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**The role of surface and volume defects in the fracture of glass  
under quasi-static and dynamic loadings**

by

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# **The role of surface and volume defects in the fracture of glass under quasi-static and dynamic loadings**

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## **Abstract**

In this paper, the role of surface and volume defects on fracture in soda-lime glass is analyzed when samples are submitted to quasi-static or dynamic loadings. To investigate fracture, different experiments are carried out, namely, quasi-static compression of glass spheres and edge-on-impacts. The first test series aims at studying crack initiation. Different surface treatments are performed to study their influence on the failure load, from which it is concluded that initiation of cracks occurs in the vicinity of the contact surface. The second series is concerned with the examination of crack patterns under dynamic loadings with two different strikers (i.e., soft/flat and hard/perforating projectiles). Crack initiation under dynamic load histories is investigated near and far from the impact zone and it is concluded that it cannot take place within the volume, except in a very small zone close to the impact point. Conversely, initiation of damage from the surface, at a location far from the impact point, is possible and clearly present with a soft projectile.

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**PACS.** 81.05.Kf Glasses, 62.20.Mk Fatigue, brittleness, fracture and cracks, 83.50.Tq Wave propagation, shocks, fracture and healing.

## **Keywords:**

Compression test, Dynamic fragmentation, Edge-on-impact, Soda-lime glass, Weibull model.

## Introduction

The use of glass in transparent armors against ballistic threats (e.g., windshields) requires analyses of its response to impact. For ceramic materials, it could be shown that the cracking pattern observed in edge-on-impact experiments [1,2,3] could be reproduced by a damage model whose parameters are related to those of a Weibull model [4] tuned under quasi-static loading condition and modeling the distribution of volume defects [5]. On the other hand, it is well known that under quasi-static loading, defects on the surface of glass play a key role in the fracture process [6,7], whereas the high level of stresses induced by an impact may nucleate volume defects, which seldom occurs under quasi-static loading [8,9], except under particular conditions such as high temperature tests on glass fibers [10]. According to Kshinka et al. [11], quasi-static compression tests of glass spheres may allow for nucleation sites located in the bulk of samples.

The work reported in the present paper aims at:

- first, checking whether quasi-static compression experiments on glass spheres can indeed give a volume defect distribution useful for the damage modeling of glass under ballistic impact;
- second, observing the actual damage process in glass undergoing edge-on impact experiments and obtaining insights into the respective role of surface and volume defects in the damage process.

## I. Quasi-static compression of glass spheres

First, let us consider a so-called quasi-static case. Experiments on glass spheres under “point load” compression are used to investigate the location of crack initiation and the scatter in ultimate failure loads related to the Weibull properties.

### Experiments

Glass spheres are compressed between two anvils made of high strength steel. Compression tests were performed with an INSTRON 4505 servo-hydraulic testing machine. On average, more than 12 experiments per sphere diameter were carried out. Sphere diameters ranging from 1 mm to 8 mm were tested (Table 1). The stroke velocity was controlled so that the load was applied during 2 min per sample (i.e., for the small samples it was of the order of 50  $\mu\text{m}/\text{min}$  and for the large ones of the order of 150  $\mu\text{m}/\text{min}$ ). Aluminum foils are inserted between the anvils and the spheres. When failure occurs, the load decreases instantaneously and the test is stopped. The ultimate strength corresponds to the maximum load recorded during the test. Kshinka et al. [11] performed such type of tests on glass spheres and concluded that failure occurred within the bulk of the material. This explanation is possible since this type of experiment on cylinders (also referred to as Brazilian test [12]) is used to determine the tensile strength of brittle samples [13]. When applied to glass, this test would allow for the determination of the bulk failure properties that correspond to the intrinsic strength of glass.

To check this hypothesis, different diameters of spheres ranging from 1 mm to 8 mm are used. The spheres are made of commercial soda-lime glass. Some spheres are chemically tempered thereby inducing compressive stresses in the vicinity of the surface. It allows us to evaluate the role of the stress state of the surface on the failure level. Glass spheres are put in a  $\text{KNO}_3$  bath and heated at different temperatures. The first treatment is performed during 24

hours at 460°C (CT1) whereas the second is carried out during 48 hours at 405°C (CT2). The maximum residual stress induced by the heat treatment is a compression at the surface of the order of 600 MPa for the CT1 treatment decreasing to zero over a depth of the order of 30µm, which is related to the diffusion length, and 200 MPa for the CT2 treatment over 100µm. These estimates are based on photoelastic measurements on flat glass samples of approximately the same chemical composition performed with a stratorefractometer [14].

