



HAL
open science

Proposal of a method for real cables EMC modeling

Olivier Maurice

► **To cite this version:**

| Olivier Maurice. Proposal of a method for real cables EMC modeling. 2008. hal-00311651

HAL Id: hal-00311651

<https://hal.science/hal-00311651>

Preprint submitted on 19 Aug 2008

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Proposal of a method for real cables EMC modeling

Olivier Maurice

23. juillet 2008

Abstract

This paper deals with a method for the real cables EMC (ElectroMagnetic Compatibility) modeling. This method gives a simple and efficient approach available for the electromagnetic compatibility (EMC). The objective is firstly to take into account the continuous interlacing of the wires. To do that we start from an analytical law of the location of a wire inside the cable. From this law it is possible to construct relations for the characteristic impedance and the wave vector of the real cable. In a second step we compute the cross talk between the wires inside the cable using the relations developed by Vabre. The final equations of the lines can be used both in the time and frequency domain, based on a Branin's circuit. Finally we can study the case of a harness which is a connection of various cables.

1. Introduction

When we measure the characteristic impedance of a wire inside a cable (a group of wires), we can see that it changes quite fastly from the center to the edge between two values. We first show a simple method to evaluate this change. This can be done for a single wire or for a pair of wires in a cable. By a similar approach we can make an estimation of the wave vector computing the capacitance for each location of the wire. Having these two parameters and making the hypothesis that from a location to the next one the mismatched of the line is weak, we can obtain the whole line propagation and input impedance from an energy argument. After what, using the Vabre's relations we can estimate the cross talk between the line in the cable.

2. Frame, location and geometry of the cable

To define a law of the location of a wire inside the cable, we have to define a frame attached with the cable. Figure 1 shows a choice of frame for a typical cable.

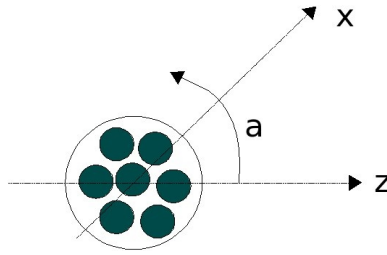


Figure 1

The location of a wire inside the cable can be defined in distance from the center -z- and rotation -a- for a length on the direction x. Each wire has its own diameter. For each wire n we can define a law giving its location all along the cable. For example:

$$\begin{cases} z(n, x) = z_0(n) \cos \left(2\pi \frac{x}{T_z} + z_p(n) \right) \\ a(n, x) = a(n) \sin \left(2\pi \frac{x}{T_a} + a_p(n) \right) \end{cases}$$

equation 1

$Z_0(n)$ and $a(n)$ are the amplitude of movement of the wire in the cable. $Z_p(n)$ and $a_p(n)$ are their values at the beginning of the cable, and T_z and T_a are the periods of interlacing. The location of the wire n depending on x in the directions z and a define completely the cable.

3. Characteristic impedance

We know the formulas giving the characteristic impedance of one or two wires inside a metallic enclosure or upon a ground plane [1]. Figure 2 presents a typical configuration of a line made of two wires inside a circular shield.

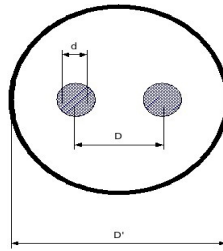


Figure 2

The characteristic impedance increases from the center to the edge. We can obtain an evaluation of its values in the center and on the edge (here for a single wire line):

$$\text{internal wire: } Z_c \approx 60 \ln \left(\frac{d_e}{d_i} \right)$$

$$\text{external wire: } Z_c \approx 60 \ln \left(\frac{4h}{d} \right)$$

equation 2

Figure 3 shows the signification of the parameters used in the equation 2 (for $e=0$). In some cases, from the inductances L and capacitances C the characteristic impedance can be evaluated directly using :

$$Z_c(\mathbf{x}) = \sqrt{\frac{L(\mathbf{x})}{C(\mathbf{x})}}$$

equation 3

Some special software can be used too in order to compute the characteristic impedance of the wire for various location inside the cable. We can compute the evolution of the characteristic impedance using the relations 2 from the center to the edge of the cable. The python program used to compute Z_c is given in appendix B.

The values computed are given on the curve shown figure 4. This curve is the law for a single wire of the change of its characteristic impedance depending on its location in a shield.

