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Model simplification and system approximation with constrained Laguerre models

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Abstract

A functional approximation method is presented based on Laguerre functions. This paper illustrates the use of supplementary time constraints (e.g., interpolation of the value of the impulse response of a system at a given time) and frequency constraints (e.g., interpolation of the value of the system's transfer function at a given frequency) in order to improve the quality of approximation. The proposed method may be employed either in the reduction of a known system or in the identification of an unknown system. A number of numerical results further clearly show its advantage over unconstrained least squares approximations.

Keywords: Orthonormal functions, Laguerre functions, Kautz functions, functional approximation, model reduction, identification with constraints.

1 Introduction

The use of Laguerre models in least squares approximation theory is classical [1]-[3]. The Laguerre functions have a simple structure which eases their network representation as shown in fig.1 and it is well known that their truncated series capture the behaviour of a large class of stable linear systems [3]. Moreover, Gilbert [4], follows for example by Calvez et al. [5] have introduced the constrained least squares techniques using orthonormal functions. Our purpose here is then to define for Laguerre models, some particularly interesting constraint equations and to show how their use in the model simplification and system approximation contexts considerably improved the quality of the obtained results.

In the next section, principles of model simplification and system approximation based on the Laguerre least squares decomposition are presented and developed. We show that those very simple but powerful methods lead to similar one-parameter optimisation problems (determination of the Laguerre pole for a given Laguerre model order followed by minimisation of a quadratic error criterion), because once that parameter is specified, i.e. the Laguerre pole, the approximation coefficients are completely defined in least squares sense. In section 3, we consider in addition the use of those methods under some typical linear constraints, which would considerably broaden the field of application of the aforementioned methods. Simple and quite illustrative examples are presented in section 4. Finally, the section 5 concludes the paper.

2 Laguerre based model simplification and system approximation

The time normalised Laguerre functions form a complete orthonormal set in $L^2[0, \infty[$. Their Laplace transforms which

are known as Laguerre filters and are given by (1), also form a complete orthonormal set in $H^2(C+)$, the Hardy space of all analytical functions on the open right half complex plane that are squares integrable [6].

$$L_n(s) = \sqrt{2\alpha} \frac{(s-\alpha)^n}{(s+\alpha)^{n+1}} \quad n = 0, \dots, \infty \text{ and } \alpha > 0 \quad (1)$$

Consider the transfer function of a prescribed stable system noted $F(s)$, for a fixed Laguerre pole α (actually the pole is located at $(-\alpha)$ however for sake of simplicity we will note it α), that is there exists an infinite real sequence (a_n^*) such that:

$$F(s) = \sum_{n=0}^{\infty} a_n^* L_n(s) \quad (2)$$

Given the orthonormality property of the Laguerre filters, the coefficients (a_n^*) straightforward follow in the frequency domain from

$$a_n^* = \frac{\sqrt{2\alpha}}{2\pi} \int_{-\infty}^{\infty} -F(j\omega) \frac{(j\omega + \alpha)^n}{(j\omega - \alpha)^{n+1}} d\omega \quad n = 0, \dots, \infty \quad (3)$$

which lead in time domain by virtue to the Parseval's theorem to

$$a_n^* = \int_0^{\infty} f(t) l_n(t) dt \quad n = 0, \dots, \infty \quad (4)$$

where $f(t)$ is the system's impulse response and where $l_n(t)$, $n = 0$ to ∞ , represent the corresponding time domain Laguerre functions.

It is also obvious that $f(t) \in L^2[0, \infty[$ and that the so-called system's energy is finite and given by

$$E = \int_0^{\infty} (f(t))^2 dt = \sum_{n=0}^{\infty} (a_n^*)^2 \quad (5)$$

2.1 Model simplification

In control theory and applications, model simplification is interesting for systems with known but complex and high order transfer functions. A large number of techniques has been developed in this field [7]-[9], see in [9] their ordinary classification. The Laguerre based simplification method may be classified into sub-optimal ones (determination of the optimal Laguerre pole followed by minimisation of the least squares criterion), as we are going to show it.

Consider a stable system with known rational N th order transfer function $F(s)$. A reduced r th order Laguerre model of

the system (for a given Laguerre pole), i.e. $r < N$, is simply obtained by truncated the series (2) as

$$F_r(s) = \sum_{n=0}^{r-1} a_n^* L_n(s) \quad (6)$$

Furthermore, the approximation accuracy is improved by a judicious choice of the Laguerre pole. Typically [10]-[11], that is done by maximising with respect to α the energizing criterion

$$J_r(\alpha) = \left[\sum_{n=0}^{r-1} (a_n^*)^2 \right] / E \quad (7a)$$

Courtesy of the Laguerre filters orthonormality propriety, maximise the energizing criterion (7a) is equivalent to minimise the least squares criterion

$$I_r(\alpha) = \frac{1}{2} \|F(s) - F_r(s)\|^2 \quad (7b)$$

with $F_r(s)$ given by (6) and where the formalism $\| \cdot \|$ means the standard norm in $H^2(C+)$.