Other experiments were performed by changing the contact conditions (Table 2):

- no aluminum foil was inserted between flat anvils and the spheres,
- spherical anvils were used.

## **Results**

The main results are summarized in Tables 1 and 2. When failure occurs, the ultimate load is recorded and, generally, the glass spheres are totally comminuted (i.e., reduced into a very thin powder). Some spheres, which are not comminuted because failure occurred prematurely, have been analyzed. In this case, the strength was too small and the stored elastic energy was not sufficient to fully comminute the sample. Figure 1 shows that failure occurs just under the contact surface and a Hertz theory reveals that high tensile stresses exist at the loss of contact [15].

## **Discussion**

Since failure loads are very different for as-received and tempered spheres (Table 1), one may argue that the surface plays a major role in the failure process. Chemical tempering does not affect significantly the bulk of spheres and the same ultimate load should have been observed, had fracture been caused by volume defects.

Furthermore, the tensile stresses at the loss of contact between anvils and spheres seem to cause cracks to initiate around this point. When aluminum foils are used, a significant increase in ultimate load is obtained. This constitutes yet another proof that failure is due to the contact zone. The ultimate load is even higher when the contact surface is larger, which corresponds to spherical anvils. We can also note that an aluminum foil “softens” the contact conditions and has a similar effect as spherical anvils.

## Probabilistic Analysis

In the following study, it is assumed that damage is caused by tensile loading (i.e., mode I cracking). The behavior of soda-lime glass under a tensile loading is elastic and brittle. The cumulative failure probability  $P_F$  is modeled by a Weibull law [4]

$$P_F = 1 - \exp \left[ - \frac{Z_{eff}}{Z_0} \left( \frac{\sigma_F}{\sigma_0} \right)^m \right], \quad (1)$$

where  $Z_{eff}$  is the effective volume or surface [16],  $m$  the Weibull modulus,  $\sigma_0$  the scaling stress relative to a reference volume or surface,  $Z_0$ , and  $\sigma_F$  the failure stress, i.e., the maximum value within a volume or a surface of the local maximum principal stress. Under quasi-static loading conditions, most failures of glass bodies of macroscopic size are caused by surface defects. However, for instance Gy and Guillemet [10] have shown that failure can also be initiated by bulk defects for glass fibers at 610°C. The failure scatter is very different in both cases [10,17] and typical Weibull parameters are given in Table 3. A classical way of determining the Weibull parameters is to rewrite Eq. (1) in the so-called Weibull coordinate system

$$\ln \left[ \ln \left( \frac{1}{1-P_F} \right) \right] = m \ln(\sigma_F) + A(m, Z_0 \sigma_0^m), \quad (2)$$

where  $A(m, Z_0 \sigma_0^m)$  is a parameter dependent on the Weibull parameters and the effective volume or surface. The problem is to evaluate the failure stress due to the compression test and the corresponding effective volume or surface. It has been shown that, if all bodies behave elastically, contact stresses can be described by a Hertz solution [15,18,19]. The present analysis aims at deriving the main dependence of each quantity with the relevant parameters of the problem, following the spirit of the method discussed by Tsoungui et al. [20]. The contact area is assumed to be a circle of radius  $a$  and the maximum contact pressure  $P_0$  is scaled by

$$P_0 \propto \left( \frac{E^2 F}{R^2} \right)^{1/3}, \quad (3)$$

where  $F$  is the failure load,  $E$  the equivalent Young's modulus, and  $R$  the radius of the glass sphere. All stresses are proportional to the maximum pressure and, in particular, the failure stress  $\sigma_F$  is a radial stress on the edge of the contact circle. Classical results [15] show that

$$\sigma_F \approx 0.16 P_0. \quad (4)$$

According to the Weibull theory, the average failure stress  $\bar{\sigma}_F$  is related to the effective volume or surface  $Z_{eff}$  by [16]

$$\bar{\sigma}_F \propto \sigma_0 \left( \frac{Z_0}{Z_{eff}} \right)^{1/m}. \quad (5)$$

