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► **To cite this version:**

François Konschelle, Jérôme Cayssol, Alexandre I. Buzdin. Superconducting fluctuations near the FFLO phase. 25th International Conference on Low Temperature Physics (LT25), Aug 2008, Amsterdam, Netherlands. pp.052121 (1-4), 10.1088/1742-6596/150/5/052121 . hal-00319673

HAL Id: hal-00319673

<https://hal.science/hal-00319673>

Submitted on 9 Sep 2008

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Superconducting fluctuations near the FFLO state

François Konschelle, Jérôme Cayssol and Alexandre I. Buzdin*

Abstract. The Fulde, Ferrell, Larkin & Ovchinnikov (FFLO) state consists in a modulation of the superconducting order parameter due to Zeeman effect. In a Ginzburg-Landau approach, higher order terms than usual in the gradient expansion, *i.e.* quartic terms, are needed to take into account the FFLO modulation. The role of this quartic term in addition to the quadratic usual term in a Gaussian fluctuation spectrum have been investigated for heat capacity C and paraconductivity σ near a FFLO state for both isotropic and anisotropic cases. In the isotropic (resp. anisotropic) case, the power laws are drastically different (resp. similar) in comparison with the homogeneous superconductivity case. Nevertheless, for the anisotropic case the anisotropic ratio σ_{xx}/σ_{yy} is quite different for a FFLO phase than for a BCS one. In addition, we predict anomalous power laws near the tricritical point where the normal phase and the two superconducting (uniform and FFLO) phases are merging. The multiple crossovers associated with the phase transitions between homogeneous, tricritical and inhomogeneous fluctuation regimes thus may serve as a powerful tool to identify the FFLO phases.

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Despite of an important number of possible candidates suggested for about 40 years, the quest for the mysterious FFLO phase (for Fulde, Ferrell, Larkin and Ovchinnikov [1, 2]) has been subjected to a total renewal with the discovery of an intriguing phase in CeCoIn_5 compound (for a recent review, see [3]). The key ingredient of such phases is the Zeeman effect. Owing to the Zeeman effect, the singlet Cooper pair acquires a finite global momentum, inducing a spatial modulation over the whole superconducting state [4]. More recently, cold atom gases with unbalanced population of fermions have also attracted a lot of interest (for a review, see [5]).

We focus ourselves on the Gaussian fluctuation properties of the normal phase, in the vicinity of a second order phase transition towards the FFLO phase (see Fig.1.a). Near a modulated phase, the coefficient of the gradient term in a Ginzburg-Landau (GL) functional disappears at a certain ratio between the temperature T and the magnetic exchange field h [6]. This tricritical point (T^*, h^*) separates the normal phase to the both homogeneous and inhomogeneous superconducting states. It follows from the disappearance of the condensate stiffness that a finite wave vector appears, inducing several new physical properties associated with modulated phases [3, 4, 5]. In the isotropic case, the corresponding modulation is degenerate and lies within a sphere or a ring, depending on the dimensionality. In the anisotropic case, the wave vector is located along the high symmetry lines of the crystal lattice (or of the gap symmetry).

Previously, we have identified three regimes characterized by different fluctuation properties [8]. The regime I (see Fig.1.a) corresponds to the usual BCS transition (with uniform condensate) [9, 10], while the regimes II and III exhibit different properties due to the vicinity of the