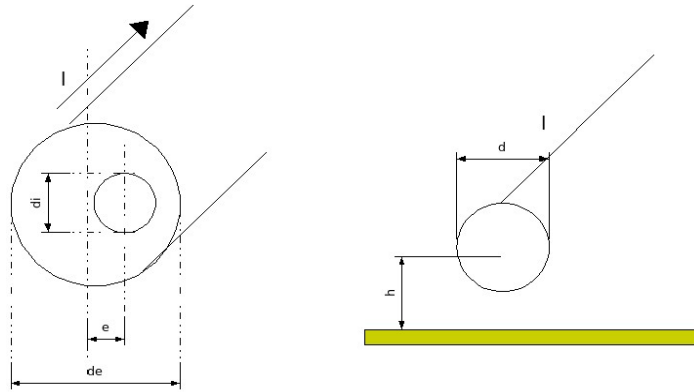


Figure 3

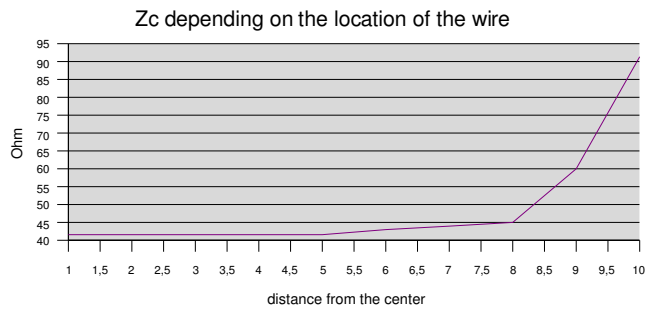


Figure 4

From this curve we can think that the characteristic impedance follows in general an exponential law depending on the axe z:

$$Z_c(z) = Z_{c_0} \exp(\alpha z)$$

equation 4

This kind of law can be obtain too for a two wires line inside a cable. Using equation 1 (here, under our hypothesis, we don't have an influence of a) we can define a characteristic impedance which depends on x. We have here:

$$Z_c(n, x) = Z_{c_0} \exp \left(\alpha z_0(n) \cos \left(2\pi \frac{x}{T_z} + z_p(n) \right) \right)$$

equation 5

Using this relation we can compute a coefficient of reflexion seen from the input of the

line. If we accept that the characteristic impedance is near to be purely real (it means that the spectrum of the reflected wave is reduced to a Dirac's pulse), we can describe the wave $O(t)$ seen from the input (for a normalized input voltage):

$$O(t) = \sigma^+(\mathbf{dx}) \delta_{\frac{2\mathbf{dx}}{v_g}} + \left(1 + \sigma^+(\mathbf{dx})\right) \sigma^+(2\mathbf{dx}) \left(1 + \sigma^-(\mathbf{dx})\right) \delta_{\frac{4\mathbf{dx}}{v_g}} + \dots$$

equation 6

v_g is the group speed we will define later, σ^+ being a wave going from left to right and σ^- a wave going from right to left. As the reflexion between two location separated of dx is very weak (this is in accordance with the hypothesis of continuous change in the location of the wire), we can neglect the products of these coefficients. So the relation 6 can be reduced to:

$$O(t) \approx \sigma^+(\mathbf{dx}) \delta_{\frac{2\mathbf{dx}}{v_g}} + \sigma^+(2\mathbf{dx}) \delta_{\frac{4\mathbf{dx}}{v_g}} + \dots \Rightarrow O(t) = \int_t dt \sigma^+(\mathbf{x} = v_g t) \delta_{\frac{\mathbf{dx}}{v_g}}$$

equation 7

The value of σ^+ can be computed too under the same hypothesis:

$$d\sigma^+(\mathbf{x}) = \frac{Zc(\mathbf{x} + \mathbf{dx}) - Zc(\mathbf{x})}{Zc(\mathbf{x} + \mathbf{dx}) + Zc(\mathbf{x})} \approx \frac{\mathbf{dx}}{2Zc(\mathbf{x})} \frac{\partial Zc(\mathbf{x})}{\partial \mathbf{x}} \Rightarrow \sigma^+ = \frac{1}{2} \int_x \left(\frac{\mathbf{dx}}{Zc(\mathbf{x})} \frac{\partial Zc(\mathbf{x})}{\partial \mathbf{x}} \right)$$

equation 8

Having define $Zc(x)$, $\sigma^+(x)$ is defined using relation 8.

