



Statistical Properties of Line Centroid Velocities and Centroid Velocity Increments in Compressible Turbulence

D. C. Lis, J. Pety, T. G. Phillips, E. Falgarone

► To cite this version:

D. C. Lis, J. Pety, T. G. Phillips, E. Falgarone. Statistical Properties of Line Centroid Velocities and Centroid Velocity Increments in Compressible Turbulence. The Astrophysical Journal, 1996, 463, pp.623. <10.1086/177276>. <hal-00315752>

HAL Id: hal-00315752

<https://hal.science/hal-00315752v1>

Submitted on 18 May 2014

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



HAL Authorization

STATISTICAL PROPERTIES OF LINE CENTROID VELOCITIES AND CENTROID VELOCITY INCREMENTS IN COMPRESSIBLE TURBULENCE

D. C. LIS, J. PETY, AND T. G. PHILLIPS

California Institute of Technology, Downs Laboratory of Physics, 320-47, Pasadena, CA 91125

AND

E. FALGARONE

CNRS URA 336, Ecole Normale Supérieure, 24 rue Lhomond, 75005, Paris, France

Received 1995 September 13; accepted 1995 December 8

ABSTRACT

We have calculated probability density functions (PDFs) of centroid velocities and centroid velocity increments of line profiles computed from the output of a 512^3 simulation of compressible turbulence. The PDFs of centroid velocities calculated over the whole data cube are roughly Gaussian. On a smaller scale, non-Gaussian PDFs are observed in some cases. However, this is far from being the rule, and most of the distributions show relatively minor deviations from a Gaussian. By contrast, PDFs of *centroid velocity increments* clearly show non-Gaussian wings that are associated with regions of increased vorticity in the flow and thus appear related to the phenomenon of intermittency. Investigations of PDFs of centroid velocity increments in non-star-forming regions thus seem a promising avenue for studying the intermittency in the interstellar medium, as an alternative to the line-shape approach.

Subject headings: hydrodynamics — ISM: clouds — ISM: kinematics and dynamics — turbulence

1. INTRODUCTION

There is considerable evidence that turbulence plays an important role in the dynamics of interstellar molecular clouds. One of the approaches for study of turbulence in the interstellar medium is the analysis of the shapes of molecular line profiles as proposed by Falgarone & Phillips (1990). They argued that the non-Gaussian CO line wings in *inactive regions without associated star formation activity* represent a direct observational signature of the turbulent nature of the gas flow within molecular clouds and of the existence of regions of intermittent turbulent activity. In a subsequent study, Falgarone et al. (1994) made a comparison between the molecular line profiles of non-star-forming dense molecular clouds and those calculated from the output of a 512^3 hydrodynamic code for compressible turbulence computed by Porter, Pouquet, & Woodward (1994). Falgarone et al. (1994) found that the vorticity peaks ascribed to intermittency contribute to the non-Gaussian features and wings of the line profiles, but at an intensity level that is weak compared to the contribution of the bulk of the flow. Dubinski, Narayan, & Phillips (1995) showed, also, that non-Gaussian line profiles can be produced simply from a random velocity field with a Kolmogorov spectrum.

Turbulent activity and dissipation are known, from flow experiments and numerical simulations, to be highly concentrated in space and time. Finding the small regions undergoing such an intermittent activity in the interstellar medium might bring important clues to the understanding of its dynamical evolution. Our knowledge of the interstellar medium relies on observations that provide only average values of its physical properties. These average properties are not necessarily representative of the values of the gas characteristics (temperature, density, etc.) that are of relevance in describing the physical processes and the evolution of the medium. Indeed, the fluctuations of some properties can be so large that the average value provides inadequate information. For instance, a very rich and specific chemistry can be driven in atomic interstellar clouds in

the small regions transiently heated by intermittent turbulent dissipation (Falgarone & Puget 1995; Falgarone, Pineau des Forêts, & Roueff 1995). Intermittency also locally affects the relative velocities and possibly the size distribution of very small dust particles and large molecules. These changes, in turn, modify the thermal local properties of the gas, and may be the seed of major changes in the subsequent evolution of turbulent clouds.

In a recent observational study, Miesch & Scalo (1995) analyzed probability density functions (PDFs) of the ^{13}CO centroid velocities in several star-forming molecular clouds. They concluded that, while some of the PDFs are approximately Gaussian, most of them exhibit relatively broad, often nearly exponential wings, similar to those in the PDFs of velocity increments and derivatives found in experiments and numerical simulations of incompressible turbulence. Since none of the existing theoretical studies or numerical simulations considered the characteristics of the PDFs of the line-of-sight centroid velocity in turbulent flows, to help evaluate the Miesch & Scalo method and results, we study here the statistical properties of the centroid velocity and *centroid velocity increments* based on the simulation results of Porter et al. (1994). It is interesting to try to find improved ways to isolate the signatures of intermittency. It is known that the effects are greater for the higher line moments and for differential measurements such as velocity increments. Falgarone & Phillips (1990) studied line wings, but intermittency can probably be better found in a velocity-increment study, so centroid velocity increments should provide a new window on the effect. Studies of higher line moments are difficult from the observational viewpoint, since they require mapping of extended regions with very high signal-to-noise ratio.