Let us now assume that  $Z_{eff}$  is scaled by  $a^n$ , where  $a$  is the contact radius between the sphere and the anvils and  $n = 2$  or  $3$ . In the present analysis, only the defects located in the vicinity of



the contact surface are considered. However, they are not necessarily located at the surface. An estimate of the contact radius  $a$  is given by

$$a \propto \left( \frac{FR}{E} \right)^{1/3}, \quad (6)$$

so that the failure load, and its mean  $\bar{F}$ , are scaled by

$$\bar{F} \propto R^{\frac{2m-n}{m+n}}. \quad (7)$$

Consequently, if one knows the change of the mean failure load with the sphere diameter, then one can relate the power to the underlying Weibull modulus. Figure 2a shows, in a log-log plot, that reasonable power-law dependence is found between the average failure load and the sphere diameter. A power of 1.44 is obtained for as-received spheres, leading to a Weibull modulus of the order of 9 when  $n = 2$  and 13 when  $n = 3$ , respectively.

For a given sphere diameter, say 1mm, let us analyze the scatter. Figure 2b shows, in Weibull plot, the failure load versus the cumulative failure probability. We can calculate the scaling of the term in brackets of Eq. (1)

$$\frac{Z_{eff}}{Z_0} \left( \frac{\sigma_F}{\sigma_0} \right)^m \propto F^{\frac{m+n}{3}}. \quad (8)$$

Equation (8) shows that for a given radius  $R$ , the apparent Weibull modulus of the failure load versus the failure probability is not identical to the underlying Weibull modulus, but is about one third of the latter, because of the load dependence of the contact zone. This constitutes yet another way of identifying the Weibull modulus of the material. In the present experiments, apparent Weibull moduli are found to be of the order of 5 (Fig. 2b), leading to a Weibull modulus of 13 when  $n = 2$  and 12 when  $n = 3$ . These values are close to those found by using

the scaling of the failure load with the sphere radius. The most consistent values are obtained with  $n = 3$ .

Moreover, by using Eqs. (4) and (5), the maximum tensile stress level can be calculated for as-received spheres as well as tempered ones

$$\bar{\sigma}_F \approx 0.1 \left( \frac{FE^2}{R^2} \right)^{1/3} . \quad (9)$$

By using the superposition principle, we can assume that the difference between the failure stresses is due to the residual stresses induced by the tempering process. Since the maximum stresses are on the edge of the contact circle, we can compare these values with the theoretical residual stresses (Fig. 3). For the CT1 treatment, an estimate of the average residual stress is  $440 \pm 50 \text{MPa}$  and  $185 \pm 60 \text{MPa}$  for CT2. Even though the scatter is high, one can note a significant effect of chemical tempering on the initiation level in glass spheres. Consequently, initiation occurred in a small zone around the contact zone (i.e., a Hertz type of failure). Furthermore, these stress levels can be compared to the previous estimates (i.e., maximum value:  $600 \text{MPa}$  for CT1 and  $200 \text{MPa}$  for CT2). The agreement can be considered as satisfactory and is not inconsistent with fracture initiation close to the contact surface, given the uncertainty in the estimation of residual stresses for chemically tempered glass spheres. All these results allow us to conclude that failure initiation is located close to the contact zone.

The Weibull moduli found here are high compared to those found in the literature for surface defects (Table 3). Moreover, this evaluation of the Weibull modulus is likely to be an underestimate since other causes, besides the flaw size distribution, contribute to the apparent Weibull modulus related to the scatter of failure loads:

- scatter in the radius of spheres,
- the glass samples are not perfectly spherical,
- measurement accuracy.

