



# Rheological behaviour probed by vibrating microcantilevers

Naser Belmiloud, Isabelle Dufour, Annie Colin, Liviu Nicu

## ► To cite this version:

Naser Belmiloud, Isabelle Dufour, Annie Colin, Liviu Nicu. Rheological behaviour probed by vibrating microcantilevers. Applied Physics Letters, 2008, 92 (Issue 4), pp.041907. hal-00322050

**HAL Id: hal-00322050**

**<https://hal.science/hal-00322050>**

Submitted on 16 Sep 2008

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Rheological behavior probed by vibrating microcantilevers

Naser Belmiloud<sup>a)</sup> and Isabelle Dufour

University of Bordeaux, IMS Laboratory, 351 cours de la Libération, F-33405 Talence, France

Annie Colin

University of Bordeaux, LOF-Rhodia, 178 avenue du Docteur Schweitzer, F-33608 Pessac, France

Liviu Nicu

LAAS/CNRS, 7 avenue du Colonel Roche, F-31077 Toulouse, France

(Received 12 December 2007; accepted 2 January 2008; published online 30 January 2008)

The aim of this paper is to demonstrate that vibrating microcantilevers can be used to quantify fluid properties such as density and viscosity. Contrary to classical rheological measurements using microcantilevers, the development of the proposed microrheometer is based on the measurement of fluid properties over a range of vibration frequencies, without necessarily being restricted to resonant phenomena. To this end, an analytical model is implemented and, when combined with measurements, allows the determination of the viscosity as a function of frequency. The preliminary results are encouraging for the development of a useful microrheometer on a silicon chip for microfluidic applications. © 2008 American Institute of Physics. [DOI: 10.1063/1.2837181]

Simultaneous measurements of both phase and amplitude lead to the rapid determination of both unknown fluid parameters (viscosity and density). First, a brief review of the theory of fluid-microcantilever interaction is made to explain the proposed method of fluid property extraction from the beam's dynamic response. This is then followed by a discussion of our first measurements and fluid parameter determination for Newtonian silicon oils.

Under the incompressible and Newtonian assumption, viscosity and density are intrinsic constants. When a microcantilever vibrates in a viscous fluid, the fluid offers resistance to the motion of the beam. The fluid loading can be interpreted as the sum of two forces: an inertial force that is proportional to the cantilever acceleration and a viscous force that is proportional to the cantilever velocity. Thus, a viscous fluid exerts on the beam surface the drag force:<sup>1-3</sup>

$$F_{\text{fluid}} = -g_1(f) \frac{\partial w(t)}{\partial t} - g_2(f) \frac{\partial^2 w(t)}{\partial t^2}, \quad (1)$$

with  $w$  the deflection at the end of the beam,  $f$  the vibration frequency, and  $t$  is time. Terms  $g_1$  and  $g_2$  are due to the dissipative and inertial parts, respectively, classically written as follows:

$$g_1 = \frac{\pi}{2} \rho b^2 f \Gamma''(f), \quad (2)$$

$$g_2 = \frac{\pi}{4} \rho b^2 \Gamma'(f), \quad (3)$$

where  $\Gamma'$  and  $\Gamma''$  are the real and imaginary parts of the hydrodynamic function, respectively. The hydrodynamic function  $\Gamma' + j\Gamma''$  largely depends on frequency, beam geometry, and fluid mechanical properties.

In Fig. 1, the analytical solution of the equation governing the microcantilever dynamic demonstrates that the evo-

lution of both, the amplitude and phase, as a function of the frequency, depends on the fluid properties. The illustrated behavior suggests that, despite the absence of a resonant phenomenon for high viscosities, the measurement of the microcantilever deflection can be used to determine the fluid viscosity by measuring the deflection amplitude at a given frequency (chosen in accordance with the viscosity range). A unique viscosity corresponds to a given vibration spectrum. Phase and amplitude are controlled by two unknown parameters which are independent of each other; thus, we may extract values of both density and viscosity at each frequency.