2. ANALYSIS

Our study of the statistical properties of the centroid velocities is based on the output of the 512^3 hydrodynamic simulation computed by Porter et al. (1994). The simulation and its relevance to the interstellar medium is described in

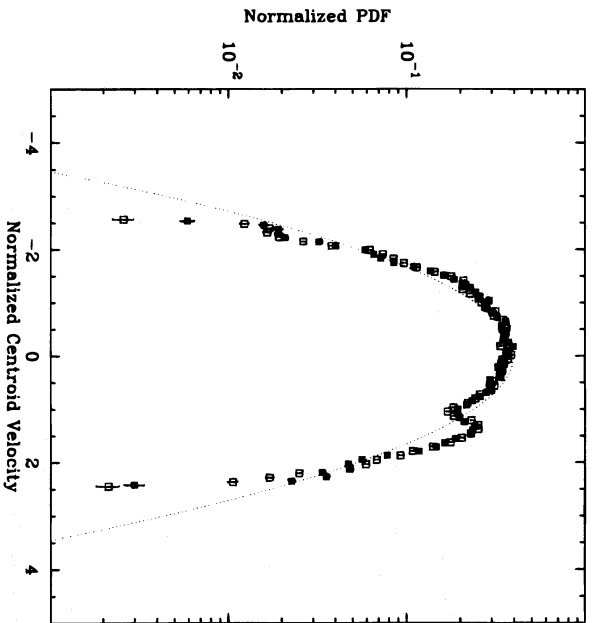


Fig. 1.—PDFs of the centroid velocity for 512^2 lines of sight on the face of the data cube. The velocity scale has been normalized so that the mean velocity $v_{\text{mean}} = 0$ and the dispersion $\sigma = 1$. Filled and open squares correspond to uniformly weighted and density-weighted centroid velocities, respectively. The dotted curve represents a Gaussian distribution with $v_{\text{mean}} = 0$ and $\sigma = 1$.

detail in Falgarone et al. (1994). The results presented here correspond to the simulation epoch that best represents the interstellar medium spectra ($t = 1.2 \tau_{ac}$, where the acoustic time, τ_{ac} , is the sound crossing time of the energy-containing scale). The power spectrum during this epoch is close to that predicted by Kolmogorov for incompressible turbulence [$E(k) \approx k^{-5/3}$], and the density contrast is $\rho_{\text{max}}/\rho_{\text{min}} \sim 40$.

In the computations of the centroid velocities,¹ $\langle v_x \rangle$, where v_x is the line-of-sight velocity component, we treated all the 512^2 lines of sight on the face of the data cube as independent spectra. We used both density weighting, which is appropriate for optically thin lines, and uniform weighting, which is more appropriate for optically thick lines originating in a clumpy, supersonic medium. The average difference between the centroid velocities calculated with a uniform and density weighting is $\lesssim 3\%$ of the range in the centroid velocity and the maximum difference is $\lesssim 25\%$.

2.1. Probability Density Functions of Centroid Velocities and Centroid Velocity Increments

PDFs of the centroid velocities for all 512^2 lines of sight on the face of the data cube are shown in Figure 1. To facilitate the shape comparison, all the PDFs of the centroid velocities and the centroid velocity increments discussed here are normalized to a zero mean and a standard deviation of unity. A normalized Gaussian distribution is also shown as a dotted line. The PDFs based on centroid velocities calculated with uniform weighting and density weighting are plotted as filled and open squares, respectively. The error bars due to the limited number of points in a given histogram bin are also shown. The centroid velocity PDFs for the uniform and density weighting are similar and close to a Gaussian distribution; there is no evidence for the

presence of exponential wings. In order to determine whether such wings may arise on a smaller scale, we show in Figure 2 centroid velocity PDFs for 16 subcubes of the size $128 \times 128 \times 512$. Although most of the distributions in Figure 2 do not diverge significantly from a Gaussian, in some cases non-Gaussian wings may be found. Only in one case (Fig. 2, *upper right panel*), an exponential wing is present on one side of the histogram.

Both experimental studies of laboratory and atmospheric flows (e.g., Anselmet et al. 1984; Gagne 1987) and the numerical simulations of turbulence (e.g., Vincent & Meneguzzi 1991) have demonstrated that broad, near-exponential wings are found in the PDFs of velocity increments and derivatives, while the PDFs of the velocity components built over the whole system are approximately Gaussian. We therefore calculated PDFs of the centroid velocity increments between pairs of nearby lines of sight [$\langle v_x \rangle(r_1) - \langle v_x \rangle(r_2)$]. For a given line of sight, r_1 , we considered all lines of sight with a separation $|r_1 - r_2|$ between $\Delta - 0.5$ and $\Delta + 0.5$ pixels. The resulting PDFs for a separation $\Delta = 5$ pixels are shown in Figure 3. In contrast to the PDFs of the centroid velocities (Fig. 1), the PDFs of the centroid velocity increments (Fig. 3) clearly show wings above the Gaussian distribution. However, these wings are much less pronounced than those in the PDFs of the *velocity increments* calculated for the same separation over the whole data cube (Fig. 3, *solid gray line*).