4. Group speed

The average speed for all the line length X must be the integration of all the delays all along the line. We can write (n being the numbering of one line on the cable):

$$\frac{X}{v_g} = \frac{1}{N} \sum_{n=1}^N \left(\int_{x=0}^X \frac{\mathbf{dx}}{v_n(\mathbf{x})} \right) \Rightarrow v_g = \frac{X}{\frac{1}{N} \sum_{n=1}^N \left(\int_{x=0}^X \frac{\mathbf{dx}}{v_n(\mathbf{x})} \right)}$$

equation 9

The local speed at a distance x being given by:

$$v_n(\mathbf{x}) = \frac{1}{C_n(\mathbf{x}) Zc_n(\mathbf{x})}$$

equation 10

The computation of $C_n(x)$ can be made using a finite element software which resolve the Poisson's law. The values of C_n can be computed at various x in order to find, using a polynomial regression, the law for $C_n(x)$.

5. Transmission

For a matched line, we can write the energy assessment between the source (V_s), the reflected energy (O) and the transmitted one (T) for a lossless line:

$$\int_t dt \left(\frac{V_s^2(t)}{Z_c} - \frac{O^2(t)}{Z_c} \right) = \int_t dt \frac{T^2(t)}{Z_c}$$

equation 11

As Z_c is real and positive and $O^2(t)$ must be inferior to $V_s^2(t)$, we can write:

$$T(t) = \sqrt{V_s^2(t) - O^2(t)}$$

equation 12

For a normalized source we obtain the classical transmitted power relation for real values of the loads and of the characteristic impedances:

$$T(t) = \sqrt{1 - (\sigma^+(t))^2}$$

equation 13

This relation is always available only in the frequency domain, but when the impedances are purely real, the white noise band of a fixed impedance is a Dirac's pulse. The convolution product becomes a simple product where the real impedance value is a simple coefficient. Equation 13 gives the value of the incident wave on a matched load. This is sufficient to develop a Branin's model for the line. This model is based on the temporal response of one line inside the cable for a normalized Dirac's source. The response to any temporal signal is obtain making the convolution product between this signal and the pulse response of the line. If we remember that the coefficient of reflexion is very weak, we can evaluate the voltage value for each location in the direction x (here we consider Δx as dx). Writing:

$$V(x) \approx V_0 (1 + \sigma^+(dx))(1 + \sigma^+(2dx)) \dots = V_0 \prod_n (1 + \sigma^+(ndx))$$

equation 14

A similar equality is obtained for the value of current at a location x :

$$I(\mathbf{x}) \approx I_0 (1 - \sigma^+(\mathbf{dx})) (1 - \sigma^+(2\mathbf{dx})) \dots = I_0 \prod_n (1 - \sigma^+(n\mathbf{dx}))$$

equation 15

From these relations we can compute the electric and magnetic coupling.

6. The temporal Branin's model

As $O(t)$ and $T(t)$ are computed for a Dirac source, the response to any signal is obtained using a convolution product. Figure 5 shows a schematic of the model and of its operations. This model can be transformed in the frequency domain. In this case, the convolution product is replaced by a simple one. The figure 5 is the beginning of a more complete schematic that will include the cross talk between the wires of the cable.

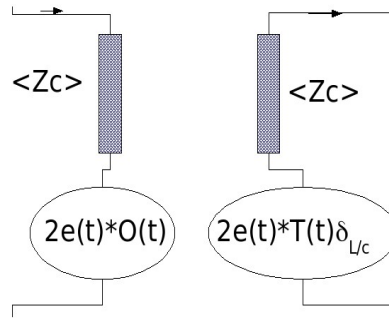


Figure 5

7. Cross talk between wires

To define completely the cable we have to find the relations to compute the cross talk between the wires in the cable.

The first work is to start from the Vabre's relation and to extend them for non homogeneous lines. Appendix A recall the demonstration made by Vabre in his study of pulsed electronics [2].

The near end cross talk for a step source on the culprit line is a step function too but the far end cross talk is the time derivation of the step function. To compute the signal cross talked on the culprit lines we must define a step function with a finite time rise.

With this knowledge we can conclude that the near end cross talk can be a copy of the modulated step obtained with a non homogeneous culprit line. By the same approach we can think that the far end cross talk should be a time derivation of the modulated step. But to make these hypothesis, we suppose that the victim line is homogeneous, which is not the

case.

We have to define the coupling factor between the lines at each location of the cable. It means to define the inductance and capacitance coupling for all values of the line x direction.

As previously we can obtain laws for $Zc\#(x)$ and $v\#(x)$, where the $\#$ symbol means that the computation is made between the two lines culprit and victim ones.