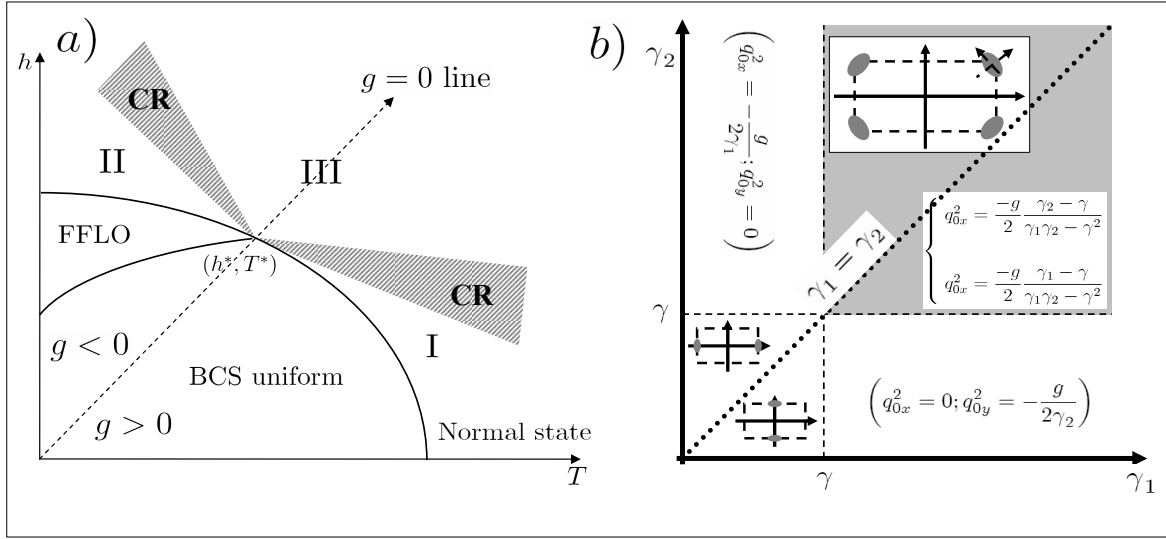


Figure 1. *a)* Schematic (h, T) phase diagram for FFLO superconductors, where h is the magnetic field and T is the temperature. The solid lines represent the transition between the normal metal, the spatially uniform BCS superconductor, and the non uniform FFLO state. The fluctuation regimes I, II and III correspond respectively to the uniform BCS state ($g > 0$), to the FFLO state ($g < 0$) and to the tricritical regime ($g \rightarrow 0$) [8]. Gray hatched regions represent the crossover regimes between the various fluctuation cases. *b)* Mean-field treatment of the rectangular anisotropy model with respect to γ_1 and γ_2 . The gray area represents the diagonal location of the modulation wave vector \mathbf{q}_0 whereas the white areas represent the locations of \mathbf{q}_0 on the axis. The corresponding \mathbf{q}_0 are also explicitly given in each case.

inhomogeneous superconducting state. More precisely, the regime II corresponds to a well established modulation over the superconducting condensate, whereas the regime III corresponds to the region near the tricritical point, where homogeneous and inhomogeneous phases compete. In the present communication, we first briefly outline the results of [8] for an isotropic fluctuation spectrum. Then, we provide additional results in the case of anisotropic systems which is the most relevant for real compounds. Finally, we show that the critical region is rather small for superconductors, hence justifying the use of a Gaussian model.

The Gaussian functional

$$H[\Psi] = \sum_{\mathbf{k}} \left[a \left(T - \tilde{T}_c \right) + g_i k_i^2 + \gamma_{ij} k_i^2 k_j^2 \right] |\Psi_{\mathbf{k}}|^2 = \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} |\Psi_{\mathbf{k}}|^2 \quad (1)$$

will be used in the following, where \tilde{T}_c is the critical temperature associated with the normal to homogeneous superconducting phase transition, and the tensors g_i and γ_{ij} characterize the stiffness of the condensate. As the $g_i \rightarrow 0$ near the tricritical point, and become negative for high magnetic fields and low temperatures, higher order terms (like the γ_{ij} ones) are required in the GL functional [6, 7]. In the standard GL approach, where $\gamma_{ij} = 0$, there are just the g_i components that characterize the anisotropy, and the isotropy of the GL functional can be restored by simply rescaling the axis in momentum space. In contrast, near a FFLO phase, the presence of the γ_{ij} terms change the behavior of the fluctuation specific heat and paraconductivity. Moreover, this γ_{ij} tensor is very sensitive to the detailed form of the Fermi surface.