An analogy between beam dynamics in viscous fluids and second order low-pass band allows the simplification of the mechanical transfer function. The assumption that the contribution of higher modes in the transfer function value is negligible has been made, i.e., the first mode is considered dominant. The mechanical transfer function can then be written as

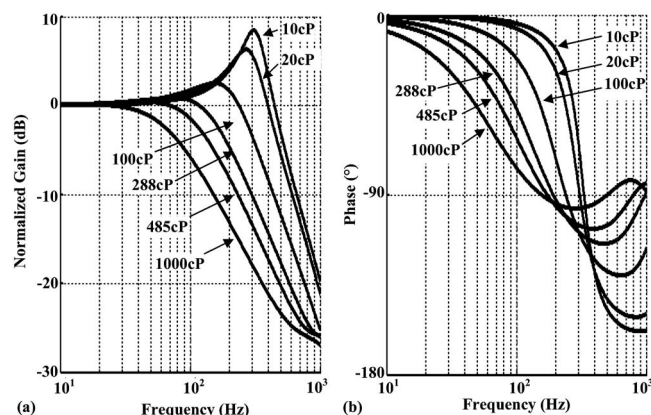


FIG. 1. Theoretical amplitude (a) and phase (b) of a silicon cantilever ( $18 \mu\text{m}$ ,  $600 \mu\text{m}$ ,  $4 \text{ mm}$ ) oscillating in fluids with different viscosities.

<sup>a)</sup>Electronic mail: naser.belmiloud@ims-bordeaux.fr.

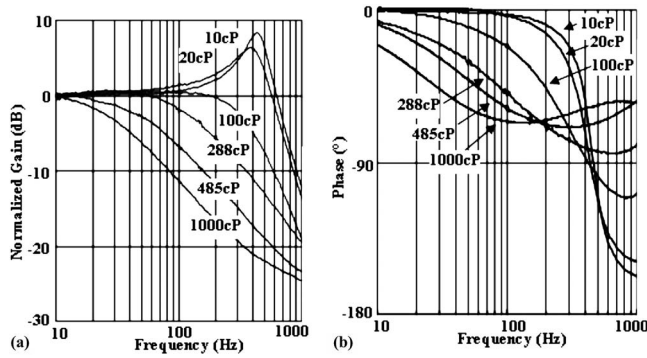


FIG. 2. Amplitude (a) and phase (b) measurement corresponding to the simulation presented in Fig. 1 for a silicon cantilever ( $18 \times 600 \times 4000 \mu\text{m}^3$ ) immersed in various silicon oils: 10, 20, 100, 288, 485, and 1000 cP.

$$H = \frac{w(f)}{F} \approx \frac{H_0}{1 - \left(\frac{f}{f_0}\right)^2 + 2\xi\left(\frac{f}{f_0}\right)j}, \quad (4)$$

with  $\xi$  the damping ratio,  $f_0$  the natural frequency of the system {beam+fluid}, and  $H_0$  the static value of the transfer function.

Referring to the differential equation governing the microcantilever deflection, the amplitude and phase measurements ( $|H|, \phi$ ) may be converted to the drag force components at each frequency:

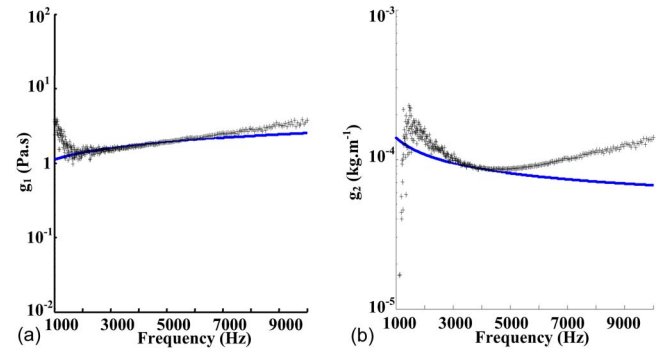


FIG. 3. (Color online) Extracted drag force components  $g_1$  (a) and  $g_2$  (b) exerted on a silicon cantilever ( $70 \times 200 \times 3000 \mu\text{m}^3$ ) by a silicon oil whose viscosity and density are 96.5 cP and  $1000 \text{ kg m}^{-3}$ , respectively, compared with Maali's theoretical components (blue line).

$$g_1 = -2\pi m_L \left| \frac{H_0}{H} \right| \frac{f_{0,\text{vac}}^2}{f} \sin \phi, \quad (5)$$

$$g_2 = m_L \left| \left( 1 - \left| \frac{H_0}{H} \right| \cos \phi \right) \left( \frac{f_{0,\text{vac}}}{f} \right)^2 - 1 \right|, \quad (6)$$

where  $m_L$  is the cantilever mass per unit length,  $f_{0,\text{vac}}$  is the resonant frequency in vacuum,  $\phi$  is the phase, and  $f$  is the frequency.

