On the stabilizing and destabilizing effects of damping in a non-conservative pin-disc system

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J.-J. Sinou, L. Jezequel

Laboratoire de Tribologie et Dynamique des Systèmes, Equipe Dynamique des Structures et des Systèmes, Ecole Centrale de Lyon, Ecully, France

Summary. Friction-induced vibration due to mode coupling is a major concern in a wide variety of mechanical systems. Though there are numerous papers on both linear and non-linear stability analyses of dynamical systems with friction-induced vibrations, the effects of damping on the evolution of the stable–unstable regions is not yet fully understood. The purpose of this work is to study a pin-disc model describing a typical friction-induced vibration due to a geometric constraint in order to illustrate the effects of damping on linear stability analyses. The results not only illustrate the influence of damping for determining the stable and unstable zones, but they also indicate that the "most efficient damping factor of the coupling modes" is an important factor to be taken into account. The physical mechanism causing this change in instability as a consequence of the damping factor is also suggested to help avoid bad design.

1 Introduction

Recently, the effect of damping on the stability of distributed non-conservative systems with friction has been investigated by some researchers [1], [2] by considering theoretical systems with 2 *dof*. For example, Hoffmann and Gaul [1] studied the effects of damping on mode coupling or coalescence between two modes in friction induced oscillations. They demonstrated that linear viscous structural damping changes the mode coupling and that an imperfect merging of modes may be observed. They developed a feedback-loop formalism in order to allow a more detailed understanding of the underlying mechanical processes. Nevertheless, they concluded that viscous instability in the field of friction-induced oscillations is in itself a surprising phenomenon. Moreover, Sinou and Jézéquel [2] indicated that neglecting damping in a stability analysis or adding damping on only one part of the system may lead to a misunderstanding of the phenomenon referred to as mode coupling or coalescence between two modes. This effect of damping, called the destabilization paradox, was previously described and theoretically explained by Kirillov and Seyranian [3], [4].

Correspondence: Jean-Jacques Sinou, Laboratoire de Tribologie et Dynamique des Systèmes UMR CNRS 5513, Equipe Dynamique des Structures et des Systèmes, Ecole Centrale de Lyon, Bâtiment E6, 36 Avenue Guy de Collongue, 69134 Ecully, France
e-mail: jean-jacques.sinou@ec-lyon.fr

Despite these previous works, problems generated by the destabilization paradox have not yet been considered for physical–mechanical systems subjected to friction-induced vibration such as pin-disc systems. However, friction-induced oscillations due to mode coupling (i.e. coalescence between modes) occur in a wide variety of mechanical systems and are known to lead to very annoying phenomena such as brake squeal [5]–[11].

Given this fact, we will seek to describe the effects of damping on a pin-disc system in order to avoid bad design in mechanical systems subject to flutter instability. The main objective of this paper will be to clarify the influence of structural damping for the well-known pin-disc system because this system includes all the essential features for generating friction-induced vibrations due to geometric constraint and for showing the role of damping. First, the mechanical system will be presented and the associated equations of motion will be described. Second, the effects of damping on the size of the stable and unstable regions will be investigated. It will be shown that the "most efficient damping factor of the coupled modes" is one of the most important parameters to be taken into account for pin disc systems and mechanical systems subject to flutter instability.

2 Description of the mechanical model

In this study, a dynamic model of a beam rubbing against a rotating disc with a constant friction coefficient is considered, as indicated in Fig. 1. In this case, flutter instability is governed by modal interaction between the cantilever and disc bending modes and variations of normal and tangential contact forces, due to the configuration of the system based on "geometrically-induced vibrations" (sprag-slip mechanism [10], [12]). This mechanical system [13] is composed of a disc, rubbed by a flexible beam, inclined at angle θ in relation to the plane of the disc. As illustrated in Fig. 1, this dynamic system is composed of only three vibration modes: the flexure of the disc (defined by the physical parameters (m_3, c_3, k_3) and the associated displacement X_3), the stretching–compressing of the beam (defined by the physical parameters (m_1, c_1, k_1) and the associated displacement X_1), and the flexure of the beam (defined by the physical parameters (m_2, c_2, k_2) and the associated displacement X_2). The governing equations of motion of this system may be written as follows:

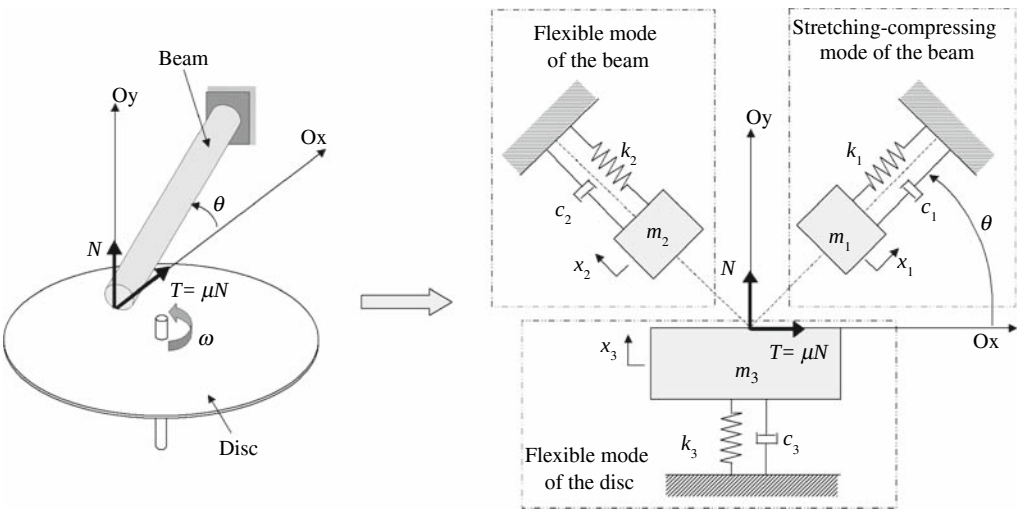

Fig. 1. Mechanical model of a beam rubbing against a disc

$$
\begin{aligned}
&m_1\ddot{X}_1 + c_1\dot{X}_1 + k_1X_1 = N\sin\theta + T\cos\theta,\\
&m_2\ddot{X}_2 + c_2\dot{X}_2 + k_2X_2 = N\cos\theta - T\sin\theta,\\
&m_3\ddot{X}_3 + c_3\dot{X}_3 + k_3X_3 = -N.
\end{aligned} \tag{1}
$$

Assuming constant pin and disc contact, the following geometric constraint may be obtained: $X_3 = X_1 \sin\ \theta + X_2 \cos\ \theta$. Eliminating X_3 and N and assuming Coulomb's law $T = \mu N$, the equations of motion (1) may be reduced to the form

$$
\mathbf{M}\ddot{\mathbf{X}} + \mathbf{C}\dot{\mathbf{X}} + \mathbf{K}\mathbf{X} = \mathbf{0} \tag{2}
$$

with $\mathbf{X} = (X_1\ X_2)^{\mathrm{T}}$. $\ddot{\mathbf{X}}$, $\dot{\mathbf{X}}$ and $\mathbf{X}$ are the acceleration, velocity, and displacement response two-dimensional vectors of the degrees-of-freedom, respectively. The mass matrix $\mathbf{M}$, the damping matrix $\mathbf{C}$, the stiffness matrix $\mathbf{K}$ are given by

$$
\mathbf{M} = \begin{bmatrix} m_1 + m_3\sin\theta\cdot f(\theta,\mu) & m_3\cos\theta\cdot f(\theta,\mu) \\ m_3\sin\theta\cdot g(\theta,\mu) & m_2 + m_3\cos\theta\cdot g(\theta,\mu) \end{bmatrix}, \tag{3}
$$

$$
\mathbf{C} = \begin{bmatrix} c_1 + c_3\sin\theta\cdot f(\theta,\mu) & c_3\cos\theta\cdot f(\theta,\mu) \\ c_3\sin\theta\cdot g(\theta,\mu) & c_2 + c_3\cos\theta\cdot g(\theta,\mu) \end{bmatrix}, \tag{4}
$$

$$
\mathbf{K} = \begin{bmatrix} k_1 + k_3\sin\theta\cdot f(\theta,\mu) & k_3\cos\theta\cdot f(\theta,\mu) \\ k_3\sin\theta\cdot g(\theta,\mu) & k_2 + k_3\cos\theta\cdot g(\theta,\mu) \end{bmatrix} \tag{5}
$$