Near a FFLO isotropic phase, where $g_i = g$ and $\gamma_{ij} = \gamma \delta_{ij}$, the following fluctuation heat

capacity C and paraconductivity σ_{xx} have been obtained [8]:

$$C = \frac{\pi A_d k_B}{4} \sqrt{\frac{aT_c}{\gamma}} \left(\frac{|g|}{2\gamma}\right)^{\frac{d-2}{2}} \left[\frac{T-T_c}{T_c}\right]^{-3/2}; \quad \sigma_{xx} = \frac{(\pi e)^2 A_d k_B}{4\hbar d} \sqrt{\frac{\gamma}{aT_c}} \left(\frac{|g|}{2\gamma}\right)^d \left[\frac{T-T_c}{T_c}\right]^{-3/2} \quad (2)$$

where $A_d = 1/\pi, 1/2\pi, 1/2\pi^2$ for $d = 1, 2, 3$ respectively. In strong contrast with the GL case [9, 10], the phase transition to a modulated phase exhibits a $-3/2$ Gaussian exponent independent on the dimensionality d . Although the above results are somewhat academic for superconductors, they may be relevant for cold fermionic gases which have spherical, or elliptical Fermi surface.

For real superconductors, the isotropic model is not sufficient. We now consider two-dimensional compounds with tetragonal anisotropy, taking in mind that orbital effects is reduced in two-dimensional layered compounds with in-plane magnetic field. In tetragonal anisotropy, the fluctuation spectrum is given by $\varepsilon_{\mathbf{k}} = a(T - \tilde{T}_c) + gk^2 + \gamma_1 k_x^4 + \gamma_2 k_y^4 + 2\gamma k_x^2 k_y^2$. The mean-field diagram associated with this spectrum with respect to γ_1 and γ_2 is given in Fig.1.b. It exhibits either four minima on the diagonal (gray region in Fig.1.b), or only two on the axis (white regions in Fig.1.b). In the on- Ox axis case, the fluctuation spectrum is expanded as $\varepsilon_{\mathbf{k}} \approx a(T - T_c) - 2g\kappa_x^2 - g(\gamma - \gamma_1)\kappa_y^2/\gamma_1$ with $T_c = \tilde{T}_c + g^2/4a\gamma_1$.

The fluctuation heat capacity and paraconductivity are finally obtained as:

$$C = \frac{k_B}{4\pi} \frac{aT_c}{\sqrt{2g^2}} \sqrt{\frac{\gamma_1}{\gamma - \gamma_1}} \left[\frac{T - T_c}{T_c}\right]^{-1} \quad \text{and} \quad \sigma_{xx} = \frac{e^2 a k_B}{8\hbar} \sqrt{\frac{2\gamma_1}{\gamma - \gamma_1}} \left[\frac{T - T_c}{T_c}\right]^{-1} \quad (3)$$

where the Gaussian exponent -1 is exactly the same as for the homogeneous case [10]. The particular behavior of the FFLO transition is thus lost when switching from the ideally isotropic model to the more realistic anisotropic case: the universal exponents of the homogeneous case are recovered. As the low-energy fluctuations are located within small islands centered at several isolated points, it is natural to find the BCS exponents. Indeed, in the usual BCS case, there is just one pocket of low-energy fluctuation modes centered at the origin of the momentum space.

Nevertheless, the anisotropic ratio $\sigma_{xx}/\sigma_{yy} = 2\gamma_1/(\gamma\eta - \gamma_1)$ is very different near a FFLO phase than near a uniform one. In fact, for the latter case, and for a fluctuation spectrum taking into account non-local effects proportional to k^4 , there would be no anisotropy ($\sigma_{xx}/\sigma_{yy} = 1$), whereas $\sigma_{xx}/\sigma_{yy} \neq 1$ in the former case.

Moreover, the most peculiar properties of the MGL functional is the fact that $g_i \rightarrow 0$ inducing FFLO phase at $(T < T^*, h > h^*)$. When $g_i \rightarrow 0$, the fluctuation spectrum (1) becomes purely quartic (regime III on Fig.1.a). Note that this quartic regime separates regimes I and II, inducing crossovers in the (h, T) phase diagram, represented in Fig.1.a by hatched gray regions. Using the spectrum: $\varepsilon_{\mathbf{k}} = a(T - T^*) + \gamma_{ij}k_i^2 k_j^2$ yields