To explain this apparent high value, we can compare a 4-point flexural test with a compression test. If one stops bending just before rupture and observes the surface in tension, one does not see any cracks. Yet, in the compression tests, we have observed Hertz microcracks just under the contact zone. As loading increases, some stable cracks then develop and change the flaw distribution. The distribution then becomes narrower as the load increases, and this could explain why a high value was found for  $m$ . The above-estimated Weibull modulus characterizes the distribution when an unstable crack forms, whereas the values currently reported (Table 3) relate to a case where cracks propagate immediately in an unstable manner. The same type of explanation was proposed to rationalize that the critical load for Hertzian cracking on flat glass samples does not depend significantly on the initial quality of the glass surfaces [21].

Besides, if one assumes that failure is due to volume defects, the maximum tensile stress in the bulk of spheres is given by [22]

$$\sigma_F \approx 0.22 \frac{F}{R^2} \quad (10)$$

and the effective volume can be assumed to be of the order of  $Z_{eff} \approx 2R^3$  so that the average ultimate load  $\bar{F}_u$  becomes [20]

$$\bar{F}_u \approx 4.55 R^{\frac{2m-3}{m}} \sigma_0 \left( \frac{V_0}{2} \right)^{1/m} \quad (11)$$

where  $V_0$  denotes the reference volume. By using the Weibull parameters given in Table 3, the ultimate load  $\bar{F}_u$  would be one order of magnitude larger than the one measured experimentally and reported in Table 1. Furthermore, the scatter in failure stresses is too high to involve volume flaws. Consequently, failure is first due to stable crack growth in the vicinity of the contact zone then followed by unstable propagation as shown in Fig. 1. The contact zone is

surrounded by Hertz cracks that were probably created upon loading. Since these cracks interact with a defect on the surface, a spall has been created.

## **II. Edge-on-impact of glass**

The effect of dynamic load histories is now analyzed. The previous section has shown that crack initiation was located close to the contact zone of the applied load and that no volume defect could be nucleated even under high tensile stresses in the bulk of spheres. The same question is studied in the present section for impact conditions. Furthermore, the cracking pattern is examined when two different types of strikers are used.

### **Experiments and results**

To visualize damage, a so-called Edge-On-Impact configuration is used [1]. This configuration allows the user to observe the cracking pattern. Bullets are fired by a gun and impact a float glass target of size  $100 \times 100 \times 10 \text{ mm}^3$ . The projectile speed is measured by two optical cells one meter apart. An “open” configuration is used, enabling for in-situ observations by utilizing a high-speed camera. The interframe time can be as low as  $0.5 \mu\text{s}$ . When the bullet reaches the second cell, flashlights are triggered. When the bullet impacts the target, it activates the camera to take pictures. Two different impactors are used. According to the European standard EN1063 [23] a “soft” bullet (i.e., magnum 44) further referred to as “BR4”, traveling at a speed of 430m/s and a hard, penetrating, bullet further referred to as “BR7”, traveling at a speed of 820 m/s, are used. “BR4” bullets have a core made of lead and an envelope of brass wire. BR7 bullets have a core made of steel, an envelope of brass wire and a small head of lead.

Typical results are shown in Figs. 4 and 5. When the bullet impacts the glass sample and induces cracking a few microseconds thereafter, dark zones appear on pictures. They

correspond to damaged zones and glass is no longer transparent. One can observe the damage zone propagating in the sample and the crack front is reasonably well delimited by this type of inspection and consequently propagation velocities can be determined with a good accuracy. After impact, fragments of the impacted sample are scattered and we generally found an intact BR7 core (made of steel) whereas the BR4 core (made of lead) is fully sublimated.

A first zone in the vicinity of the impact is totally comminuted. A second area displays a high density of radial and orthoradial cracks. A third zone appears in which long radial cracks propagate. These different zones show us different cracking features:

- multiple fragmentation in the first zone with mainly closed cracks (glass is totally comminuted),
- multiple fragmentation under mode I loading in the second zone,
- single fragmentation in the third zone (crack propagation only).

## **Discussion**

When a bullet impacts a glass target, a compressive stress wave propagates ahead of the impacted area, with a speed of a longitudinal acoustic wave. The radial displacement also induces hoop tensile stresses that nucleate defects inducing microcracking. The latter can be described by an anisotropic damage model (see, e.g., Refs. [3,5]). The level of stress and stress rate determine the type of fragmentation, namely single or multiple fragmentation. This analysis shows that under dynamic loading conditions, there is also a need for understanding the conditions related to crack initiation, propagation and arrest.