The evolution of the PDFs of the centroid velocity increments with the separation Δ is shown in Figure 4. The magnitude of the wings decreases monotonically with increasing separation. For separations $\Delta \gtrsim 40$ of PDFs are very close to a Gaussian. This behavior is the same as seen in the PDFs of velocity increments (e.g., Vincent & Meneguzzi 1991). The difference between the PDFs built from centroid velocities computed with uniform and density weighting is relatively small, although the density-weighted PDFs have consistently more pronounced wings. This indicates that the lines of sight with the largest centroid velocity increments are also characterized by an increased density compared to the bulk of the flow. This correlation may be partly due to shocks that are still present in the epoch of the simulation we study. As discussed by Porter et al. (1994), vorticity is generated by shock interactions and amplified by stretchings. Therefore, some of the high-vorticity regions must be associated with high densities.

The simulation of Porter et al. (1994) is for a fully developed compressible turbulence without gravity, star formation, etc. As a result, there is no need to filter any “systematic” motions. However, to make our analysis consistent with that in the observational study of Miesch & Scalo (1995), we examined the effect of filtering the “large-scale” motions on the resulting PDFs. Figure 5 shows the centroid velocity of PDFs calculated from the filtered centroid map as a function of the size of the boxcar filter, 2β . The evolution of these PDFs with β is comparable to that of the PDFs of the centroid velocity increments with the separation Δ shown in Figure 4. For a given increment Δ , the increment method selects a narrow range of wavenumbers around the wavenumber k_Δ from the power spectrum of the centroid velocity. The moving filter average is equivalent to a high-pass filter, which, for a given value of β , passes all wavenumbers above the cutoff wavenumber k_β . However, as the velocity power spectrum in a turbulent flow is steep, the spectrum of the centroid velocity is also

¹ The computations presented here were carried out using the Cray X-MP2E/232 supercomputer located at the Jet Propulsion Laboratory.

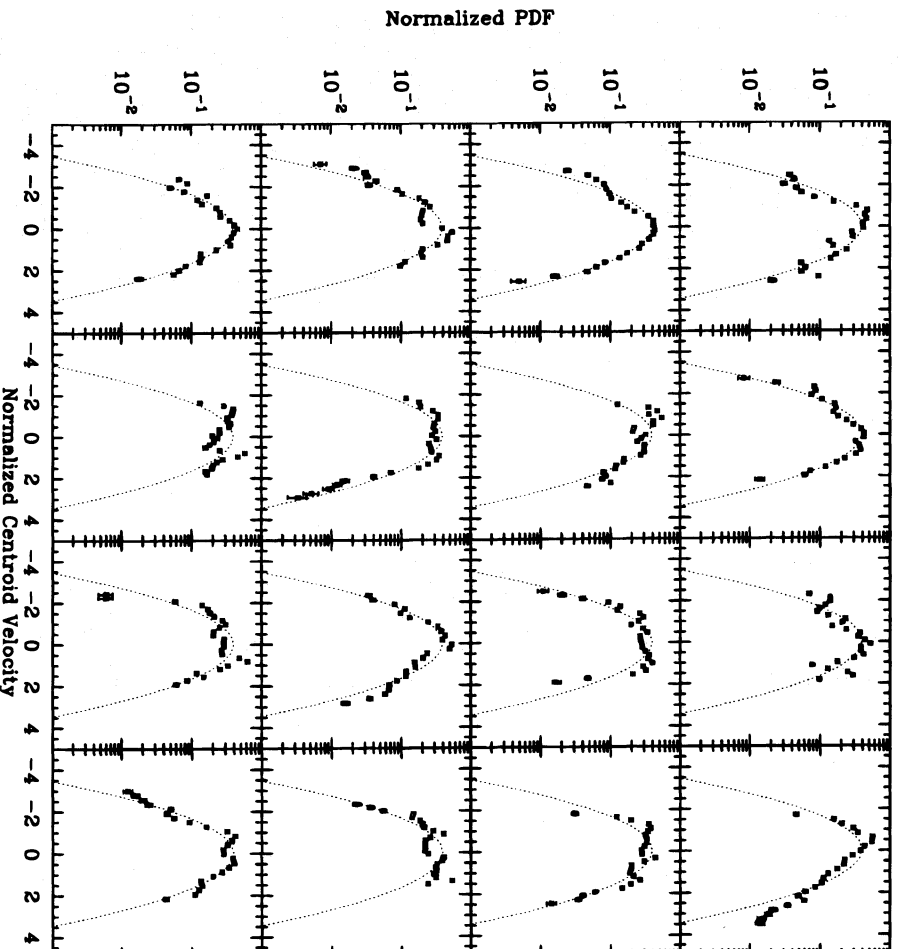


Fig. 2.—PDFs of the *centroid velocity* for 16 subcubes of size $128 \times 128 \times 512$. The velocity scale is normalized as in Fig. 1. Only PDFs for uniformly weighted centroid velocities are shown.

steep. Thus wavenumbers close to the cutoff wavenumber k_β dominate the spectrum that goes through the high-pass filter. Accordingly, in a velocity field with a steep power spectrum, the two methods give similar results for $\Delta = \beta$.