$$C^* \approx \frac{k_B}{\det \gamma} (aT^*)^{d/4} \left[\frac{T - T^*}{T^*}\right]^{\frac{d-8}{4}} \quad \text{and} \quad \sigma_{\alpha\beta} \approx \frac{\pi e^2 k_B}{\hbar} \frac{(aT^*)^{\frac{d-2}{4}}}{\sqrt[4]{\det \gamma}} \left[\frac{T - T^*}{T^*}\right]^{\frac{d-6}{4}} \quad (4)$$

for fluctuation heat capacity and paraconductivity. Gaussian divergencies of modulated systems are thus characterized by a transitive regime between the FFLO and the BCS ones wherein the stiffness g_i disappears. This regime is characterized by very soft (quartic) fluctuating modes which reveal the change of the sign of g , and hence the presence of the tricritical point.

Let us now note that a complete treatment of the Ginzburg-Levanyuk parameter [9, 10] will be given in [11]. It follows from this study that the Gaussian approach is valid in temperature

regions given by

$$\frac{T - T_c}{T_c} \gg \tilde{g}^{2/3} \left(\frac{T_c}{E_F} \right)^{2/3} \quad \text{and} \quad \frac{T - T^*}{T^*} \gg \frac{T^*}{E_F} \quad (5)$$

for the two-dimensional isotropic case in regime II and for the two-dimensional tricritical regime III, respectively. In Eq.(5), \tilde{g} is a dimensionless quantity which vanishes at the tricritical point, and E_F is the Fermi energy. Note that the anisotropic case exhibits the same Ginzburg-Levanyuk parameter than for BCS superconductors [9, 10]. Thus, the critical region, even though modified by the fluctuation spectrum discussed in this communication, remains quasi-inobservable for typical superconducting compounds. This finally justify the use of Gaussian approximation in studying fluctuation properties of FFLO states.

In conclusion, we have derived the fluctuation properties of the normal state in the vicinity of modulated phases. We have found two other regimes in addition to the conventional BCS one (see Fig.1.a). These two regimes are characterized by a quartic term in the fluctuation spectrum in addition to the usual quadratic one [9, 10]. This quartic term is required by the disappearance of the stiffness condensate g at the tricritical point. Moreover, as $g = 0$ near a constant h/T ratio in the phase diagram [6], a characteristic regime (which is universal of all systems with vanishing stiffness) of purely quartic expansion of the fluctuation spectrum has been pointed out [8, 11]. The crossovers between these three regimes may be observed by measuring the specific heat and the anisotropy of paraconductivity along the normal to superconducting phase transition line

The authors would like to acknowledge Drs. Jean-Noël FUCHS, Manuel HOUZET, Catherine PÉPIN and Yann LOUYER for enlightening discussions. This work was supported by ANR Extreme Conditions Correlated Electrons (ANR-06-BLAN-0220).

- [1] Fulde P and Ferrell R A 1964 *Phys. Rev.* **135** A550–A563
- [2] Larkin A and Ovchinnikov Y 1965 *Sov. Phys. JETP* **20** 762–769
- [3] Matsuda Y and Shimahara H 2007 *J. Phys. Soc. Jap.* **76** 051005
- [4] Casalbuoni R and Nardulli G 2004 *Rev. Mod. Phys.* **76** 263–320
- [5] Sheehy D and Radzihovsky L 2007 *Ann. Phys.* **322** 1790–1924
- [6] Buzdin A I and Kachkachi H 1997 *Phys. Lett. A* **225** 341–348
- [7] Agterberg D and Yang K 2001 *J. Phys.: Condens. Matter* **13** 9259–9270
- [8] Korschelle F, Cayssol J and Buzdin A I 2007 *Europhys. Lett.* **79** 67001
- [9] Landau L D and Lifshitz E M 1980 *Course of theoretical physics, Volume 5 : Statistical Physics, 3rd edition* (Butterworth-Heinemann)
- [10] Larkin A and Varlamov A 2003 *The physics of superconductors, Vol. I* (K.H. Bennemann and J.B. Ketterson, Springer-Verlag) chap 3, Fluctuation phenomena in superconductors
- [11] Korschelle F, Cayssol J and Buzdin A I 2008 *To be published*