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► **To cite this version:**

Gautier Meyer, Vincent Louis, Jean-Paul Sansonnet, Yannick Larvor. Two logical operators for representing and handling preferences. IJCAI-05 Multidisciplinary Workshop on Advances in Preference Handling, Jul 2005, Edinburg, United Kingdom. hal-00321525

HAL Id: hal-00321525

<https://hal.science/hal-00321525>

Submitted on 15 Sep 2008

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Two logical operators for representing and handling preferences

Gautier Meyer[†] and Vincent Louis[†] and Jean-Paul Sansonnet* and Yannick Larvor[†]

[†]France Telecom Research & Development

2, av. Pierre Marzin - 22307 Lannion cedex - France

{gautier.meyer,vincent.louis,yannick.larvor}@francetelecom.com

*LIMSI-CNRS

Bât. 508, BP 133 - 91403 Orsay cedex - France

jean-paul.sansonnet@limsi.fr

Abstract

Models of cognitive rational agents too seldom specify a decision step. This article aims at bridging this gap by proposing a qualitative formalization of the *preference* concept. We define two logical operators: the first one to specify preferences naturally; the second one to compare the agent's reactions with each other.

1 Introduction

The agent paradigm proposes an attractive point of view to model complex information processing systems, all entities being represented in an abstract but nevertheless intuitive way. In particular, rational agents are generally formalized as cognitive entities defined upon primitive concepts (so-called *mental attitudes*) such as belief, desire or intention [Rao and Georgeff, 1991], and can be implemented by automatic inference mechanisms [Bretier and Sadek, 1997]. Unfortunately, these models do not completely specify the agent's reactions. This is unsuitable in complex situations where s/he has several motivations (or intentions) and/or several means for satisfying them (action plans). For example, as part of a multimodal interaction, an assistant agent has to provide users with itineraries either through a vocal interface (by speaking instructions) or through a graphic interface (by showing a map) according to the context (case of several action plans satisfying the same intention).



Figure 1: General link between mental state and reaction

This has led the authors of [Sadek, 1991; Haddawy and Hanks, 1993] to affirm the need of establishing explicitly the link between the mental state and the reaction of an agent. Thereafter, the authors of [Haddawy and Hanks, 1993; Louis, 2002] have shown that this link could be done in two distinct steps (see figure 1): a planning step [Allen et al., 1991; Louis, 2002] and a decision step [Doyle and Wellman, 1994; Chiclana et al., 1996; Ha, 2001]. The first one aims at generating the potential reactions of the agent (alternatives); the

second one sorts them to choose the actual action to carry out. This last one requires to model preference (see [Öztürk et al., 2005] for an overview).

In a variety of fields, *Utility functions* are often used for decision steps. The utility of each alternative is computed and the highest rated alternative is selected as the reaction. This method yields a detailed representation of knowledge on the classification of alternatives. Moreover, it makes it possible to take into account the stochastic nature of the environment (concept of expected utility) and has been well studied in the context of decision theory (see [Fishburn, 1999] for a selective survey of numerical representations of preference). Unfortunately, those numerical representations suffer from the known following drawbacks and thus do not seem to be adapted to cognitive agent modeling.

- The determination of utility is difficult in practice. It requires a significant specification effort since describing each alternative requires a global view of the considered problem [Doyle and Wellman, 1994; Ha and Haddawy, 1999; Faltings et al., 2004].
- If the result “optimality” is not crucial, this method is not justified [Brafman and Tennenholtz, 1997; Bouilrier et al., 2004; Ha and Haddawy, 1999].
- In order to have a decision method more homogeneous with the agent's internal description via mental attitudes, a *qualitative* technique would be preferable to a *quantitative* one [Dubois et al., 2001].
- In the perspective of basing agents' models upon humans' cognitive ones, calling upon qualitative decision techniques seems to be a more natural way [Brafman and Tennenholtz, 1997; Rossi et al., 2004].

To bypass these limitations, we choose to rank alternatives according to a logical preference representation (see [Lang, 2004] and [Coste-Marquis et al., 2004] for a non exhaustive comparison of some ordinal representations). More precisely, we define a formal notion of preference, called “partial preference” and associated to a given “point of view”, that makes it possible to compare the alternatives with each other without any numerical representation. This notion facilitates the specification of the elementary informations necessary to the ranking and thus to the decision step.