Contrary to quasi-static experiments, the overall damage pattern is reproducible under dynamic loading conditions. Yet, it depends on the type of bullet (Fig. 5). On the one hand, impacts with BR7 bullets show mainly a circular cracking front, the speed of which, estimated from the dark zone boundaries of the pictures, has a constant value  $V_c$  of the order of 1500m/s

(which is close to the terminal velocity of a single crack propagating under a high strain energy release rate). This result indicates that numerous radial cracks propagate from the impact zone; this explanation is confirmed by post-mortem examinations that revealed mainly radial cracks propagating from the impact zone. On the other hand, different phenomena occur with BR4 bullets. Post-mortem observations reveal that there is a comminuted zone just under the contact zone. Far from the impact area, we can observe long cracks. Moreover, in particular with BR4 bullets, a “Rayleigh cone” appears (Fig. 6). The Rayleigh cone is caused by multiple nucleation of surface cracks induced by the tensile stress on the impacted surface caused by the Rayleigh wave [24]. This wave is created when the bullet impacts glass and propagates at a speed  $C_R$  of the order of 3300 m/s for glass [24]. The theoretical angle  $\theta$  of the cone reads

$$\sin \theta = \frac{V_c}{C_R}, \quad \theta \approx 28^\circ \quad (11)$$

and a value of  $28^\circ$  is found experimentally (Fig. 6). This phenomenon is not observed with BR7 bullets. A finite element (FE) simulation with the explicit code Pamshock (all materials are supposed to be elasto-plastic, see Table 4) shows that tensile stresses on the surface are higher for BR4 impacts (Fig. 7). This lower stress level is induced by the fact that the contact surface of BR7 projectiles is smaller compared to BR4 bullets during the first microseconds after impact, thereby leading to a smaller impedance. Consequently, the stress levels become too low for cracks to initiate close to the surface and cracking induced by the Rayleigh wave cannot develop when the target is impacted by a BR7 bullet.

We can also observe nucleation of cracks ahead of the main circular front, especially with a BR4 bullet (Fig. 7). This event is not observed when glass is tempered (i.e., when the surface is reinforced) or impacted with BR7 bullet [25]. In the latter cases, the main crack front is caused by flaws that were originally located in the vicinity of the impact point. These flaws nucleate cracks that propagate at a constant velocity  $V_c$ . The FE simulation shows that the level

of tension is too small to nucleate cracks on the external surface of glass when impacted with BR7 bullet (Fig. 7) or when glass is tempered. In the BR4 case, a value significantly greater than 1500 m/s can be estimated from the sequences of pictures for the apparent speed at which damage propagates from the impact point. The measured speed is an apparent speed caused by the coalescence of cracks in the zone of high stresses. This apparent speed is bound by the hoop tensile stress rate induced by the compressive wave. It can reach up to 3500 m/s, according to our experiment. This upper value is very close to the celerity of transverse waves in glass. This would be possible for damage that is supposed to be caused by transient tension of the material in the hoop direction, arising after the compressive (longitudinal) wave front. However, it might as well be coincidental since such a phenomenon can also be observed in ceramic materials for which the crack front velocity is not considered as intrinsic since it varies with the impact velocity and depends on the Weibull parameters of the material [3]. Further investigations are needed to clarify this point.

To confirm the previous interpretations, an impact is carried out on a glass tile on which a scratch was made on purpose in the middle of the lateral surface before impact (Fig. 8). This scratch constitutes indeed an initiation site at the surface of glass. The same phenomenon is observed, i.e., the scratch induces a propagating crack ahead of the main crack front. This clearly shows that surface flaws can act as initiation sites in a similar way as what is observed under quasi-static loading conditions.