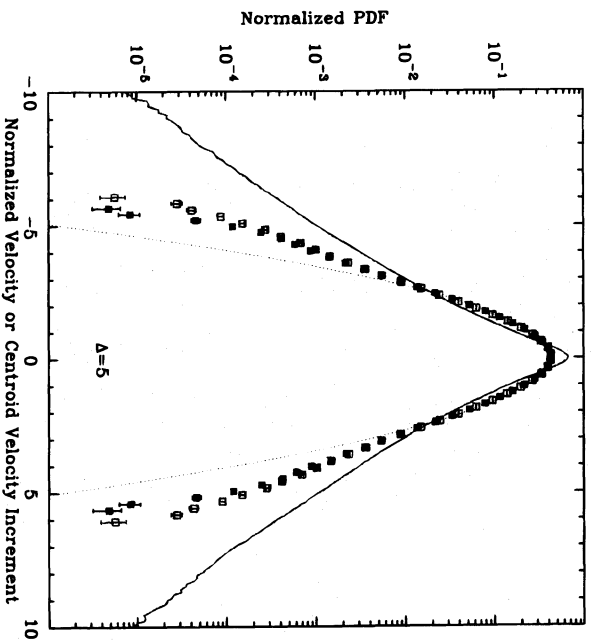


Fig. 3.—PDFs of the *centroid velocity increments* between nearby lines of sights, $\langle v_x \rangle(r + \Delta) - \langle v_x \rangle(r)$ for a separation $\Delta = 5$ pixels. Symbols are as in Fig. 1. The PDF of velocity increments, $v_x(x, y, z) - v_x(x, y + \Delta, z)$ calculated for the same separation over the whole data cube is shown as solid gray line.

One advantage of forming the PDFs of the centroid velocity increments is that the number of points used is larger and the statistics is greatly improved.

The evolution of the wings in the PDFs of the filtered centroid velocity with the size of the boxcar filter (Fig. 5) suggests that one should use the smallest possible filter size when applying this method to the observational data. In their study, Miesch & Scalo (1995) used the largest filter size for which the asymmetric lobes were no longer discernible in the autocorrelation function of the residual centroid map. The ratio of the effective scale of the observations (taken to be the square root of the total number of points) to the filter size was between 2 and 3.5. If the wings found by Miesch & Scalo represent a signature of intermittency of turbulence in their fields, our study suggests that the magnitude of these wings should be enhanced if a smaller filter size is used in the data processing. However, as discussed above, the simulation of Porter et al. (1994) does not include gravity and star formation. Therefore, our results may not be applicable directly to the fields studied by Miesch & Scalo.

2.2. *Origin of the Wings in the PDFs of the Centroid Velocity Increments*

In order to determine whether the non-Gaussian wings, found in the simulation for the PDFs of the centroid velocity increments, are related to regions of increased vorticity in the flow associated with intermittency, we studied the distribution of the lines of sight contributing to these wings on the face of the data cube. Figure 6 shows the distribution