Section 2 globally describes our proposal. Sections 3 and 4 formalize the progressive construction of our model by introducing two logical operators: an operator related to a notion of *primitive* partial preference and another one related to a notion of *extended* partial preference.

2 Overall scheme

Our goal is to provide a formal way both to specify preferences naturally and concisely, and to compare any alternative with any other. This is achieved by modeling the necessary information to make a choice thanks to the concept of partial preference.

Many researchers in AI and philosophy think that the intuitive specification of a preference implicitly makes the *Ceteris Paribus* assumption (see [Boutlier *et al.*, 2004]): all characteristics other than those explicitly specified are supposed to be equal [Hansson, 1996; Doyle and Wellman, 1994]. For example, the statement “I prefer fish dishes to meat ones” implicitly means that characteristics such as price or abundance are the same. Moreover, such an assumption allows for a succinct representation of numerous partial pre-orders [Coste-Marquis *et al.*, 2004]. However this assumption is not sufficient for comparing any two objects, in particular those whose unspecified characteristics are not equal: one cannot say whether steamed fish is preferred to grilled meat.

To avoid this problem, our main idea is to split the specification of preferences into two parts (see figure 2). Firstly, the initial data defining each partial preference is interpreted according to the *ceteris paribus* assumption, which generates a *primitive partial preference* (generation stage). Secondly, each primitive partial preference is extended (beyond the *ceteris paribus* assumption) in an *extended partial preference* (extension stage). This extension stage makes it possible to compare any alternatives. Actually, several partial preferences can be specified in parallel (which result in several primitive and extended partial preferences), each of them representing a “point of view” for classifying alternatives.

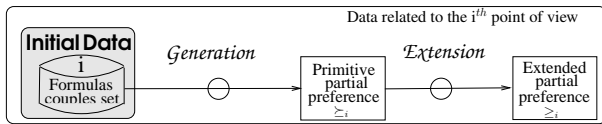


Figure 2: Overall scheme for preference construction

In our approach, each partial preference is modeled by a binary relation between the objects to compare. These objects, which are called “alternatives”, correspond in fact to states of the world. As it is difficult, if not impossible, to represent exhaustively the characteristics of a state of the world (see for example the frame problem [McCarthy and Hayes, 1969]), our model additionally associates with each partial preference an operator over couples of logical formulas that partially describe the two states to be compared. In short, each partial preference is associated with both a binary relation over alternatives and a logical operator over formulas representing alternatives. Our vision is therefore close to Von Wright’s according to which the preferences are specified by descriptions

but actually apply to “concrete” alternatives corresponding to these descriptions [von Wright, 1972].

3 Preliminaries

3.1 Language used

In this article, \mathcal{L} is a propositional language built upon a set of propositional symbols SYM , the constants \top (tautology) and \perp (contradiction) and the usual logical operators ($\wedge, \vee, \neg, \Leftrightarrow, \Rightarrow$). The symbols $\phi, \phi_1, \dots, \psi, \psi_1, \dots$ denote formulas of \mathcal{L} . The notation $\models \phi$ means that the formula ϕ is valid. For any formula $\phi \in \mathcal{L}$, $S(\phi)$ denotes the set of the propositional symbols appearing in this formula. In particular, $S(\top) = S(\perp) = \emptyset$. $S_m(\phi)$ denotes the subset of the propositional symbols in $S(\phi)$ which are relevant for ϕ ’s truth value, thus it is exactly the unique¹ set of propositional symbols appearing in the “smallest” (wrt. the number of symbols) formulas logically equivalent to ϕ :

$$S_m(\phi) = S(\psi) \quad \text{iff} \quad \begin{cases} \bullet \models \psi \Leftrightarrow \phi \\ \bullet \forall \psi' \in \mathcal{L}, \\ \quad \text{if } \models \psi' \Leftrightarrow \psi \\ \quad \text{then } S(\psi) \subseteq S(\psi') \end{cases}$$