2.2 System approximation

Let $y(t)$ be the response signal of a given stable system to a random input $u(t)$, and r the number of blocks in the Laguerre network built for a pole value α (fig. 1). By feeding the input $u(t)$ to this network and by forming a linear combination of it's various outputs noted $x_n(t)$, for $n = 0$ to $r-1$, we obtain an r th order Laguerre model of the system.

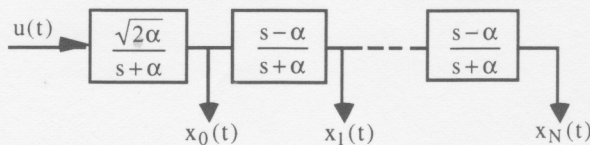


fig. 1: Continuous Laguerre network. Similar network exists for discrete filters

Let us assume that all those signals are collected over H observations from t_0 to t_{H-1} at the normalised sampling period and after attenuation of the initial condition effects. That leads to define the least squares error cost function as follows

$$I_r(\alpha) = \frac{1}{2} \sum_{k=0}^{H-1} \left[y(k) - \sum_{n=0}^{r-1} a_n x_n(k) \right]^2 \quad (8)$$

where (a_n) are the real Laguerre model coefficients in the least squares sense.

Let us rearrange the input and output data $\{y(k)$ and $x_n(k)\}$ and the Laguerre coefficients (a_n) , for $k = 0$ to $H-1$, $n = 0$ to $r-1$, as

$$Y = [y(0) \ \dots \ y(H-1)]^T, \quad A = [a_0 \ \dots \ a_{r-1}]^T \quad (9a)$$

$$\text{and } X_r = \begin{bmatrix} x_0(0) & \dots & x_{r-1}(0) \\ \vdots & \dots & \vdots \\ x_0(H-1) & \dots & x_{r-1}(H-1) \end{bmatrix} \quad (9b)$$

This permits rewriting of $I(\alpha)$ in a more contracted matrix notation as follows

$$I_r(\alpha) = \frac{1}{2} (Y - X_r A)^T (Y - X_r A) \quad (10)$$

where $e = Y - X_r A$ defines the approximation error vector.

The Laguerre coefficient vector A is then obtained by solving the so-called normal equation as

$$A = S^{-1} (X_r^T Y) \quad \text{with } S = X_r^T X_r \quad (11)$$

the so-called measure matrix.

Different authors, among whom we can state Zervos et al. [12] and Wahlberg [3], have shown that, since the input signal $u(k)$ is persistently exciting [13], the distribution of $\sqrt{H}(A - A^*)$ (where A^* contains the r first coefficients of the Laguerre series (2)) tends asymptotically to a Gaussian distribution with a zero mean and an asymptotic covariance matrix that depends on the variance of the output noise of the system. Moreover the approximation accuracy is improved by a suitable choice of the Laguerre pole which is usually done by minimising with respect to α the criterion $I_r(\alpha)$.

2.3 The pole optimisation problem

Maximise the energizing criterion (7a) or minimise the least squares approximation error (8) or (10) with respect to α are complementary problems. It is well known, see [10]-[11] and [14]-[15], that the optimum pole value satisfies in the two cases the same optimality equations which are

$$a_{r-1}^*(\alpha) = 0 \text{ or } a_r^*(\alpha) = 0 \quad \text{and} \quad a_{r-1}(\alpha) = 0 \text{ or } a_r(\alpha) = 0 \quad (12)$$

where for a given α value, a_r^* is the $(r+1)$ th coefficient of the series (2) and a_r is the $(r+1)$ th coefficient of an extended $(r+1)$ th order Laguerre model of the approximated system.

To our knowledge, there is no analytical way to solve systematically those equations. Hence to find the optimum pole, we use a simple and swift numerical method (also useful for multivariable Laguerre model optimisations) based on some Newton-Raphson's iterative algorithm [16].

3 Approximation with linear constraints

Until this stage, in the r -order system approximation by Laguerre filters, we have taken into account dominant time constants of the system which are related to the Laguerre pole involved in the approximation, as shown by different authors [17]-[18]. However, optimisation of both $J_r(\alpha)$ and $I_r(\alpha)$ criterions (relations (7) and (8)) does not necessarily lead to conservation of steady-state gain. Furthermore, if the approximation of the system transfer function from the step response imposes an exact steady-state gain to the model, the behaviour of the latter at the time-origin may not be precisely represented. To improve the approximation results, certain specificities of the actual system responses in the form of constraint equations may be taken into consideration, within the criterion optimisations.