From the differential $Zc\#$ we can estimate the capacitance g and the height of the culprit line. From the height we can compute the mutual inductance M [4]. We suppose we know $g(x)$, $M(x)$ and $v(x)$ per meter of line of given Zc . For the far end cross talk at distance X on the line 2 for a known current on the line 1 at the location x we have (the source wave comes from the « left » of the line at the value $x=0$):

$$de_2(X) = M(x) dx \frac{dI^1(x)}{dt} \prod_x^X [1 + \sigma_2^+(x)] * \delta\left(\frac{X-x}{v_g}\right)$$

equation 16

By analogy the current induced on the line 2 by a time changing of the voltage on line 1 is given by:

$$dI^2(x) = g(x) dx \frac{dV_1(x)}{dt} \prod_x^X [1 - \sigma_2^+(x)] * \delta\left(\frac{X-x}{v_g}\right)$$

equation 17

The two generators de_2 and dI^2 must be compute separately. For the whole length of the line, we must integrate the previous relations over the length X to obtain:

$$e_2(X) = \int_0^X dx \left\{ M(x) dx \frac{dI^1(x)}{dt} \prod_x^X [1 + \sigma_2^+(x)] * \delta\left(\frac{X-x}{v_g}\right) \right\}$$

$$I^2(X) = \int_0^X dx \left\{ g(x) dx \frac{dV_1(x)}{dt} \prod_x^X [1 - \sigma_2^+(x)] * \delta\left(\frac{X-x}{v_g}\right) \right\}$$

equation 18

For the near end cross talk, the physic process is an integration over the time of the wave induced locally. We obtain:

$$e_2(0) = \int_{t=0}^{\frac{2X}{v_g}} dt \left(M(x) dx \frac{dI^1(x)}{dt} \prod_x^0 [1 + \sigma_2^-(x)] * \delta \left(\frac{X-x}{v_g} \right) \right)$$

$$I^2(0) = \int_{t=0}^{\frac{2X}{v_g}} dt \left(g(x) dx \frac{dV_1(x)}{dt} \prod_x^0 [1 - \sigma_2^-(x)] * \delta \left(\frac{X-x}{v_g} \right) \right)$$

equation 19

The index added on the reflexion coefficient point out the wire considered for the propagation.

8. Complete model

In order to construct a complete model of the real cable we start from the Branin's one used in the SPICE software [3]. Figure 6 recalls this model for a basic single line.

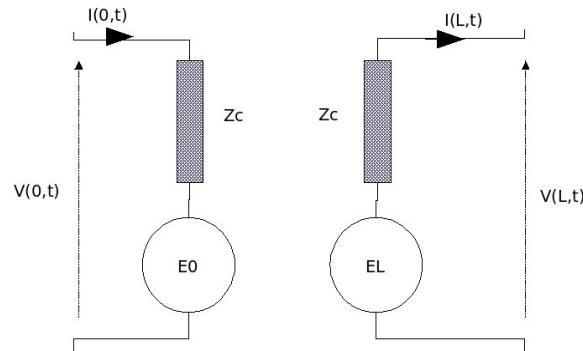


Figure 6

On the figure, the two generators E_0 and E_L are defined by ($L=X$):

$$E_0 = V(L, t-T) - ZcI(L, t-T) \quad E_L = V(0, t-T) + ZcI(0, t-T)$$

equation 20

T is the electrical line length (X/v_g). To use what we have done before we must create four operators. The operator « own » (**o**) that translates the energy transmitted from the input of a line to the output of the same line; an operator « near end cross talk » (**p**) that translates the energy cross talked from N lines to the input of another one, and an operator « far end cross talk » (**t**) that translates the energy cross talked from N lines to the output of another one. From what was discussed previously we write for the line a :

$$\begin{aligned} V_a(\mathbf{X}) &= {}^v O_{.a} V_a(0) \Rightarrow {}^v O_{.a} = \prod_{n=1}^N [1 + \sigma_a^+(\mathbf{ndx})] \\ I^a(\mathbf{X}) &= {}^I O_{.a} I^a(0) \Rightarrow {}^I O_{.a} = \prod_{n=1}^N [1 - \sigma_a^+(\mathbf{ndx})] \end{aligned}$$

equation 21

For the far end cross talk we obtain:

$$\begin{aligned} e_b(\mathbf{X}) &= t_{ba}(I^a(\mathbf{x})) \quad \text{with:} \\ t_{ba} &= \sum_a \left\{ \int_0^{\mathbf{X}} d\mathbf{x} \left[M_{ba}(\mathbf{x}) \frac{d(\cdot)}{dt} \prod_x^{\mathbf{X}} [1 - \sigma_b^+(\mathbf{x})] * \delta \left(\frac{\mathbf{X} - \mathbf{x}}{v_g} \right) \right] \right\} \\ I^b(\mathbf{X}) &= t^{ba}(V_a(\mathbf{x})) \quad \text{with:} \\ t^{ba} &= \sum_a \left\{ \int_0^{\mathbf{X}} d\mathbf{x} \left[g^{ba}(\mathbf{x}) \frac{d(\cdot)}{dt} \prod_x^{\mathbf{X}} [1 + \sigma_b^+(\mathbf{x})] * \delta \left(\frac{\mathbf{X} - \mathbf{x}}{v_g} \right) \right] \right\} \end{aligned}$$

equation 22

For the near end cross talk we have:

$$\begin{aligned} e_b(0) &= p_{ba}(I^a(v_g t)) \quad \text{with:} \\ p_{ba} &= \sum_a \left\{ \int_0^{\frac{2\mathbf{X}}{v_g}} dt \left[M_{ba}(\mathbf{x}) v_g \frac{d(\cdot)}{dt} \prod_x^0 [1 - \sigma_b^-(\mathbf{x})] * \delta \left(\frac{\mathbf{X} - \mathbf{x}}{v_g} \right) \right] \right\} \\ I^b(0) &= p^{ba}(V_a(v_g t)) \quad \text{with:} \\ p^{ba} &= \sum_a \left\{ \int_0^{\frac{2\mathbf{X}}{v_g}} dt \left[g^{ba}(\mathbf{x}) v_g \frac{d(\cdot)}{dt} \prod_x^0 [1 + \sigma_b^-(\mathbf{x})] * \delta \left(\frac{\mathbf{X} - \mathbf{x}}{v_g} \right) \right] \right\} \end{aligned}$$

equation 23

Having defined the distances, capacitances and characteristic impedances of all the line in the cable, we can compute, using equation 20 and 21 the voltage $V_a(\mathbf{x})$ and currents $I^a(\mathbf{x})$ of all the lines, from time to time for step function generators connected to these lines. For each time step the near end cross talk can be computed using 23. The global cross talk operator matrice obtained can then be used one time derivated and through a convolution product for any kind of sources.

For the wave as for the reflexion coefficient we need to specify if the wave is a direct or a backward one when we want to construct the complete relations between the waves that runs between the wires.

We add an exponent plus for the direct waves and a minus one for the backward waves.

The final relation for the far end source is:

$$\begin{aligned} \mathbf{E}_a(\mathbf{L}) = & {}^v\mathbf{O}_{a,a} \mathbf{V}_a^+(\mathbf{0}) + \mathbf{t}_{ab} \left(\mathbf{I}^{+b}(\mathbf{x}) \right) + \mathbf{p}_{ab} \left(\mathbf{I}^b(\mathbf{v}_g \mathbf{t}) \right) + \dots \\ & \dots + \mathbf{Zc} \left\{ \mathbf{I}_{a,a}^{\mathbf{O}} \mathbf{I}^{+a}(\mathbf{0}) + \mathbf{t}^{ab} \left(\mathbf{V}_b^+(\mathbf{x}) \right) + \mathbf{p}^{ab} \left(\mathbf{V}_b^-(\mathbf{v}_g \mathbf{t}) \right) \right\} \end{aligned}$$

equation 24

And for the near end source:

$$\begin{aligned} \mathbf{E}_a(\mathbf{0}) = & {}^v\mathbf{O}_{a,a} \mathbf{V}_a^-(\mathbf{L}) + \mathbf{t}_{ab} \left(\mathbf{I}^b(\mathbf{x}) \right) + \mathbf{p}_{ab} \left(\mathbf{I}^{+b}(\mathbf{v}_g \mathbf{t}) \right) + \dots \\ & \dots - \mathbf{Zc} \left\{ \mathbf{I}_{a,a}^{\mathbf{O}} \mathbf{I}^a(\mathbf{L}) + \mathbf{t}^{ab} \left(\mathbf{V}_b^-(\mathbf{x}) \right) + \mathbf{p}^{ab} \left(\mathbf{V}_b^+(\mathbf{v}_g \mathbf{t}) \right) \right\} \end{aligned}$$

equation 25

The function \mathbf{O} acts on the voltage and current at the locations \mathbf{L} or $\mathbf{0}$. To notice on what location they act, we add an index with $\mathbf{0}$ or \mathbf{L} after a comma. This allows to resume the previous relations knowing that $\mathbf{O}_{b,b}^a = \mathbf{0}$ if $a \neq b$. We write:

$$\begin{aligned} \mathbf{E}_a(\mathbf{L}) = & \left({}^v\mathbf{O}_{a,0}^{b,b} + \mathbf{Zc} \mathbf{t}^{ab} \right) \mathbf{V}_b^+(\mathbf{x}) + \mathbf{Zc} \mathbf{p}^{ab} \mathbf{V}_b^-(\mathbf{v}_g \mathbf{t}) + \dots \\ & \dots + \left(\mathbf{t}_{ab} + \mathbf{Zc} \mathbf{I}_{b,0}^{\mathbf{O}^a} \right) \mathbf{I}^{+b}(\mathbf{x}) + \mathbf{p}_{ab} \mathbf{I}^b(\mathbf{v}_g \mathbf{t}) \end{aligned}$$

equation 26

And by consequence:

$$\begin{aligned} \mathbf{E}_a(\mathbf{0}) = & \left({}^v\mathbf{O}_{a,L}^{b,b} - \mathbf{Zc} \mathbf{t}^{ab} \right) \mathbf{V}_b^-(\mathbf{x}) - \mathbf{Zc} \mathbf{p}^{ab} \mathbf{V}_b^+(\mathbf{v}_g \mathbf{t}) + \dots \\ & \dots + \left(\mathbf{t}_{ab} - \mathbf{Zc} \mathbf{I}_{b,L}^{\mathbf{O}^a} \right) \mathbf{I}^b(\mathbf{x}) + \mathbf{p}_{ab} \mathbf{I}^{+b}(\mathbf{v}_g \mathbf{t}) \end{aligned}$$

equation 27

\mathbf{Zc} is the characteristic impedance of the line a .

9. Splices on power supply

A splice on the harness appears like a narrow mismatch. In this case the junction must be studied between the two parts of the harness on each side of the splice, as could be made for a simple line with one discontinuity. The multipole previously defined is a model for each homogeneous line of the harness. The connection of the various multipole linked with each cable is made through a network including the various impedances of these connections. An help to establish the equation of the complete harness is to write a tree of the topology. Figure 7 represents such a tree for one splice on a cable.

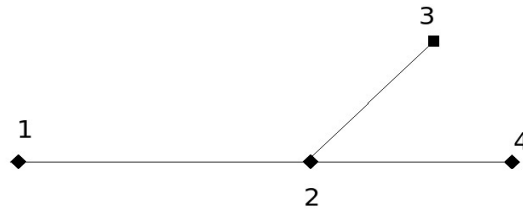


Figure 7

This harness is made of three cables connected in one point. The cables 1-2, 2-3, 2-4 are made of, respectively, 2, 1 and 2 wires. The figure 8 shows the graph of this harness.

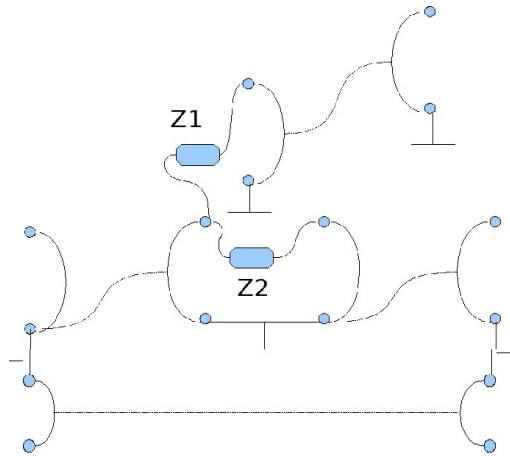


Figure 8

At the location of the splice, a network of impedances is added to take into account the effects of the connections. When a wire goes directly through the splice, it is better to add anyway an impedance of connection. Figure 9 shows this modification applied on the harness of figure 8.

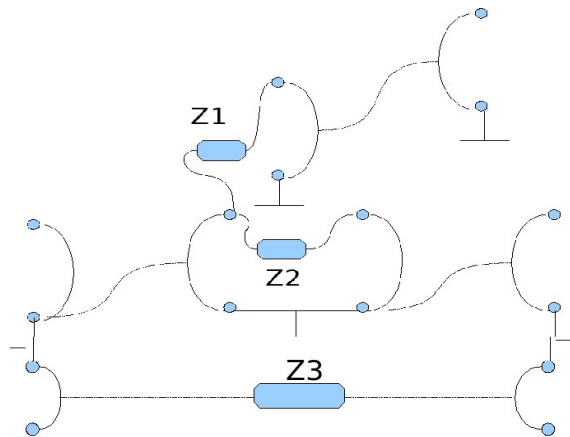


Figure 9

With this modification, each part of the harness described figure 7 is independent and the

new topology allows easily to use the relations given equations 26 and 27. The impedance Z_3 (figure 9) can be a simple inductance which represents the length of connection.