$\phi + \bar{\psi}$ denotes the formula whose relevant symbols are those of ϕ and ψ together, and such that any valuation that validates it associates (1) each relevant symbol for ϕ with the same truth value and (2) each relevant symbol for ψ but not for ϕ with the opposite value. The following properties formally define $\phi + \bar{\psi}$:

$$\begin{cases} \phi + \bar{\psi} \in \mathcal{L} \\ S_m(\phi + \bar{\psi}) = S_m(\phi) \cup S_m(\psi) \\ \forall s \in S_m(\phi), \models \phi + \bar{\psi} \Rightarrow s \quad \text{iff} \quad \models \phi \Rightarrow s \\ \forall s \in S_m(\psi) \setminus S_m(\phi), \models \phi + \bar{\psi} \Rightarrow s \quad \text{iff} \quad \models \psi \Rightarrow \neg s \end{cases}$$

3.2 Alternative

Alternatives are *states of the world*, i.e. particular interpretations of all symbols of SYM . We note them a, a_1, \dots, b, \dots . Formally, if ALT is the set of alternatives, for any alternative a of ALT and any symbol s of SYM , either $a \models s$ or $a \models \neg s$.

A formula ϕ of \mathcal{L} denotes a set of combinations of truth values for the propositional symbols of SYM , which can be seen as a set of alternatives: $\{a \text{ such that } a \in \text{ALT} \text{ and } a \models \phi\}$. In particular, the formula \top represents all the alternatives whereas \perp does not represent any alternative. Conversely, an alternative can be associated to a formula that specifies a particular truth value for every symbol of SYM . We note f_a such a formula that represents exactly the only alternative a .

3.3 Preference

A *preference* is modeled by a binary relation over the set of alternatives. In particular, a *partial preference* is a preference relation bound to a given point of view. We note the relationships associated to the partial preferences with the symbols $G_1, G_2, \dots, G_i, G'_1, \dots$ where i is a number identifying the considered point of view. The notation $a_1 G_i a_2$ means that,

¹It can be proved by contradiction that any two logically equivalent formulas have the same S_m set.

from the i^{th} point of view, the alternative a_1 is preferred to the alternative a_2 . A preference can thus be seen as a set of couples of alternatives. This set may contain contradictory couples, for example, both $aG_i b$ and $bG_i a$.

3.4 Indifference

Two alternatives are known as *indifferent* with respect to a given partial preference (or point of view) if and only if, according to this preference, each one is preferred to the other one or if none of them is preferred to the other one.

4 Primitive partial preferences

4.1 Formalization

Ceteris Paribus

The alternatives a and b of ALT are *Ceteris paribus* with respect to the formulas ϕ and ψ of \mathcal{L} (noted $CP(a, b, \phi, \psi)$), if and only if the alternatives a and b assign the same truth value to any symbol appearing neither in ϕ nor in ψ :

$$CP(a, b, \phi, \psi) \text{ iff } \begin{cases} \forall s \in \text{SYM}, \\ \text{if } s \notin S_m(\phi) \cup S_m(\psi) \\ \text{then } a \models s \text{ iff } b \models s \end{cases}$$

Definition

Each *primitive partial preference* relation is associated to a logical operator upon the formulas of \mathcal{L} . It is denoted by the symbol \succeq_i , where i is the considered point of view. The set of all \succeq_i operators define the language \mathcal{L}' : if ϕ and ψ are formulas of \mathcal{L} (but not \mathcal{L}') then $\phi \succeq_i \psi$ is a formula of \mathcal{L}' . $\phi \succeq_i \psi$ means that, from the i^{th} point of view, the alternatives verifying properties expressed in the formula ϕ and, if it is coherent, the opposite of the ones expressed in ψ , are preferred to the alternatives verifying properties expressed in the formula ψ and, if it is coherent, the opposite of the ones expressed in ϕ , if they are Ceteris Paribus with respect to the formulas ϕ and ψ (i.e. all other things, not explicitly specified by ϕ and ψ , being equal). If G_i is the primitive partial preference binary relation associated to the i^{th} point of view (as defined in 3.3), the semantics of the corresponding logical operator \succeq_i is formalized as follows:

$$\begin{aligned} \models \phi \succeq_i \psi \text{ iff } & \bullet \exists (a, b) \in \text{ALT}^2, \text{ such that } \begin{cases} a \models \phi + \bar{\psi} \\ b \models \psi + \bar{\phi} \end{cases} \\ & \bullet \left[\forall (a, b) \in \text{ALT}^2, \text{ such that } \begin{cases} a \models \phi + \bar{\psi} \\ b \models \psi + \bar{\phi} \end{cases} \right. \\ & \left. \text{if } CP(a, b, \phi, \psi) \text{ then } aG_i b \right] \end{aligned} \quad (1)$$

In practice, each primitive partial preference relation G_i is specified by an initial set of formulas of the form $\phi \succeq_i \psi$ (ϕ and ψ being themselves formulas of \mathcal{L}): these formulas are the initial data represented in figure 2. They do not necessarily induce coherent sets of preferences: they may specify both $\phi \succeq_i \psi$ and $\psi \succeq_i \phi$.