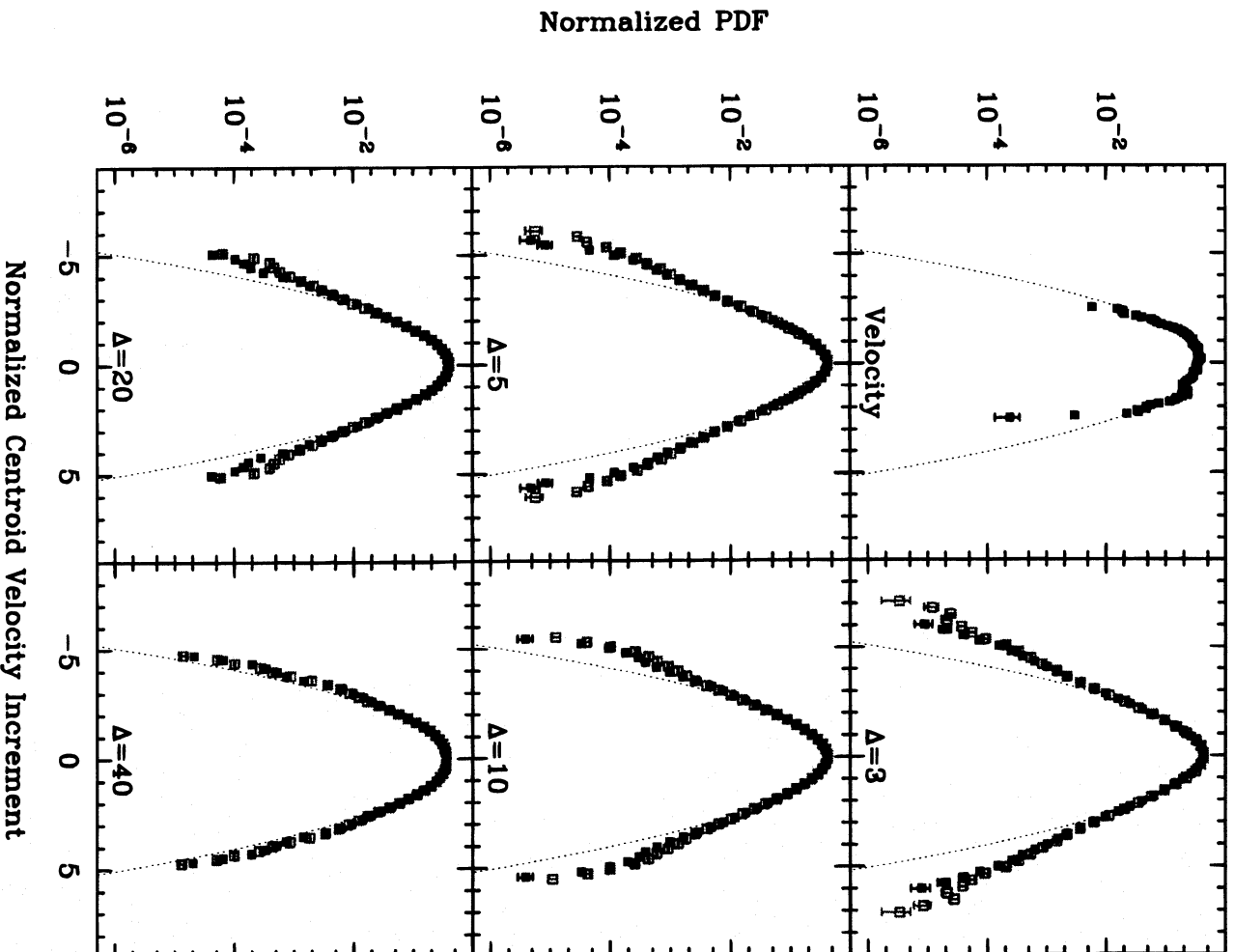


FIG. 4.—PDFs of the centroid velocity increments for separations $\Delta = 3, 5, 10, 20$, and 40 pixels. Symbols are as in Fig. 1. The magnitude of the non-Gaussian wings decreases with increasing Δ . For separations $\Delta \gtrsim 40$ pixels, the distribution is essentially Gaussian.

of the lines of sight above a threshold of 3.0 in the normalized PDF of the centroid velocity increments, for a separation $\Delta = 5$ and uniform weighting. This threshold marks the transition between Gaussian and non-Gaussian statistics for the increments (Fig. 3, *filled squares*). The lines of sight contributing to the wings of the density-weighted PDF (Fig. 3, *open squares*) have a very similar distribution (93% of the points overlap). Figure 6 shows that the lines of sight contributing to the wings of the PDFs of the centroid velocity increments are not randomly distributed. Instead, they trace a filamentary structure, which is related to the distribution of the two vorticity components, which involve cross-derivatives of the line-of-sight component of the velocity field. The distribution of the average velocity component perpendicular to the line of sight ($[\langle(\text{rot } \mathbf{v})_y\rangle^2 + \langle(\text{rot } \mathbf{v})_z\rangle^2]^{1/2}$, where $\langle(\text{rot } \mathbf{v})_y\rangle$ and $\langle(\text{rot } \mathbf{v})_z\rangle$ are the two vorticity components in the plane of the sky averaged along

the line of sight) is shown in Figure 7. The coincidence between the filaments in Figures 6 and 7 shows that the increment method can be used effectively to find the location of the maxima of the average vorticity component perpendicular to the line of sight.

It can be shown analytically that the line-of-sight average of a vorticity component i in the plane of the sky can be written as $\langle(\text{rot } \mathbf{v})_i\rangle = \partial_i \langle v_x \rangle + [v_i(0) - v_i(L)]/L$. The first term on the right-hand side is simply a cross-derivative of the centroid velocity (the quantity we sample with the increment method). The second term is a large-scale contribution of the vorticity that disappears in the simulation because of periodic boundary conditions. This term does not cancel out in the interstellar medium, therefore measurements of the centroid velocity increments may be contaminated and the correlation between the distribution of the points selected using the increments method with the