with $f(\theta,\ \mu) = \sin\ \theta + \mu\cos\ \theta$ and $g(\theta,\ \mu) = \cos\theta - \mu\sin\theta$. The base parameters are [13]: pin masses $m_1 = 0.177$ kg and $m_2 = 0.088$ kg; disc mass $m_3 = 2$ kg; natural frequency of the stretching–compressing mode of the beam $\omega_1 = 2\pi \times 8820$ rad s^{-1}, natural frequency of the first flexible mode of the beam $\omega_2 = 2 \times 658$ rad s^{-1}, natural frequency of the first flexible mode of the disc $\omega_3 = 2\pi \times 440$ rad s^{-1} (i.e. $k_i = m_i\omega_i^2$). The damping ratios are chosen to be the same in the two perpendicular directions of the beam $\zeta_1 = 0.01$ (i.e. $c_i = 2\zeta_1\sqrt{m_ik_i}$ for *i* = 1, 2); and $\zeta_2 = 0.01$ for the disc (i.e. $c_3 = 2\zeta_2\sqrt{m_3k_3}$); angle between the disc and the beam $\theta = 1.4$ rad.

3 Stability analysis and effect of damping on unstable regions

To investigate the effects of physical parameters on the stability of the equilibrium point that is equal to zero for the pin-disc system, the eigenvalues λ of the system can be found by solving the standard equation

$$
\det\left(\lambda^2\mathbf{M} + \lambda\mathbf{C} + \mathbf{K}\right) = 0. \tag{6}
$$

If we denote one or more eigenvalues λ having a positive real part, the mechanical system is unstable and the associated imaginary part of this eigenvalue defines the frequency of the unstable mode. Moreover, it may be noted that numerical methods for bifurcation analysis [14] and bifurcation software available for such systems [15] should be used for the interactive bifurcation analysis of dynamical systems.

Figures 2, 4 and 5 illustrate some general, well-known information by varying parameters; an increase of the friction coefficient μ may induce instability and increases the unstable regions; within a certain range of the angle θ of orientation between the beam and the disc, we observe that flutter instability occurs. Increasing the friction coefficient increases this range of the angle θ.

Table 1 indicates the value of the Hopf bifurcation point, defined by $\mathrm{Re}(\lambda(\mu))|_{\mu=\mu_0} = 0$ and $\frac{d}{d\mu}(\mathrm{Re}(\lambda(\mu)))\Big|_{\mu=\mu_0} \neq 0$, and the value of the associated unstable frequency for various damping ratios

Table 1. Determination of the Hopf bifurcation point and the unstable associated frequency versus the friction coefficient

Damping ratio coefficients		Hopf bifurcation point μ_0	Unstable frequency (Hz)
ζ_1	ζ_2		
0	0	0.474	1,197
0.01	0.01	0.468	1,491
0.001	0.01	0.44	1,974
0.01	0.001	0.407	688

ζ_1 and ζ_2. It appears that damping may increase the unstable zone, and if we use only the undamped mechanical system for determining the stable and unstable regions, the information about the stability of the equilibrium point of the mechanical system may be erroneous.

To further understanding the effects of damping, Fig. 2 shows the stable and unstable regions for various damping ratios ζ_1 and ζ_2 and various friction coefficients while keeping the angle θ at its