To make it possible to compare alternatives naturally, the *binary relation* supporting each primitive partial preference

must be a pre-order² (i.e. a reflexive and transitive relation). That is why the following axioms are imposed to the corresponding *logical operator* (supporting the considered primitive partial preference).

Reflexivity axiom (R)

Any consistent formula of \mathcal{L} is required to be preferred to itself:

$$\text{if } \not\models \phi \Leftrightarrow \perp \text{ then } \models \phi \succeq_i \phi \quad (\text{R})$$

This entails that the *binary relation* G_i supporting the corresponding primitive partial preference is reflexive. This ensures that two similar alternatives are indifferent.

Transitivity axiom (T)

We impose that if the formula ϕ_1 is preferred to ϕ_2 and the formula ϕ_2 is preferred to ϕ_3 , then ϕ_1 is preferred to ϕ_3 :

$$\text{if } \models \phi_1 \succeq_i \phi_2 \text{ and } \models \phi_2 \succeq_i \phi_3 \text{ then } \models \phi_1 \succeq_i \phi_3 \quad (\text{T})$$

This entails that the *binary relation* G_i supporting the corresponding primitive partial preference is transitive. This ensures that if the alternative a_1 is preferred to a_2 and the alternative a_2 is preferred to a_3 , then a_1 is preferred to a_3 .

4.2 Remarkable properties

Link between operator \succeq_i and relation G_i

As stated in section 3.2, each alternative can be identified with a formula of \mathcal{L} that specifies a truth value for each propositional symbol. Consequently, if a couple of alternatives belongs to a preference relation, then the associated formulas are connected by the corresponding logical operator. Moreover, if two formulas, that assign a value to each symbol of SYM, are connected by the logical operator, then the two associated alternatives belong to the corresponding preference relation :

$$\forall (a, b) \in \text{ALT}^2, \quad aG_i b \text{ iff } \models f_a \succeq_i f_b$$

Immediate proof by identifying the formulas ϕ and ψ with the formulas f_a and f_b in the definition (1).

Behavior of operator \succeq_i wrt. logical equivalence

If two formulas are logically equivalent, then any occurrence of the first one in a partial preference expression can be substituted for the second one and conversely. Formally:

$$\text{if } \begin{cases} \models \phi_1 \Leftrightarrow \phi_2 \\ \models \psi_1 \Leftrightarrow \psi_2 \end{cases} \text{ then } \models \phi_2 \succeq_i \psi_2 \Leftrightarrow \models \phi_1 \succeq_i \psi_1$$

As a result, if two formulas are equivalent then the former is preferred to the latter and conversely.

Proof by use of the reflexivity and transitivity properties.

Ceteris paribus (CP)

The formula ϕ is preferred to the formula ψ if one of the characteristics of ϕ is preferred to one of the characteristics of ψ and if the other characteristics of ϕ and ψ are equal. Consequently, if a formula is preferred to another one, then the specialization of the former by any criterion not yet specified is preferred to the specialization of the latter by the same criterion. Formally:

²This relation is not necessarily an order because two indifferent alternatives are not necessarily equal.

$$\text{if } \models \phi \succeq_i \psi \text{ then } \left[\begin{array}{l} \forall \gamma \in \mathcal{L} \text{ such as } \left\{ \begin{array}{l} (\gamma \wedge \phi) + \bar{\psi} \neq \perp \\ (\gamma \wedge \psi) + \bar{\phi} \neq \perp \end{array} \right. \\ \models (\phi \wedge \gamma) \succeq_i (\psi \wedge \gamma) \end{array} \right] \quad (\text{CP})$$

Proof by use of definition 1.