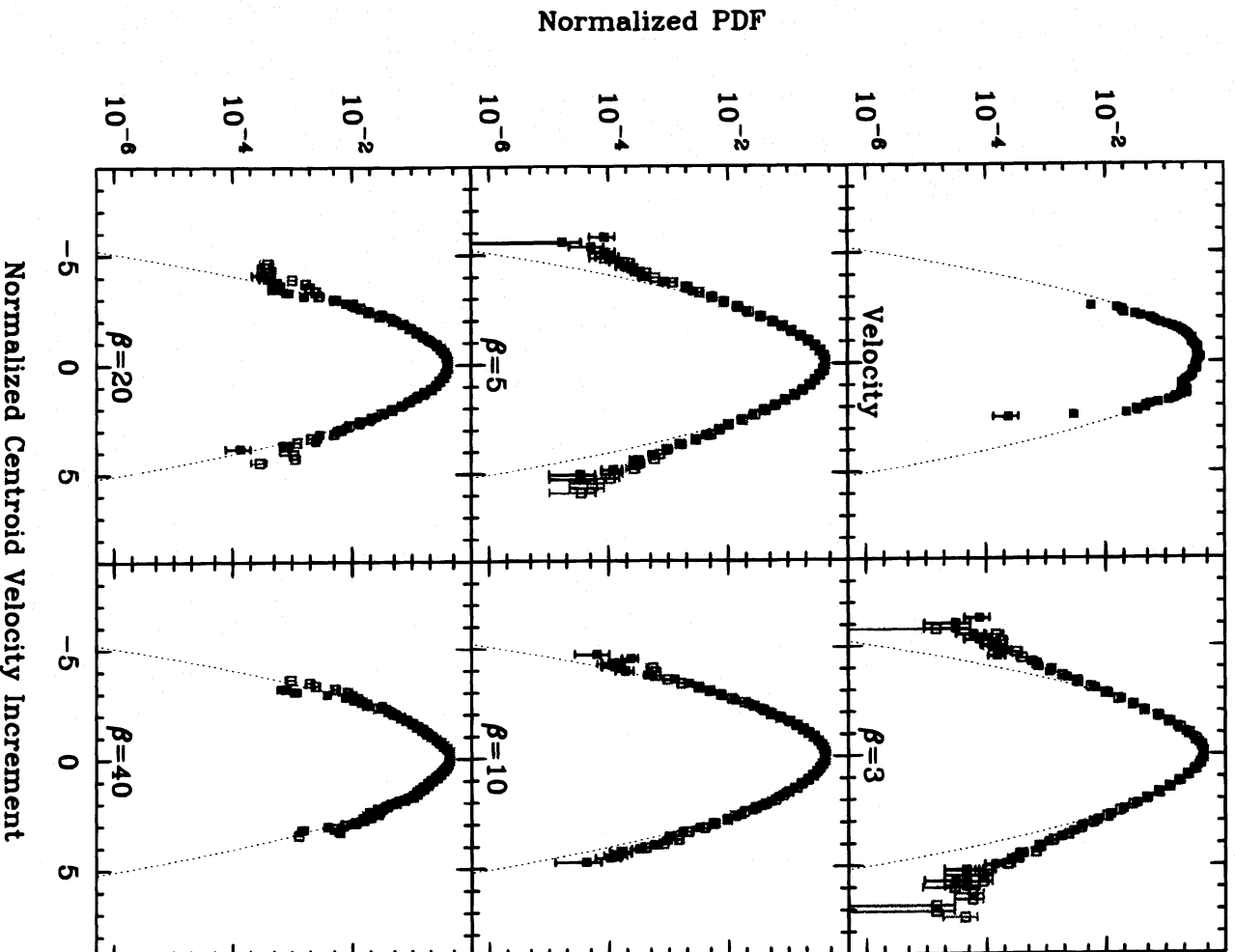


FIG. 5.—PDFs of the filtered centroid velocity after subtracting a smooth image calculated using a boxcar filter with size 2θ . Symbols are as in Fig. 1.

vorticity filaments may not be as good as in the simulation. However, in Kolmogorov turbulence $\Delta v/\Delta l \sim \Delta l^{-2/3}$, so that the large-scale contribution of the vorticity is much smaller than the small-scale contribution. In addition, intermittency further enhances the small-scale velocity differences relative to the large-scale ones. We have also verified that the correlation between the distribution of the points selected with the increment method and the filaments in the distribution of the average vorticity component in the plane of the sky does not change when the calculations are performed over a fraction of the data cube (128 points along the line of sight, instead of 512). This suggests that the boundary conditions due to the turbulent component of the velocity field do not have a significant effect on our results. The term due to the boundary conditions may not be negligible in the presence of large-scale rotation. However,

the rotation term will vary slowly with position and can thus be separated easily from the fluctuating term due to turbulence.

The distribution of the lines of sight selected with the increment method (Fig. 6) does not correlate equally well with the maxima in the vorticity modulus averaged along the line of sight, since the vorticity modulus depends also on the two velocity components in the plane of the sky which are not observed. However, the lines of sight selected using the increment method are also characterized by an *increased* line-of-sight-averaged vorticity modulus, *compared to the bulk of the flow*. Figure 8 shows the vorticity PDFs for the lines of sight above thresholds of 3.0 and 4.0 (*filled circles and squares, respectively*) in the PDF of the centroid velocity increments for a separation $\Delta = 5$ and uniform weighting (Fig. 3, *filled squares*). The overpopu-

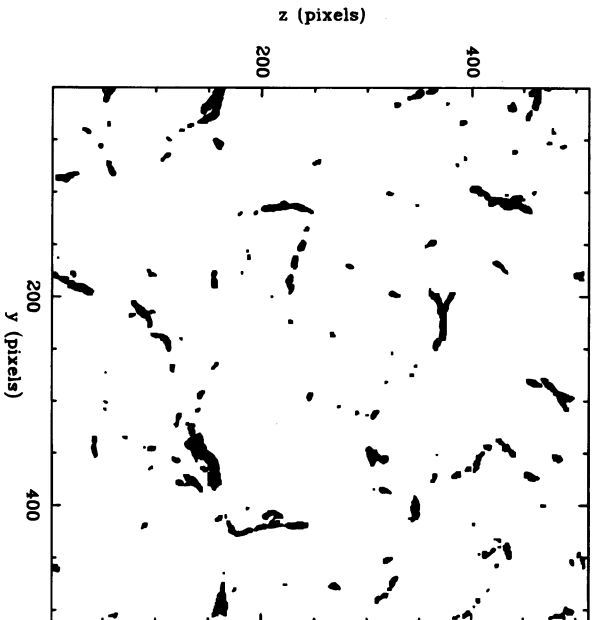


Fig. 6.—Spatial distribution of points with a centroid velocity increment greater than 3.0 in the PDF corresponding to $\Delta = 5$ with uniform weighting (Fig. 3, *filled squares*; density weighting leads to a very similar distribution).