Thus, the determination of the two parts of the drag force allows one to extract analytically at each frequency the viscosity and density, i.e., the fluid's rheological properties:

$$\rho = \frac{2\pi f g_2 (2a_1 b_2 - a_2 b_1) + a_2 [g_1 a_2 - \sqrt{(2\pi f g_2 b_1 - a_2 g_1)^2 + 8\pi f g_2 g_1 a_1 b_2}]}{\pi^2 b^2 f a_1 (a_1 b_2 - a_2 b_1)}, \quad (7)$$

$$\eta = \frac{a_1 [2\pi f g_2 b_1 + g_1 a_2 - \sqrt{(2\pi f g_2 b_1 - a_2 g_1)^2 + 8\pi f g_2 g_1 a_1 b_2}]^2}{\pi (a_1 b_2 - a_2 b_1) (2\pi f g_2 (2a_1 b_2 - a_2 b_1) + a_2 [g_1 a_2 - \sqrt{(2\pi f g_2 b_1 - a_2 g_1)^2 + 8\pi f g_2 g_1 a_1 b_2}])}, \quad (8)$$

where  $a_1=1.0553$ ,  $a_2=3.7997$ ,  $b_1=3.8018$ , and  $b_2=2.7364$  are Maali's parameters and  $b$  is the microcantilever width.<sup>1</sup>

The density and viscosity are determined using Eqs. (7) and (8), where it is evident that the beam geometry and vacuum frequency of the cantilever in question are required.

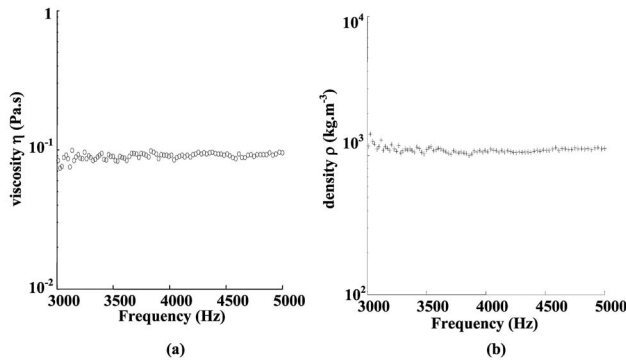


FIG. 4. Determination of viscosity (a) and density (b) based on measurements of a silicon cantilever ( $70 \times 200 \times 3000 \mu\text{m}^3$ ) immersed in silicon oil whose viscosity and density are 96 cP and  $1000 \text{ kg m}^{-3}$ , respectively.

We perform a microcantilever calibration to measure these parameters. They can be calculated easily from measure-

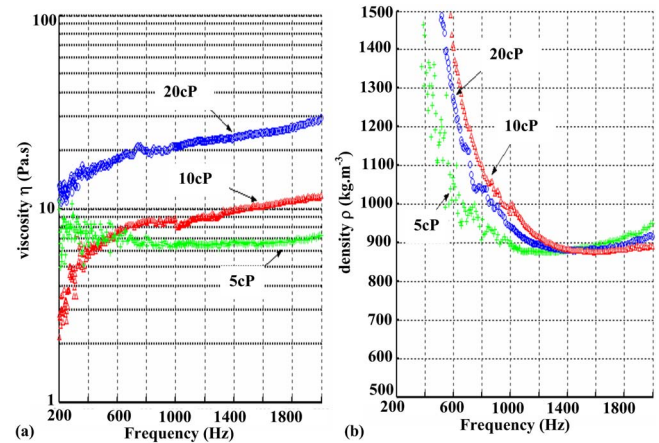


FIG. 5. (Color online) Determination of viscosity (a) and density (b) with measurements of a silicon cantilever ( $39 \times 600 \times 4000 \mu\text{m}^3$ ) immersed in three silicon oils: 5, 10, and 20 cp. Oil density is  $980 \text{ kg m}^{-3}$ .

