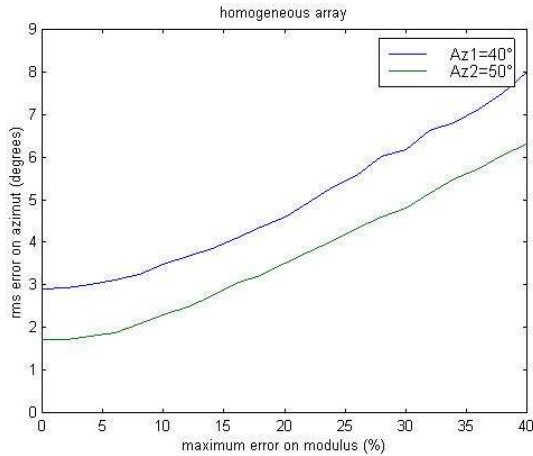


Fig. 3. RMS error of the angular estimation (homogeneous array)

For a given level of perturbation, the angular error is smaller when the direction finding is implemented on the heterogeneous array. This observation remains systematically if the parameters of the scenario are modified in a large scale: number of incoming waves, directions of arrival, carrier frequency, signal to noise ratio.

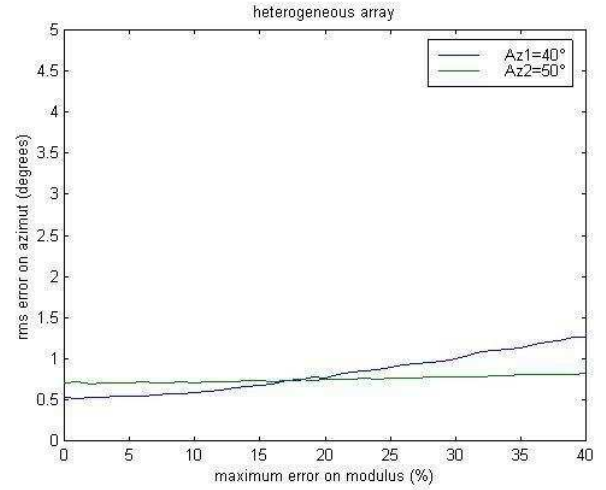


Fig. 4. RMS error of the angular estimation (heterogeneous array)

Consequently, the heterogeneous array appears more robust than the equivalent homogeneous structure. This result however assumes the same magnitude of perturbation in the two array manifolds. This point needs further work as the model of steering-vector for the heterogeneous case requires the computation of the sensor spatial responses in addition to the classical geometrical phases: the level of uncertainty may increase with the number of parameters present in the vector.

## 5. CONCLUSION

This paper investigates the robustness of the MUSIC direction finding algorithm relatively to modelling errors for two structures of arrays (homogeneous or heterogeneous). The calculation is limited to first order terms of the perturbed vectors or matrices. For the numerical simulations, examples involve trans horizon applications in the HF band (3-30 MHz). The array responses are calculated thanks to an electromagnetic simulation software, using a deterministic model of the polarization emerging from the ionosphere and taking the ground effect into account. Computations have been carried out with a standard scheme of perturbations. The results indicate a better robustness of the heterogeneous array for a given level of uncertainty.

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