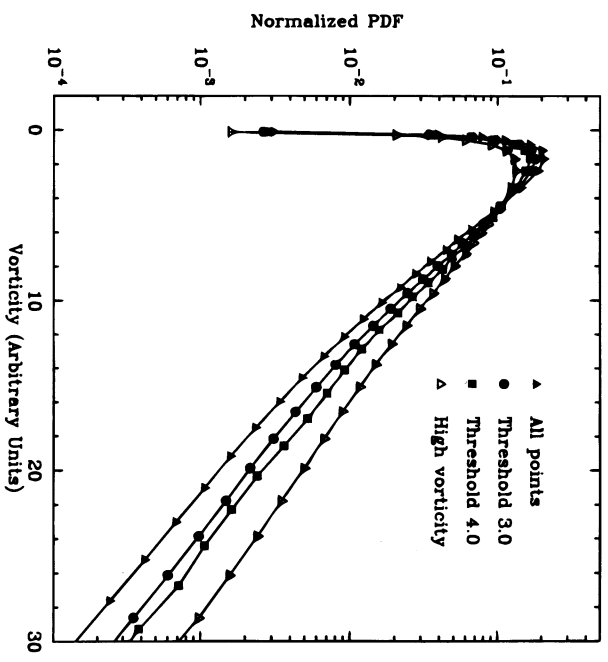


Fig. 8.—Comparison of the vorticity PDF for the whole data cube (*filled triangles*) with that for the lines of sight that contribute to the non-Gaussian wings in the PDF of the centroid velocity increments for a separation $\Delta = 5$ pixels. Filled circles and squares correspond to thresholds of 3.0 and 4.0, respectively (see Fig. 3). Open triangles show the vorticity PDF for the lines of sight with the average modulus of vorticity greater than 6.

lation of the tail of the vorticity PDFs for the lines of sight contributing to the wings in the PDF of the centroid velocity increments, compared to the average for the whole data cube (*filled triangles*), suggests that these wings are associ-

ated with some of the zones of increased vorticity assignable to intermittency (e.g., Porter, Pouquet, & Woodward 1992). The tail in the vorticity PDF for the lines of sight selected using the increment method is less pronounced than for

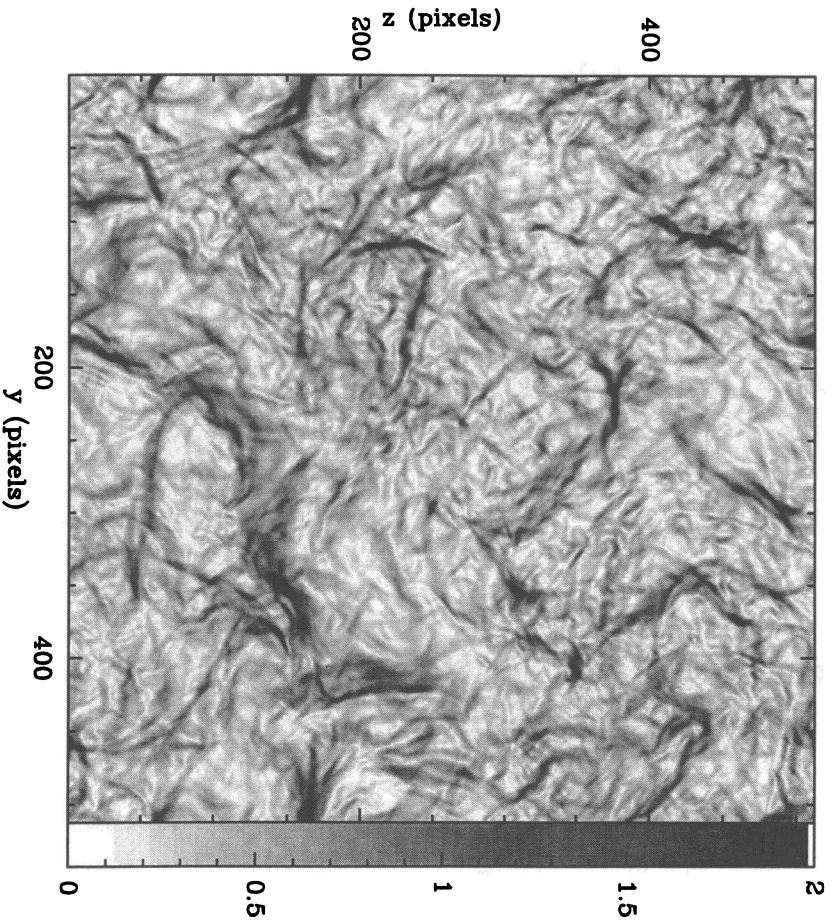


Fig. 7.—Spatial distribution of $\langle[(\text{rot } \theta)_y]^2 + [(\text{rot } \theta)_z]^2]^{1/2}$, where $\langle(\text{rot } \theta)_y\rangle$ and $\langle(\text{rot } \theta)_z\rangle$ are the two components of the vorticity in the plane of the sky averaged along the line of sight.

those with the highest average modulus of the vorticity (Fig. 8, *open squares*). As discussed above, this is related to the fact that we observe only one component of the velocity field.