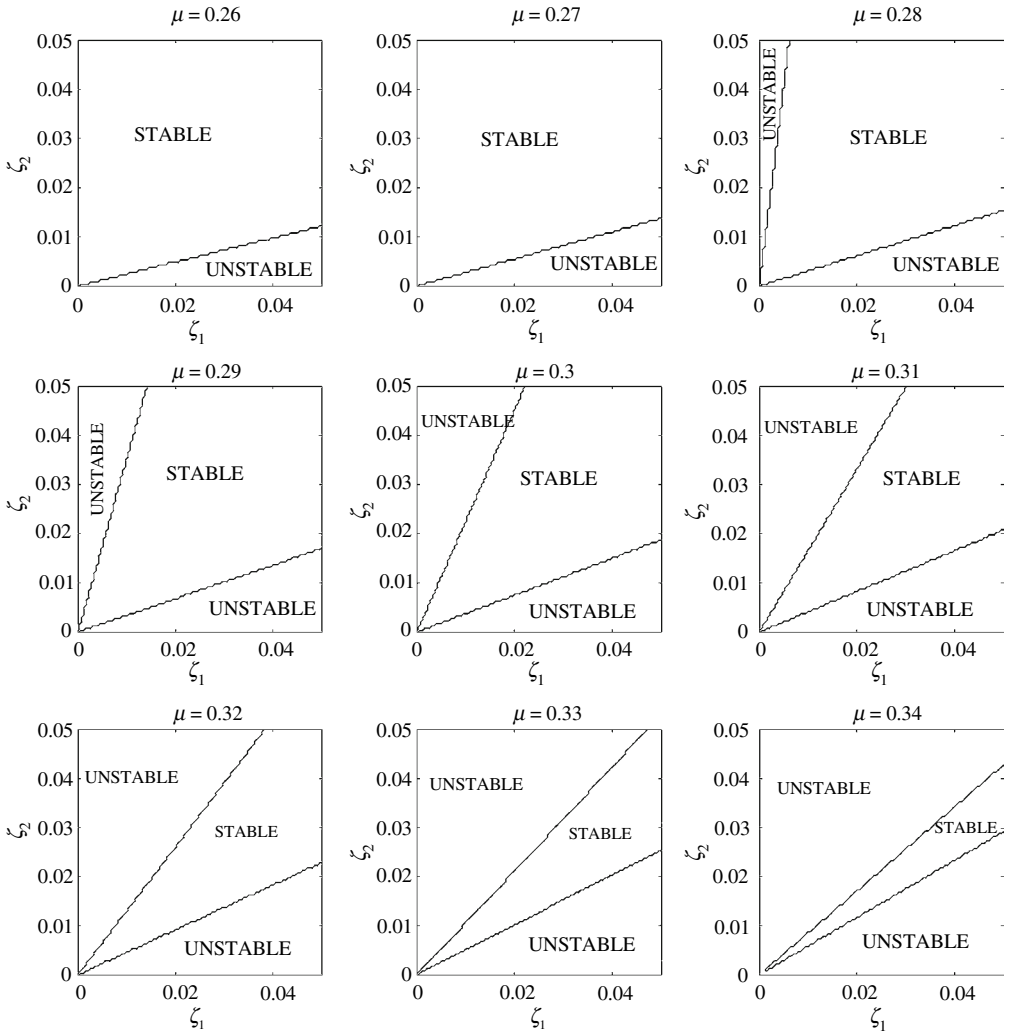

Fig. 2. Instability region (with regard to variables ζ_1, ζ_2 and μ, with $\theta = 1.2$ rad)

base value. It clearly appears that increasing damping in both the disc and the beam have beneficial effect and increase the stable region. Moreover, Fig. 2 suggests that adding damping in only the beam or the disc may induce flutter instability.

Figure 3 illustrates the influence of the factor ζ_1/ζ_2 for various values of the friction coefficient μ and the angle θ. Considering only these results, the varying effects of ζ_1/ζ_2 show that the predictions of stable and unstable regions are complex; increasing or decreasing ζ_1/ζ_2 may increase the unstable region. However, it may be observed that the smallest unstable region is obtained for the ratio damping ζ_1/ζ_2 near 1.5.

To better assess the influence of the damping factor ζ_1/ζ_2, Fig. 4 shows the effects of varying ζ_1/ζ_2 and friction coefficient μ while keeping the angle θ at its base value. Figure 5 then illustrates the effects of varying ζ_1/ζ_2 and angle θ while keeping the friction coefficient μ at its base value. The general observations from these parametric studies are:

- for all values of ζ_1/ζ_2, we note the existence of a critical value of the friction coefficient μ below which the system becomes unconditionally stable,