10. Conclusion

Starting from a geometrical and analytical description of the wires inside a cable, we can estimate the characteristic impedances, capacitances and group speed of the waves on each line made of two wires in the cable. These laws give the input to compute the transmission and cross talk of all the lines in the cable for a step source applied on one line. Using the superposition theorem, the matrix of cross talk is obtained through a time derivation and with a convolution product of the step response. To take into account the splices on the power supply lines, we create various networks, each of them being a cable without splices.

References

- [1] Ianov, « Haute tension », Dunod, 1987
- [2] Vabre, « Electronique des impulsions », Masson, 1975
- [3] Paul, « Electromagnetics for engineers », Wiley international edition, 2004
- [4] Paul, « Introduction to electromagnetic compatibility », Wiley, p573, 2006

Appendix A

We consider the Telegraph's circuit of the two lines cross talked presented figure A1.

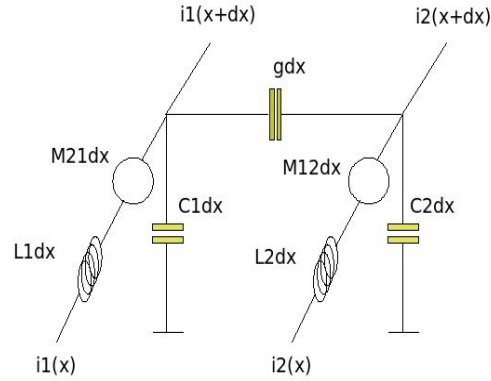


Figure A1: Telegraph's circuit

From this schematic we can write the following equations (with the hypothesis that $L_1=L_2=L$, $M_{12}=M_{21}=M$ and $C_1=C_2=C$):

$$\begin{cases} v_1(x) = Ldx \frac{di_1}{dt} + M \frac{di_2}{dt} + v_1(x+dx) \\ v_2(x) = Ldx \frac{di_2}{dt} + Mdx \frac{di_1}{dt} + v_2(x+dx) \end{cases} \quad (A1)$$

and for the currents:

$$\begin{cases} i_1(x) = Cdx \frac{dv_1}{dt} + gdx \frac{d(v_1-v_2)}{dt} + i_1(x+dx) \\ i_2(x) = Cdx \frac{dv_2}{dt} + gdx \frac{d(v_1-v_2)}{dt} + i_2(x+dx) \end{cases} \quad (A2)$$

From these systems we can obtain:

$$\left. \begin{aligned} \frac{-\partial v_1}{\partial x} &= L \frac{\partial i_1}{\partial t} + M \frac{\partial i_2}{\partial t} \\ \frac{-\partial i_1}{\partial x} &= (C+g) \frac{\partial v_1}{\partial t} - g \frac{\partial v_2}{\partial t} \end{aligned} \right\} \quad (A3)$$

The same for the current i_2 and the voltage v_2 .

We precise two relations generally verified, which we consider as hypothesis:

$$M < L \quad \text{and} \quad g < (C+g) \quad (A4)$$

Under these conditions, the system of equations becomes (using the Laplace's transform):

$$\begin{cases} \frac{-dv_1}{dx} = Lpi_1 \\ \frac{-di_1}{dx} = (C+g)pv_1 \\ \frac{-dv_2}{dx} = Lpi_2 + Mpi_1 \\ \frac{-di_2}{dx} = (C+g)pv_2 - gpv_1 \end{cases} \quad (A5)$$

With the two first equations we have:

$$\frac{d_1^{2v}}{dx^2} - L(C+g)p^2 v_1 = 0 \quad (A6)$$

The solution is:

$$v_1(x, p) = Ae^{-p\sqrt{L(C+g)}x} + Be^{p\sqrt{L(C+g)}x} \quad (A7)$$

We can defined $u = (\sqrt{L(C+g)})^{-1}$ to write:

$$v_1(x, p) = Ae^{-\frac{px}{u}} + Be^{\frac{px}{u}} \quad (A8)$$

With the last equations we obtain:

$$\frac{d^2 v_2}{dx^2} + L(C+g)p^2 v_2 = [M(C+g) - Lg]p^2 v_1 \quad (A9)$$

V_1 is given by A8. The solution of the differential equation without the second member is:

$$v_2(x, p) = De^{-\frac{px}{u}} + Ee^{\frac{px}{u}} \quad (A10)$$

Defining:

$$Q = \frac{M}{L} \frac{C+g}{g} \quad \aleph = \frac{g}{C+g} \quad (A11)$$

we obtain a particular solution:

$$\begin{cases} v_2 = \left[D - \aleph \frac{Q-1}{2} \frac{x}{u} pA \right] e^{-\frac{px}{u}} + \left[E + \aleph \frac{Q-1}{2} \frac{x}{u} pB \right] e^{\frac{px}{u}} \\ i_2 = \frac{1}{Rc} \left(e^{-\frac{px}{u}} \left[D - \aleph \frac{Q+1}{2} A - \aleph \frac{Q-1}{2} \frac{x}{u} pA \right] + e^{\frac{px}{u}} \left[-E + \aleph \frac{Q+1}{2} B - \aleph \frac{Q-1}{2} \frac{x}{u} pB \right] \right) \end{cases} \quad (A12)$$

Rc being the characteristic impedance of the line 1 taking into account the line 2:

$$Rc = \sqrt{\frac{L}{C+g}} \quad (A13)$$

To resolve this system we consider the matched case. A step function is applied on the line

1. If the step function is $U(p)$, V_1 is then equal to U delayed of x/u . In this case $A=U$, $B=0$, $i_1=V_1/Rc$. The system becomes:

$$\begin{cases} v_2 = \left(D - \aleph \frac{Q-1}{2} \frac{x}{u} pA \right) e^{-p \frac{x}{u}} + E e^{p \frac{x}{u}} \\ i_2 = \frac{1}{Rc} \left[e^{-p \frac{x}{u}} \left(D - \aleph \frac{Q+1}{2} U - \aleph \frac{Q-1}{2} \frac{x}{u} pU \right) - E e^{p \frac{x}{u}} \right] \end{cases} \quad (A14)$$

D and E can be defined from the limit conditions, at $x=0$:

$$v_2(0) = D + E = -Rc i_2(0) \quad i_2(0) = \frac{1}{Rc} \left(D - \aleph \frac{Q+1}{2} U \right) \quad (A15)$$

and at $x=X$:

$$v_2(X) = Rc i_2(X) \quad i_2(X) = \frac{1}{Rc} \left(e^{-p \frac{X}{u}} \left[D - \aleph \frac{Q+1}{2} U - \aleph \frac{Q-1}{2} \frac{X}{u} pU \right] - E e^{p \frac{X}{u}} \right) \quad (A16)$$

From these relations we conclude that:

$$D = \aleph \frac{Q+1}{4} U(p) \quad E = -\aleph \frac{Q+1}{4} U(p) e^{-2p \frac{X}{u}} \quad (A17)$$

Finally:

$$\begin{cases} v_2 = \aleph \frac{Q+1}{4} U \left[e^{-p \frac{x}{u}} - e^{-p \frac{2X-x}{u}} \right] - \aleph \frac{Q-1}{2} \frac{x}{u} pU e^{-p \frac{x}{u}} \\ i_2 = \frac{1}{Rc} \left[\aleph \frac{Q+1}{4} U \left(-e^{-p \frac{x}{u}} + e^{-p \frac{2X-x}{u}} \right) - \aleph \frac{Q-1}{2} \frac{x}{u} pU e^{-p \frac{x}{u}} \right] \end{cases} \quad (A18)$$

This last system of equations is the solution of our problem.

Appendix B

#This software compute for typical values of diameter of wires the characteristic impedance using the Ianov's book formulas

```
from cmath import *
from math import *
l=1
pi=3.14
de=10E-3
di=0.5E-3
v=3E8
Zc=[]
e=0 #e is the distance to change when the wire go to external
er=1
eo=1/(36*3.14*1E9)
for e in range(1,11):
    C=eo*er*2*pi*l/acosh((de**2+di**2-4*(e*1E-3)**2)/(2*de*di))
    Zc.append(abs(1/(v*C)))

Ce=eo*er*2*pi*l/(log(2*(di+0.1E-3)/di+sqrt((2*(di+0.1E-3)/di)**2-1)))
Zc.append(abs(1/(v*Ce)))

Zc_fichier=open('Zc_fichier.csv','w')
a=0
for a in range(1,11):
    Zc_fichier.write(str(a)+' '+str(Zc[a])+'\n')

Zc_fichier.close()
```