Another point to study is the cracking speed during the first microseconds. Several differences can be seen between the experiments carried out with the two bullets. On the one hand, the cracking speed is estimated to be 1700m/s, during the whole process for BR7 bullets (Fig. 9a). Since the stress levels are very high in the vicinity of the impact point for BR7 bullets, the cracking velocity is of the order of  $V_c$  and the circular shape corresponds to long cracks that propagate from the impact point. For BR4 bullets, values within the range 3000-3500m/s are measured during the first microseconds (Fig. 9b) and of the order of 1500m/s

thereafter. This variation can be explained in a similar way as above: for BR4 bullets, the first damage pattern is caused by coalescence of numerous microcracks. It corresponds to a collective behavior of microcracks as opposed to the propagation of long cracks. The possibility that these microcracks were nucleated from volume defects in the vicinity of the impact point cannot be completely ruled out, although it would be hard to explain why volume defects would be initiation sites for a soft projectile and not a more energetic and hard one. Eventually, the damage pattern becomes identical to that observed for BR7 bullets, i.e., long cracks propagating with a velocity equal to  $V_c$ .

## Summary

Compression tests have been performed on as-received and tempered glass spheres. Chemical tempering induces residual stresses in the vicinity of the external surface. Different failure load levels have been recorded; thereby showing that failure initiation is located in the contact zone. An *a priori* evaluation of the mean residual stress level induced by the two heat treatments could be confirmed by an *a posteriori* analysis of the difference in average failure loads. Furthermore, a Weibull analysis shows that the scatter obtained by analyzing any set of experimental data is of the same order as that obtained when using the size effect property (i.e., the dependence of the average failure load with the sphere diameter).

The experiments on glass spheres have been performed under quasi-static loading conditions. However, crack propagation cannot be considered as gradual or stable during the whole test (e.g., most of the compressed spheres were fully comminuted at the end). This is also the case for the impact experiments reported herein.

Fragmentation of impacted glass is mainly caused by the growth of cracks nucleated close to the impact zone or far from it, on the surface of the glass body hit by the projectile. Differences observed between soft and hard bullets corroborate this hypothesis. Far from bullet



impact, propagation of long cracks is always observed. The population of defects is a key parameter that governs the cracking pattern. It was also shown that when scratches exist on the surface and when the stress level is sufficiently high, they can induce cracking ahead of the main front. The role of volume defects was more difficult to quantify. It is believed that their role in the fragmentation, if there is any, is confined in the immediate vicinity of the impact area. This is consistent with observations on fibers that concluded that the volume defects are not as severe as surface defects. Similarly, for spheres, the defects close to the contact zone played a dominant role in the initiation of failure. Furthermore, the type of projectile can have a significant influence on the fragmentation pattern. Soft bullets with a flat end crash on the glass surface and are stopped whereas hard (piercing) bullets penetrate glass. The estimation of the scatter in strength is therefore the key to understanding the nucleation stage. Furthermore, as was also observed in the compression of spheres, tempering glass constitutes a way of partially preventing crack initiation.

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## References

- [1] U. Hornemann, J.F. Kalthoff, H. Rothenhäusler, H. Senf and S. Winkler, “Experimental Investigation of Wave and Fracture Propagation in Glass-Slabs Loaded by Steel Cylinders at High Impact Velocities,” EMI report **E 4/84**, Weil am Rhein, Germany (1984).
- [2] E. Straßburger and H. Senf, “Experimental Investigation of Wave and Fracture Phenomena in Impacted Ceramics and Glasses,” ARL report **ARL-CR-214** (1994).
- [3] Denoual C., *Approche probabiliste du comportement à l’impact du carbure de silicium : application aux blindages moyens*, PhD thesis, ENS de Cachan, France (1998). See also: C. Denoual and F. Hild, “A Damage Model for the Dynamic Fragmentation of Brittle Solids,” *Comp. Meth. Appl. Mech. Eng.* **183** (2000) 247-258.
- [4] W. Weibull, “A Statistical Theory of the Strength of Materials,” *Roy. Swed. Inst. Eng. Res.* **151** (1989).
- [5] C. Denoual and F. Hild, “Dynamic Fragmentation of Brittle Solids: A Multi-Scale Model” *Eur. J. Mech. A/Solids* **21** [1] (2002) 105-120.
- [6] T. Zarzycky, *Glasses and the Vitreous State*, Cambridge University Press, Cambridge, UK (1991).
- [7] B. Nghiem, *Fracture du verre et hétérogénéité à l’échelle submicronique*, PhD thesis, University of Paris 6, France (1998).
- [8] A.K. Varshneya, *Fundamentals of Inorganic Glasses*, Academic Press, Inc., Boston, USA (1994).
- [9] C. Guillemet, “Current Challenges Concerning the Mechanical Strength of Glass Products,” in *Strength of Inorganic Glass*, C.R. Kurkjian, ed., Plenum Press, New York, USA (1985).