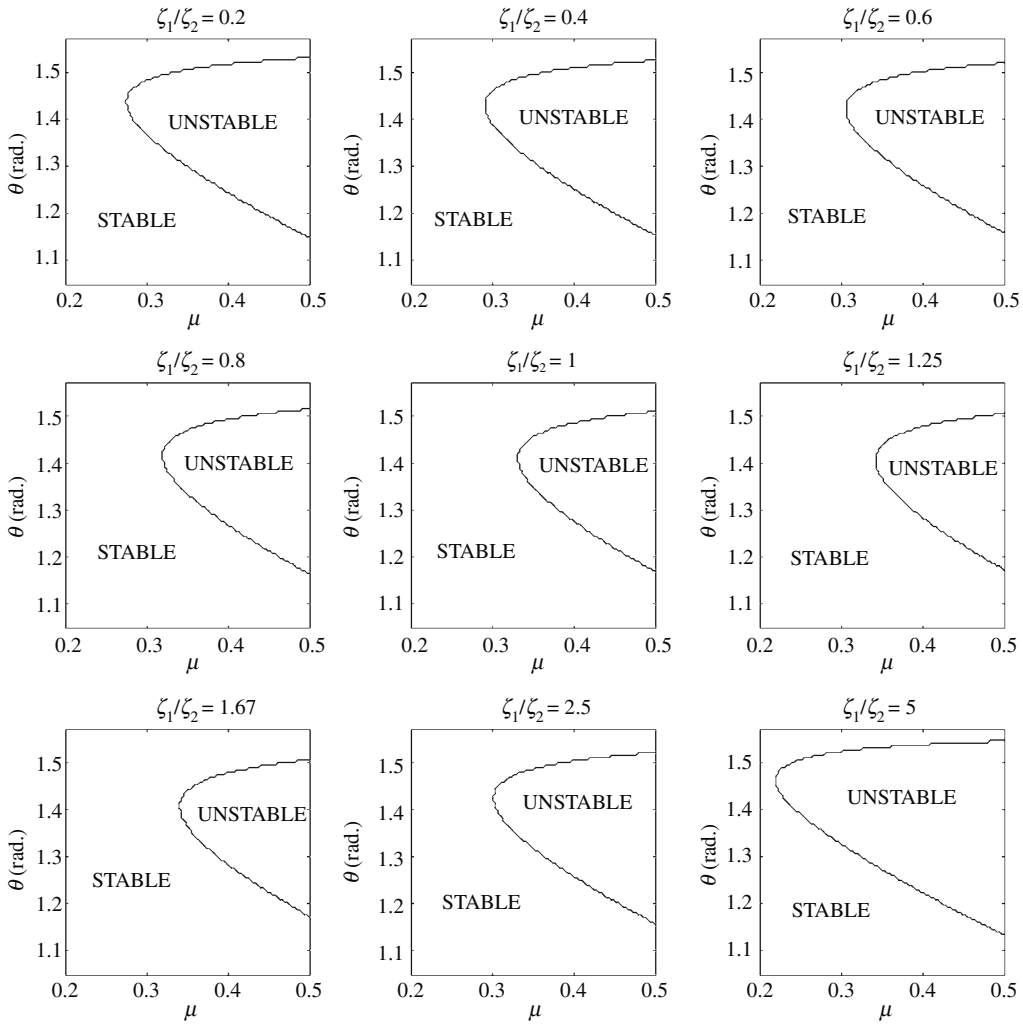

Fig. 3. Instability region (with regard to variables μ, θ and ζ_1/ζ_2, with $\zeta_1 = 0.01$)