2.3. Relevance to the Interstellar Medium

Our results are based on a simulation of decaying supersonic, compressible turbulence, which does not include gravity, magnetic fields, and star formation. All those missing effects may be important in varying degrees in interstellar molecular clouds of different types. In addition, the velocity field in molecular clouds is, in general, not statistically homogeneous. Therefore, one should be cautious when trying to extrapolate our results to the interstellar medium. Complications due to star formation can be largely avoided by studying inactive regions, like high-latitude clouds. We believe that the effect of intermittency of the velocity field is so fundamental that it should manifest itself in the interstellar medium. For instance, recent two- and three-dimensional numerical simulations of MHD turbulence with comparable magnetic and kinetic energies (e.g., Biskamp, Welter, & Walter 1990; M. Rieutord 1995, private communication) show that the existence of magnetic field does not destroy the phenomenon of intermittency in the velocity field, as revealed by the non-Gaussian wings in the velocity increments. Non-Gaussian wings in the PDFs of the centroid velocity increments may possibly be attributed to effects other than the intermittency of turbulence. However, the filamentary distribution of the lines of sight selected by the increment method is a solid prediction of the present work that can be tested observationally.

3. SUMMARY

Our study of the statistical properties of the line centroid velocity in a turbulent, compressible but *non-star-forming and gravitationless* simulation shows the following:

1. The PDF of the *centroid velocity* calculated for all lines of sight on the face of the data cube is close to a Gaussian, although deviations from a Gaussian are present on smaller scale.

2. The PDFs of the *centroid velocity increments* clearly show non-Gaussian wings. The magnitude of these wings decreases with increasing separation Δ . For a given separation, the magnitude of these wings is smaller than those found in PDFs of *velocity increments* calculated over the whole data cube.

3. Non-Gaussian wings also appear in PDFs of the centroid velocity after “large-scale” motions have been filtered out by subtracting a smooth image. In a turbulent velocity field with a steep power spectrum, this approach is partially equivalent to calculating centroid velocity increments. However, the increment method gives much better statistics.

4. The lines of sight contributing to the non-Gaussian wings of the PDFs of centroid velocity increments trace a filamentary structure, which follows the distribution of the two vorticity components involving cross-derivatives of the line-of-sight component of the velocity field.

5. The increment method allows for selecting lines of sight with *increased* average vorticity compared to the bulk of the flow (although not those with the *highest* vorticity, which cannot be localized with observations of only one velocity component). The non-Gaussian wings in the PDFs of centroid velocity increments thus appear to have their origin in the phenomenon of intermittency.

Although one has to be cautious when extrapolating these results to the interstellar medium, we believe that the methods discussed in this paper provide a useful new tool for studying the effects of intermittency of turbulence on physics and chemistry of the interstellar medium and should be applicable to non-star-forming quiescent molecular clouds.

We thank D. H. Porter, A. Pouquet, and P. R. Woodward for providing us with the result of their hydrodynamic simulation. This research has been supported by NSF grant AST 93-13929 to the Caltech Submillimeter Observatory and by the JPL Superconducting Project, which is sponsored by JPL and the NASA Office of Space Science and Applications.

REFERENCES

- Anselmet, F., Gagne, Y., Hopfinger, E. J., & Antonia, R. A. 1984, *J. Fluid Mech.*, 140, 63
- Biskamp, D., Welter, H., & Walter, M. 1990, *Phys. Fluids B*, 2, 3024
- Dubinski, J., Narayan, R., & Phillips, T. G. 1995, *ApJ*, 448, 226
- Falgarone, E., Lis, D. C., Phillips, T. G., Pouquet, A., Porter, D. H., & Woodward, P. R. 1994, *ApJ*, 436, 728
- Falgarone, E., & Phillips, T. G. 1990, *ApJ*, 359, 344
- Falgarone, E., Pineau des Forêts, G., & Roueff, E. 1995, *A&A*, 300, 870
- Falgarone, E., & Puget, J. L. 1995, *A&A*, 293, 830
- Gagne, Y. 1987, Thèse d'Etat, Université de Grenoble
- Miesch, M. S., & Scalo, J. M. 1995, *ApJ*, 450, L27
- Porter, D. H., Pouquet, A., & Woodward, P. R. 1992, *Theor. Comp. Fluid Dyn.*, 4, 13
- Vincent, A., & Meneguzzi, M. 1991, *J. Fluid Mech.*, 225, 1