Addition of preferences

If the alternatives satisfying a formula ϕ are preferred to those satisfying a formula ψ when all other criteria not explicitly specified by ϕ and ψ are equal, and if, similarly, the alternatives satisfying a formula γ are preferred to those satisfying a formula ω when all other criteria are equal, then the alternatives satisfying both ϕ and γ are preferred to those satisfying both ψ and ω providing that all other criteria are equal:

$$\text{if } \left\{ \begin{array}{l} \models \phi \succeq_i \psi \\ \models \gamma \succeq_i \omega \end{array} \right. \text{ and } \left\{ \begin{array}{l} (\gamma \wedge \phi) + \bar{\psi} \neq \perp \\ (\gamma \wedge \psi) + \bar{\phi} \neq \perp \\ (\psi \wedge \gamma) + \bar{\omega} \neq \perp \\ (\psi \wedge \omega) + \bar{\gamma} \neq \perp \end{array} \right. \text{ then } \models (\phi \wedge \gamma) \succeq_i (\psi \wedge \omega)$$

Proof by use of the CP and transitivity properties.

4.3 Example

Let \mathcal{L} be a propositional language to describe some properties of the world regarding colours, which is built, as specified in section 3.1, upon the set of five propositional symbols $\text{SYM} = \{C_g, C_b, C_r, C_o, C_y\}$ (meaning respectively “green”, “blue”, “red”, “orange”, “yellow”).

Let us consider a partial preference supporting a “colour” point of view (identified with $i = 1$) and defined by the information gathered in the table of figure 3. The resulting language \mathcal{L}' (see the previous section) includes the \succeq_1 logical operator, which makes it possible to express preferences on colours under the *Ceteris Paribus* assumption. For example, $C_g \succeq_1 C_b$ means that any green object is preferred to any blue object if all other characteristics are equal.

Colour		
C_g	\succeq_1	C_b
C_r	\succeq_1	C_g
C_o	\succeq_1	C_y
C_y	\succeq_1	C_g
C_o	\succeq_1	C_g

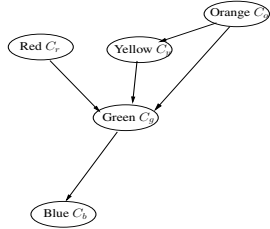


Figure 3: Specified data Figure 4: Graphical representation

Thanks to the properties formalizing primitive partial preferences, the basic data specified in figure 3 additionally entails the following formulas:

- $C_r \succeq_1 C_r$; $C_b \succeq_1 C_b$; ... (R)
- $C_r \succeq_1 C_b$; $C_o \succeq_1 C_b$; $C_j \succeq_1 C_b$ (T)
- $C_g \wedge C_r \succeq_1 C_b \wedge C_r$; $C_g \wedge C_o \succeq_1 C_b \wedge C_o$; ... (CP)

For instance, the fact “from the colour point of view, red objects are preferred to blue ones” (i.e. $C_r \succeq_1 C_b$) can be explicitly derived from the first two lines of the table and the transitivity property of the primitive partial preference operator. The initial data for specifying a partial preference (according to a given point of view) could also be represented graphically, which is easier to read for humans (see figure 4).

5 Extended partial preferences

5.1 Motivation

The properties we stated in the previous sections make it possible to practically build primitive partial preference operators (upon formulas) and thus to conveniently specify a partial preference binary relation (upon alternatives).

Nevertheless, in practice the alternatives to be compared are seldom explicitly in relation via a primitive partial preference. Indeed, the description criteria are generally interdependent, so that a difference between alternatives on a criterion implies differences on the other criteria³. Consequently, it is necessary to be able to compare two alternatives such that the former satisfies a formula $\phi \wedge \gamma$ and the latter satisfies a formula $\psi \wedge \omega$, and such that only the formula $\phi \succeq_i \psi$ of \mathcal{L}' is initially specified. This means one needs a logical operator that can handle couples of formulas of the form $(\phi \wedge \gamma, \psi \wedge \omega)$ while knowing only that $\phi \succeq_i \psi$ but without knowing anything about the couple of formulas (γ, ω) .

5.2 Example (continued)

Let us add the C and M propositional symbols (respectively meaning “car” and “motorcycle”) to the language \mathcal{L} introduced in the previous example. The logical framework we have defined for handling primitive partial preferences makes it possible to formally derive facts such as “from the colour point of view, red cars are preferred to blue cars” (i.e. $C_r \wedge C \succeq_1 C_b \wedge C$) from the previously inferred formula $C_r \succeq_1 C_b$ and the CP property. In other words, it can be explicitly deduced that an alternative verifying the property “red” is preferred to any alternative verifying the property “blue”, provided they assign the same truth values to other criteria (*Ceteris Paribus* = other things being equal). Unfortunately, if other things are *not* equal, such a deduction cannot be derived. In particular, nothing can be said about the preference between alternatives verifying the properties “red” and “car” and alternatives verifying “blue” and “motorcycle”.