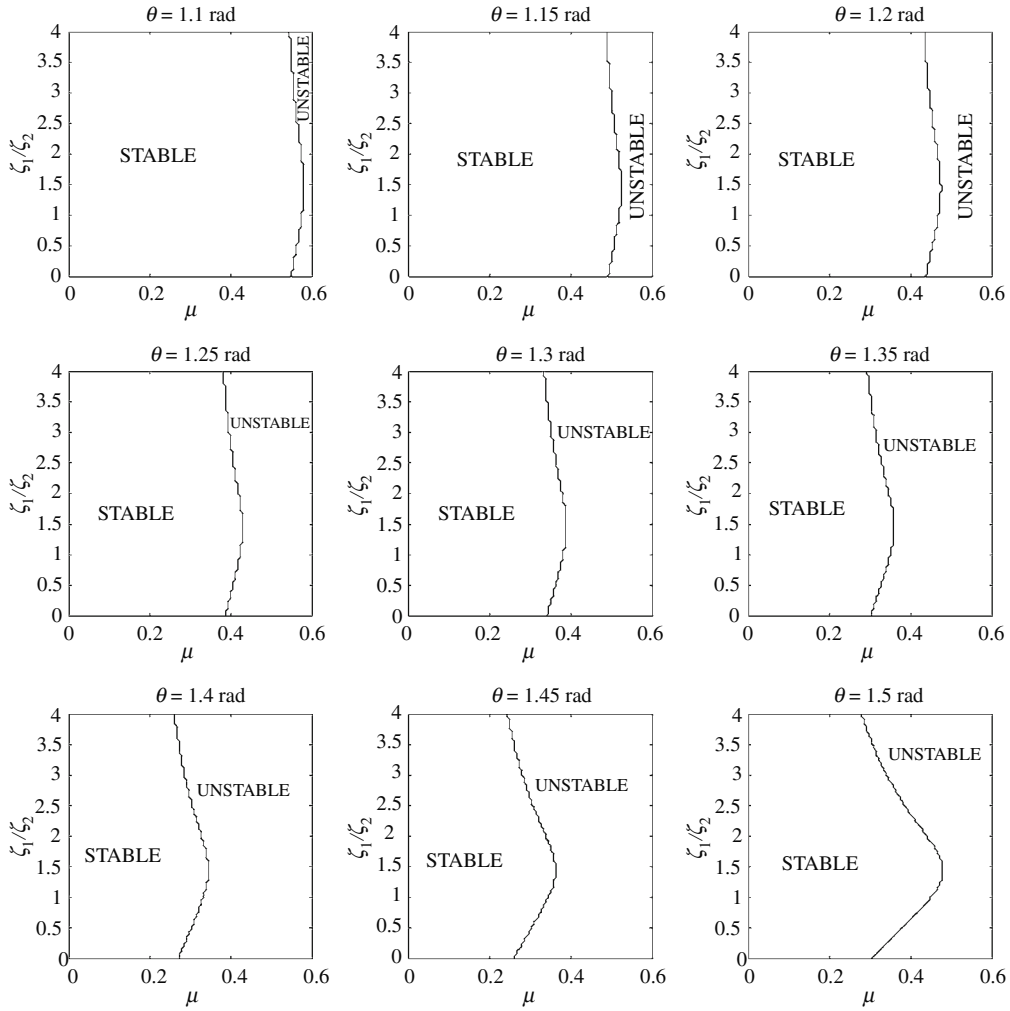

Fig. 4. Instability region (with regard to variables μ, ζ_1/ζ_2 and θ, with $\zeta_1 = 0.01$)

- for all the values of ζ_1/ζ_2, within a certain range of the angle θ, the system is unstable,
- increasing or decreasing only the damping ratio ζ_1 or ζ_2 may increase the unstable region.

These results indicate an additional significant fact: there exists a value of ζ_1/ζ_2 for which the smallest unstable region is observed, regardless of the values of the friction coefficient μ and the angle θ. This value is estimated near $\zeta_1/\zeta_2 = 1.5$. As explained previously, Fig. 3 also describes values of ζ_1/ζ_2 near 1.5 for the largest stable region. Throughout various computational simulations, it has been observed that the value of the most efficient damping factor ζ_1/ζ_2 appears to be the same for various values of ζ_1 or ζ_2.

Finally, we have shown the importance of the damping ratio in order to predict the stable and unstable regions. The effects of damping appear to be a key factor for flutter instability. These results not only indicate that increasing or decreasing damping of either the disc or the beam alone may have dangerous effects on the stability of the equilibrium point of the pin-disc system, but also show