- [10] R. Gy and C. Guillemet, "Characterization of a mode of rupture of glass at 610°C," in L.D. Pye, W.C. La Course and H.J. Stevens, eds., *The Physics of Non-Crystalline Solids*, Taylor & Francis, London, UK, (1992).
- [11] B.A. Kshinka, S. Perrela, H. Nguyen and R.C. Bradt, "Strengths of Glass Spheres in Compression," *J. Am. Ceram. Soc.* **69** [6] (1986) 467-472.
- [12] F.C. Lobo Carneiro, "Um novo método para determinação da resistência à tração dos concretos," in Anais 5<sup>a</sup> reunião da Associação Brasileira de Normas Técnicas (ABNT) em São Paulo (1943) 127-129. See also: F.C. Lobo Carneiro, "Une nouvelle méthode pour la détermination de la résistance à la traction des bétons," *Bull. RILEM* **13** (1953) 103-108.
- [13] Ö Vardar and I. Finnie, "An Analysis of the Brazilian disk fracture test using the Weibull probabilistic treatment of brittle strength," *Int. J. Fract.* **11** [3] (1975) 495-508.
- [14] H. Aben and C. Guillemet, *Photoelasticity of Glass*, Springer Verlag, Berlin, Germany (1993).
- [15] R. Brzesowsky, *Micromechanics of Sand Grain Failure and Sand Compaction*, PhD thesis, Utrecht Faculteit Aardwetenschappen, the Netherlands (1994).
- [16] D.G.S. Davies, "The Statistical Approach to Engineering Design in Ceramics," *Proc. Brit. Ceram. Soc.* **22** (1973) 429-452.
- [17] D.R. Oakley, "An Empirical Study of the Effect of Stressed Area on the Strength of Float Glass Surfaces," *J. Non-Cryst. Solids* **196** (1996) 134-138.
- [18] Y. Hiramitsu and Y. Oka, "Determination of the Tensile Strength of Rock by a Compression Test of an Irregular Test Piece," *Int. J. Rock. Mech. Min. Sci.* **3** (1966) 89-99.
- [19] R. Kienzler and W. Schmitt, "On Single Particle Communion: Numerical Analysis of Compressed Spheres," *Powder Tech.* **61** (1990) 29-38.

- [20] O. Tsoungui, D. Vallet, J.-C. Charmet and S. Roux, "Size Effect in Single Grain Fragmentation," *Gran. Matter J.* **2** (1999) 19-27.
- [21] B. Lawn, "Indentation of Ceramics with Spheres: a century after Hertz," *J. Am. Ceram. Soc.* **81** [8] (1998) 1977-1993.
- [22] Y. Hiramitsu and Y. Oka, "Determination of the Tensile Strength of Rock by a Compression Test of an Irregular Piece," *Int. J. Rock. Min. Sci.* **3** (1966) 89-99.
- [23] NF EN1063 standard: "Verre dans la construction. Vitrage de sécurité. Mise à essai et classification de la résistance à l'attaque par balle."
- [24] K.F. Graff, *Wave Motion in Elastic Solids*, Clarendon Press, Oxford, UK (1975).
- [25] X. Brajer, *Endommagement du verre sous impact de projectile : application aux vitrages blindés civils*, M.Sc. report, LMT-Cachan, France (2001).

## Table Captions

Table 1: Mean failure load for different sphere diameters and temperings. In parentheses are given the corresponding standard deviation and in brackets the number of samples. The load was applied with planar anvils and an aluminum foil was inserted between the spheres and the anvils. At least 10 spheres have been tested per case.

Table 2: Mean failure load for different diameters of as-received spheres. In parentheses are given the corresponding standard deviation. The load was applied with different anvils. At least 10 spheres have been tested per case.