5.3 Extension principle

Nevertheless, it should be possible to compare any alternatives on the basis of the initial data. That is why we make a new assumption to extend each *primitive partial preference* in an *extended partial preference*. This assumption expresses that a preference is an “argument” for preferring alternatives verifying a formula ϕ to alternatives verifying a formula ψ .

5.4 Formalization

Definition

Each *extended partial preference* relation is associated to a logical operator upon the formulas of \mathcal{L} . It is denoted by the symbol \succeq_i , where i is the considered point of view. $\phi \succeq_i \psi$ means that, from the i^{th} point of view, the alternatives verifying properties expressed in the formula ϕ and, if it is coherent, the opposite of the ones expressed in ψ , are preferred (without the *Ceteris Paribus* restriction) to the alternatives verifying properties expressed in the formula ψ and, if it is coherent,

³For example, the exact starting point of a trip often depends on the used transportation: a trip from Paris may start from the Montparnasse railway station or the Roissy airport.

the opposite of the ones expressed in ϕ . Formally, if G'_i is the extended partial preference binary relation associated to the i^{th} point of view (as defined in section 3.3), the semantics of the corresponding \geq_i operator is the following:

$$\models \phi \geq_i \psi \quad \text{iff} \quad \bullet \exists (a, b) \in \text{ALT}_i^2, \text{ such that } \left\{ \begin{array}{l} a \models \phi + \bar{\psi} \\ b \models \psi + \bar{\phi} \end{array} \right.$$

$$\bullet \left[\forall (a, b) \in \text{ALT}_i^2, \text{ such that } \left\{ \begin{array}{l} a \models \phi + \bar{\psi} \\ b \models \psi + \bar{\phi} \end{array} \right. \right. \\ \left. \left. a G'_i b \right. \right]$$

Ceteris Imparibus

The formula ϕ is *Ceteris Imparibus* preferred to ψ , according to the i^{th} point of view (noted $\text{CI}_i(\phi, \psi)$), if and only if there is a couple of formulas (ϕ', ψ') such that ϕ' is preferred to ψ' according to the i^{th} primitive partial preference (i.e. $\phi' \succeq_i \psi'$) but not conversely, and the formula ϕ subsumes the formula ϕ' and the formula ψ subsumes the formula ψ' . Formally:

$$\text{CI}_i(\phi, \psi) \quad \text{iff} \quad \left[\begin{array}{l} \exists (\phi', \psi') \in \mathcal{L}^2 \\ \text{such that } \left\{ \begin{array}{l} \models \phi' \succeq_i \psi' \\ \models \neg(\psi' \succeq_i \phi') \\ \models \phi \Rightarrow \phi' \\ \models \psi \Rightarrow \psi' \end{array} \right. \end{array} \right]$$

Construction rule

Each *extended partial preference* operator is constructed as the *ceteris imparibus* extension of the corresponding *primitive partial preference*:

$$\models \phi \geq_i \psi \quad \text{iff} \quad \left[\models \phi \succeq_i \psi \quad \text{or} \quad \text{CI}_i(\phi, \psi) \right]$$

Properties CI and T should not be imposed together on extended partial preference operators. If they are imposed together, any alternative verifying a formula ϕ would be preferred (according to the corresponding preference relation) to any other alternative verifying ψ as soon as there would be two formulas ϕ_1 and ψ_1 such that $\models \phi \geq_i \phi_1$ and $\models \psi_1 \geq_i \psi$ (proof by using the CI and T properties).

The $\neg(\psi \succeq_i \phi)$ condition in the *ceteris imparibus* property blocks the construction rule when the formulas ϕ and ψ are both preferred to each other (according to the considered primitive partial preference). Without this condition, such a rule would involve the symmetry of operators \geq_i and thus of binary relations G'_i (proof by contradiction with the R and T properties).

Note also that the binary relations induced by the operators \succeq_i and \geq_i are different. Whereas the first one, G_i , is transitive and reflexive, the second one, G'_i , is only reflexive. This is justified by the fact that G_i makes it possible to interpret the initial data and that G'_i makes it possible to exploit this information for the decision.