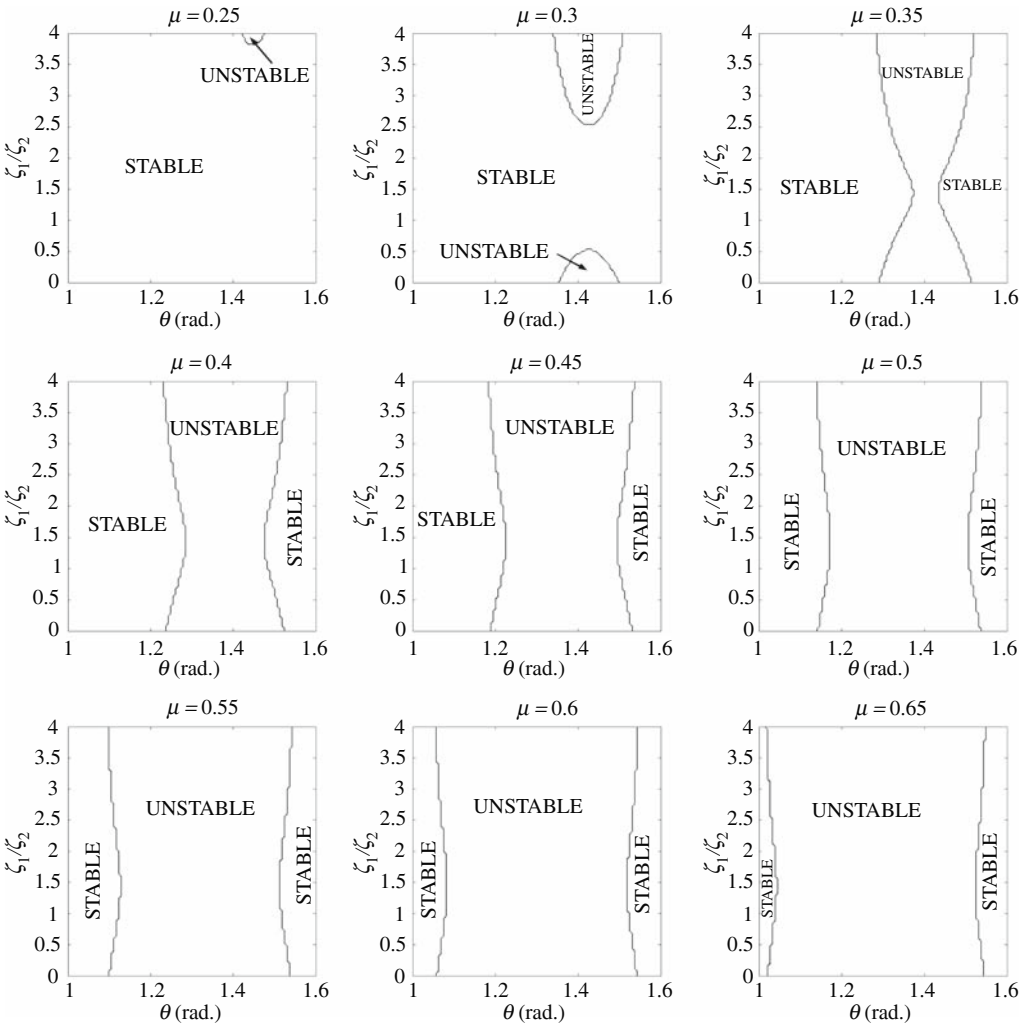

Fig. 5. Instability region (with regard to variables θ, ζ_1/ζ_2 and μ, with $\zeta_1 = 0.01$)

the important fact that the system is more stable when a particular value of the damping factor ζ_1/ζ_2 is used ($\zeta_1/\zeta_2 \approx 1.5$ for this mechanical model).

In order to identify the physical mechanics, which are causing instability as a consequence of the damping ratio ζ_1/ζ_2 in the study above, we recall that the effect of damping depends on whether energy dissipation occurs in the different parts of the mechanical system. This is a very well-known phenomenon observed in the field of rotor-dynamics where the non-rotating damping is always stabilizing, while the damping associated with the rotating parts has a destabilizing effect [16]. In this study, the introduction of unequal damping coefficients for each part of the mechanical system changes the merging motion for the mechanical system, as illustrated in Fig. 6. We recall that flutter instability is composed of two modes of which only one is unstable. By adding unequal damping, the lesser-damped mode defines the potentially unstable mode and the other mode corresponds to the stable one. Therefore, by introducing various damping ratios, the merging scenario and the unstable mode may change, as indicated in Fig. 6 where the