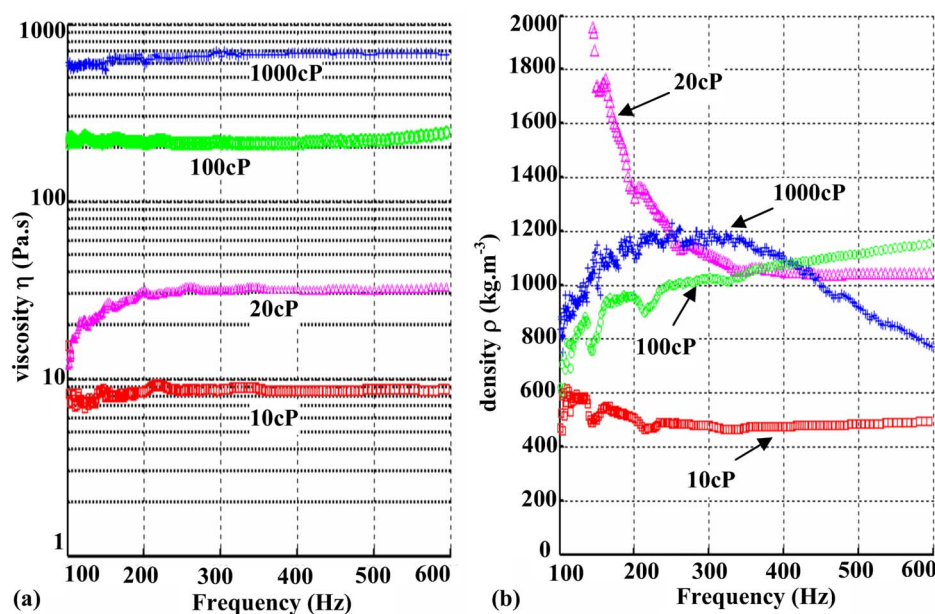


FIG. 6. (Color online) Determination of viscosity (a) and density (b) corresponding to the spectra presented in Fig. 2.

ments of the resonant frequency and quality factor of the cantilever in a single reference fluid (e.g., air).

Experiments were carried out with three different microcantilever designs. The deflection amplitude and phase have been measured using an optical readout scheme. An alternating current circulates on the beam, which, in presence of a magnet, generates the beam vibration by electromagnetic actuation. Measurements are done in a thermoregulated room in which temperature is controlled at 20 °C. The vibration amplitude and phase have been measured as functions of frequency on a range of 10–10 000 Hz. The beam has been immersed in oils of known viscosities ranging from 10 to 30 000 cP. In Fig. 2, no resonant peaks occur for viscosities above 100 cP because of the excessive damping. Using microcantilevers of other geometries, similar curves are obtained. The measurements are reproducible and confirm that, even without resonant phenomena, the fluid properties govern the vibrational characteristics of the beam oscillation.

Using Eqs. (5) and (6), it is possible to estimate from the spectrum measurement both components  $g_1$  and  $g_2$  of the hydrodynamic force. In Fig. 3, the experimental results are compared to the theoretical expressions of the drag force (model of Maali *et al.*<sup>1</sup>). The model predicts the dissipative part better than the inertial part. Typically, with the microcantilever geometry and the range of frequency, the Reynolds number value is relatively small. This confirms that viscous losses are more predominant than inertial effect. That is why, they are more easier estimated. In Fig. 3, in the range of 3–5 kHz, both measured components are in good agreement with theoretical predictions. Below this range, the low-frequency noise interferes with measurements. Above this range, the higher modes contribute significantly in both phase and gain values. The model is able to accurately ex-

tract (or predict) fluid properties at a given frequency.

Using Eqs. (7) and (8), viscosity and density can be calculated after extraction of the drag force components. Preliminary measurements (Fig. 4) are in good agreement with the rheological behavior of the silicon oils for this frequency range.<sup>4</sup> We measure a mean density of 1100 kg/m<sup>3</sup> and a mean viscosity equal to 96 cP (instead of 96.5 cP with a cone/plate rheometer). As is implicit in the Newtonian and incompressible model, we observe viscosity and density as frequency-independent constants.

For other beam geometries, experimental data continue to be reproducible (Figs. 5 and 6). The model works well for high viscosities despite the absence of a resonant peak. For that reason, this represents an alternative to classical microcantilever-based rheometer.<sup>5,6</sup> In Fig. 6, the viscosity measurements are more accurate than the density values. This shows that, for those cases in which the density is known, the proposed method may be used to accurately quantify the dynamic viscosity. In fact, the accuracy of both, the beam geometry and the static value of the transfer function determines the quality of the measurements.

We thank the “Région Aquitaine” for its financial sponsorship.

<sup>1</sup>A. Maali, C. Hurth, R. Boisgard, C. Jai, T. Cohen-Bouhacina, and J. P. Aimé, *J. Appl. Phys.* **97**, 074907 (2005).

<sup>2</sup>L. Rosenhead, *Laminar Boundary Layers* (Clarendon, Oxford, 1963).

<sup>3</sup>J. E. Sader, *J. Appl. Phys.* **84**, 64 (1998).

<sup>4</sup>J. Lamb and P. Lindon, *J. Acoust. Soc. Am.* **41**, 1032 (1966).

<sup>5</sup>C. Bergaud and L. Nicu, *Rev. Sci. Instrum.* **71**, 2487 (2000).

<sup>6</sup>S. Boskovic, J. W. M. Chon, P. Mulvaney, and J. E. Sader, *J. Rheol.* **46**, 891 (2002).