Indifference

The *ceteris imparibus* property implies that two alternatives are indifferent if they have both at least one strictly preferred component according to the corresponding primitive partial preference:

$$\text{if } \left\{ \begin{array}{l} \models \phi \succeq_i \psi \wedge \neg(\phi \succeq_i \psi) \\ \models \gamma \succeq_i \omega \wedge \neg(\gamma \succeq_i \omega) \end{array} \right. \quad \text{then } \left\{ \begin{array}{l} \models (\phi \wedge \omega) \geq_i (\psi \wedge \gamma) \\ \models (\psi \wedge \gamma) \geq_i (\phi \wedge \omega) \end{array} \right.$$

5.5 Example (end)

The new extended partial preference operator preserves all previous primitive partial preferences: red cars are still preferred to blue ones, i.e. $C_r \wedge C \geq_1 C_b \wedge C$ holds. Moreover, new interesting preferences can be now inferred between alternatives whose differences are not initially specified. For example, from the colour point of view, red cars are preferred to blue motorcycles (i.e. $C_r \wedge C \geq_1 C_b \wedge M$), meaning that any alternative verifying the properties “red” and “car” is preferred to any alternative verifying “blue” and “motorcycle”. The first operator (\succeq_1) is useful to deduce the “strictly” logical consequences of the initial information given about colours (see table 3). The second one (\geq_1) relies on these deductions, which only focus on differences in colour between alternatives to compare, to classify (from the colour point of view) alternatives that may differ from each other in other criteria than colour.

6 Conclusion

The preference concept is often numerically formalized in the literature. Unfortunately, such formalisms are not adapted to our cognitive agent model. In order to overcome this limitation of classical decision theory, we have chosen to formalize the preference concept logically similarly to [von Wright, 1972]. Even if information logically represented are less precise than those numerically represented, there are more easily elicited⁴.

Recently, the preference concept has been dealt with in such a qualitative way. In [Boutilier et al., 2004; Rossi et al., 2004], the concept of *Flip* (elementary change) makes it possible to compare two alternatives whose differences are not explicitly specified by a single preference expression. This work is based on the following definition: an alternative is preferred to another one if the latter results from the former by a worsening succession of elementary changes: a is preferred to b if there is a sequence of alternatives $\{a_i\}$ such that $a_1 = a$, $a_n = b$ and for all i , a_i is obtained from a_{i-1} via a single elementary change and a_{i-1} is preferred to a_i . Unfortunately, the existence of a worsening succession of elementary changes does not imply the absence of an improving one. What should be concluded in such cases? Also, as the comparisons are computed through all the conceivable alternatives within a worsening sequence, this approach does not seem to be adapted to deal with real cases, where the characteristics describing alternatives, and therefore the alternatives to consider within a sequence, are potentially numerous.

This article provides a new perspective to the problem of preference management by dividing the specification phase in two. The initial intuitive preference specifications are first interpreted by some *primitive partial preferences* according to the *Peteris Paribus* assumption. Each of them is then extended in an *extended partial preference* according to the *Ceteris Imparibus* assumption. This extended preference makes it possible to compare alternatives whose differences are not

⁴It is often natural to specify a concept of “desirability” with qualitative information upon standard alternatives (e.g. “I prefer meals with wine to those with water”).

explicitly specified by the initial data. Moreover, no explicit definition of equivalence between characteristics is needed.

Next developments concern defining the preferences of an agent by a set of partial preferences (corresponding to different points of view) within our framework. In this view, we consider dynamically solving possible contradictions between these points of view thanks to an aggregation phase (defined for example as in [Rossi *et al.*, 2004]), in order to build a new mental attitude that would represent the agent's global opinion. This method is justified by the idea developed in [Dubois *et al.*, 2001] according to which the multicriteria decision and the multi-agents decision are two different approaches of the same problem. Each of our *partial preferences* can be identified with an agent's preference in the model proposed by Rossi *et al.* Another research direction is to take into account some hierarchical information to specify the partial preferences. For example, the preference "green 4x4 are preferred to red 4x4" may be more important than the preference "red cars are preferred to green cars" because it is more specific. This is related to the problems of default and incomplete information handling in the field of knowledge representation. Finally, it would be also interesting to study the link between preference and desire, and in particular to check whether desiring something could be seen as preferring it to its opposite.

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