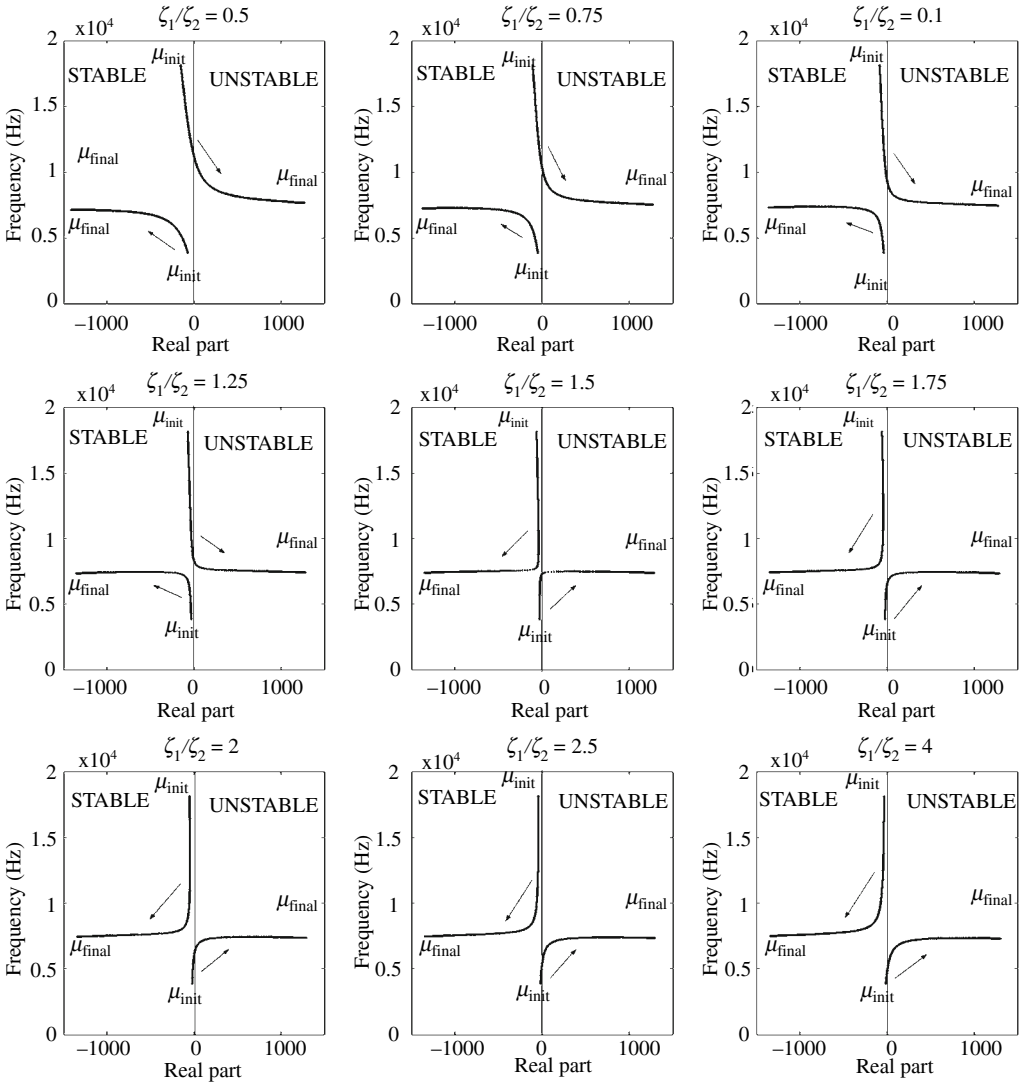

Fig. 6. Evolution of the stable and unstable modes versus the damping ratio ζ_1/ζ_2

merging phenomenon is observed for ζ_1/ζ_2 near 1.25 and 1.5. Effectively, if the damping ratio ζ_1/ζ_2 is lower than 1.5, the unstable mode is the mode that has the higher frequency. If the damping ratio ζ_1/ζ_2 is bigger than 1.5, the unstable mode is the mode that has the lower frequency.

Moreover, the results that have been previously shown in Fig. 2 are due to this phenomenon. When the damping factor ζ_1 is much weaker than the damping factor ζ_2, the stretching–compressing of the beam [defined by the physical parameters (m_1, c_1, k_1)] corresponds to the unstable mode whereas if the damping factor ζ_2 is much lower than the damping factor ζ_1, the unstable mode becomes the flexure of the beam [defined by the physical parameters (m_2, c_2, k_2)]. Moreover, Fig. 2 indicates that the system is always stable if the damping ratio ζ_1/ζ_2 is approximately 1.5. It may be noted that this optimal value of the damping ratio $\zeta_1/\zeta_2 = 1.5$ corresponds physically to the case of adding equal structural damping (i.e. $c_1 = c_2$ with $c_i = 2\zeta_1\sqrt{m_i k_i}$ for $i = 1,\ 2$). This last

observation is also shown in Figs. 3, 4 and 5 where the largest stable area is obtained approximately for $\zeta_1/\zeta_2 = 1.5$.

Physically speaking, when the damping ratio ζ_1/ζ_2 reaches the value 1.5, the unstable mode becomes stable (and the stable mode becomes unstable). So before the damping ratio ζ_1/ζ_2 exceeds a certain value (approximately $\zeta_1/\zeta_2 = 1.5$), adding damping has the effect of delaying the energy source driving instability (i.e. damping is added on the unstable mode). After the damping ratio ζ_1/ζ_2 has exceeded this optimal value, the system becomes more unstable again (i.e. the unstable area grows again). This is only due to the fact that the stable and unstable modes are reversed. Now, damping is added to the stable mode and the system becomes comparable to a less constrained system (i.e. damping gives the mechanical system additional freedom to extract the energy source driving instability).

4 Conclusion

A dynamic model describing elementary mechanisms of friction-induced vibrations due to a geometric constraint is used to investigate the effects of damping on flutter instability. This study illustrates that damping is one of the most important parameters for friction-induced vibrations. Moreover, neglecting damping in a stability analysis may result in worse design and a misunderstanding of flutter instability in a mechanical system. Therefore, if too much damping is added to only one part of the mechanical system, instability may occur. Throughout the computational simulations, it has be found that there exists a damping factor ζ_1/ζ_2 which allows us to obtain the largest stable region with regard to the effects of damping.

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