Table 3: Typical Weibull parameters for soda-lime glass.

Table 4: Material parameters used in the FE simulations.

Table 1

Diameter, $D$ (mm)	As-received	CT1	CT2
1	290 (60) [10]	540 (125) [12]	350 (120) [14]
1.5	700 (70) [15]	1400 (455) [13]	720 (45) [15]
2	910 (290) [9]	2590 (260) [19]	1370 (240) [12]
2.2	860 (145) [14]	2160 (425) [13]	1900 (220) [14]
4	2770 (345) [14]	5940 (1180) [14]	4090 (830) [15]
8	6030 (1000) [10]	11060 (3500) [8]	8430 (1000) [7]

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Table 2

Diameter, $D$ (mm)	planar anvils	planar anvils + aluminum foils	spherical anvils
2*	640 (51) [10]	640 (58) [10]	-
2	877 (240) [9]	910 <sup>‡</sup> (290) [9]	-
4	1880 (461) [10]	2770 <sup>‡</sup> (345) [14]	-
8	3820 (1000) [10]	5780 <sup>†</sup> (1080) [10]	6180 (1000) [10]

\*different sphere manufacturer

<sup>‡</sup>same test series as in Table 1

<sup>†</sup>test series different from that of Table 1

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Table 3

Failure location	Weibull modulus $m$	Scale parameter $\sigma_0$ (MPa)	Gauge volume or surface $Z_0$
surface [15]	$\cong 7$	$\cong 100$	$\cong 100 \text{ cm}^2$
volume [5]	28.5	2750	$10^{-6} \text{ mm}^3$

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Table 4

Material property	Glass	Soft bullet (core)	Hard bullet (core)
Young's modulus (GPa)	70	16.7	220
Poisson's ratio	0.22	0.3	0.3
Mass density (kg/m <sup>3</sup> )	2500	11500	7900
Yield stress (MPa)	4000 <sup>†</sup>	13.2	≈300

<sup>†</sup>Hugoniot elastic limit

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## Figure Captions

- Figure 1: Fractography of a sphere 8 mm in diameter. The contact zone is depicted with a dotted circle. The first arrow shows the likely initiation site. A crack is created and turns around the contact zone (dashed line) to interact with Hertz cracks to form a spall.
- Figure 2: -a- Mean failure load versus sphere diameter. The lines are least squares fits (correlation coefficient greater than 0.98). The estimated errors are depicted by the symbol size.  
-b- Weibull plot for as-received spheres 1mm in diameter;  $P_F$  denotes the failure probability and  $F$  the applied load (in N). The lines correspond to a least squares fit (correlation coefficient: 0.97). The estimated errors are depicted by the symbol size.
- Figure 3: Estimates of the mean residual stresses for two different chemical temperings based upon the analysis of the mean failure load. The lines correspond to the best fits.
- Figure 4: Cracking pattern 10  $\mu$ s after impact of glass by a BR4 bullet in an edge-on-impact configuration.
- Figure 5: Cracking pattern 10  $\mu$ s after impact of glass by a BR7 bullet in an edge-on-impact configuration.
- Figure 6: Observation of a so-called Rayleigh cone 10  $\mu$ s after impact with a BR4 bullet. Depiction of the cone and the corresponding cracking pattern.

Figure 7: FE simulations of the EOI experiments. Comparison of maximum principal stress levels 5  $\mu\text{s}$  after impact (for the edge, a) and 10  $\mu\text{s}$  after impact (for the observation surface, b); left part: BR4 bullet, right part: BR7 bullet.

Figure 8: Sequence of pictures taken when a BR4 bullet hits a glass tile with a scratch. One can note that a crack nucleates from the scratch ahead of the main cracking front (first photography: 9  $\mu\text{s}$  after impact, interframe time: 1.5  $\mu\text{s}$ ). Each picture consists of the view of the tile surface and one edge.

Figure 9: Sequence of pictures taken when a BR4 (a) or BR7 (b) bullet hits a glass tile. Two different cracking pattern are observed during the first 3.25  $\mu\text{s}$  after impact (interframe time: 0.65  $\mu\text{s}$ ).