3.1 Constraint equation generalities

Let $f(t)$ be a function to be approximated by a linear combination of r other functions $(\psi_i(t))$ as

$$f_r(t) = \sum_{i=0}^{r-1} a_i \psi_i(t) \quad (13)$$

such that the real coefficients a_i minimise the least squares cost function I_r , and also satisfy the linear constraint matrix equation below

$$L A - y = 0 \quad (14)$$

where the vector A contains the coefficients a_i and the matrices L and y of appropriate dimensions define the constraints. The number of constraint relationships, namely

the number of rows of L , must of course be smaller than (r) so as to solve the problem. This requires determination of the extrema for the following criterion [19]

$$I(A, \lambda) = I_r + \lambda^T (L A - y) \quad (15)$$

where λ is the vector containing the Lagrange parameters to determine with a_i .

Derivation of (15) yields the following system

$$\begin{cases} S A + L^T \lambda = B \\ L A - y = 0 \end{cases} \quad (16)$$

Remarks : In the Laguerre model simplification context, the matrix S becomes identity and the elements of B directly come from (3) or (4). Whereas in the Laguerre system approximation context, the matrix S and B are defined by (11); $S = X_r^T X_r$ and $B = X_r^T Y$.

It is easy to prove that the solution to the general problem (with constraints), consists of the solution without constraint denoted A_0 , e.g. the coefficient vector obtained after Laguerre pole optimisation in the previous section, to which is added a corrective term introducing constraint effects on the coefficients (a_i) denoted \bar{A} . That is

$$A = A_0 + \bar{A} \quad (17a)$$

and it can straightforwardly be shown that

$$\bar{A} = S^{-1} L^T (L S^{-1} L^T)^{-1} (y - L A_0) \quad (17b)$$

3.2 Constraint equation for Laguerre models (for a given α value)

Here the functions ($\psi_i(t)$) are the Laguerre ones ($l_i(t)$), i.e. the time domain functions coming from the Laguerre filters (1). Then among the existing constraints, the most common are as follows:

- an initial value of $f_r(t)$ identical to that of the original model $f(t)$,
- identical slope at the time-origin for both $f_r(t)$ and $f(t)$,
- imposed steady-state gain,
- known frequency characteristics of the system (phase and gain margins, ...) used to impose the frequency response of the approximation. This nature of constraint implies the identity: $F_r(j\omega_i) = Z_i$; where $F_r(j\omega_i)$ denotes the Fourier transform of $f_r(t)$ and with $Z_i \in \mathbb{C}$. It is equivalent, in the real number space, to a double equality.

Constraint on the initial value of $f(t)$

The Laplace transform of $f_r(t)$ is written as: $F_r(s) = \sum_{i=0}^{r-1} a_i L_i(s)$. The constraint is $f_r(0) = f(0)$, and respecting the theorem of the initial value implies

$$f_r(0) = \lim_{s \rightarrow \infty} s F_r(s) = f(0) \quad (18a)$$

This constraint can simpler be written in the following matrix form

$$\begin{cases} L A - y = 0 \\ \text{with } L = \sqrt{2\alpha} [1 \ 1 \ \dots \ 1]_{(r)} \text{ and } y = f(0) \end{cases} \quad (18b)$$

Constraint on initial value of derivative of $f(t)$

Let $f_r(t) = \sum_{i=0}^{r-1} a_i l_i(t)$, the matrix form of the constraint

$$\left. \frac{df_r(t)}{dt} \right|_{t=0} = f'(0) \text{ is thus:}$$

$$\begin{cases} L A - y = 0 \\ L = -\sqrt{2\alpha} [\alpha \ \dots \ \alpha + (2\alpha)i \ \dots \ \alpha + 2\alpha(r-1)] \text{ and } y = f'(0) \end{cases} \quad (19)$$

Constraint on the steady-state gain of the system to be approximated

If K denotes the steady-state gain of the system, it is easily shown that this constraint can be written in the following matrix form

$$\begin{cases} L A - y = 0 \\ L = \sqrt{\frac{2}{\alpha}} [1 \ \dots \ (-1)^i \ \dots \ (-1)^{r-1}] \text{ and } y = K \end{cases} \quad (20)$$

Frequency-type constraints

In the frequency domain, the Fourier transform of $f_r(t)$ is defined by:

$$F_r(j\omega) = \sqrt{2\alpha} \sum_{i=0}^{r-1} a_i \frac{(j\omega - \alpha)^i}{(j\omega + \alpha)^{i+1}} \quad (21a)$$

Let θ and ρ respectively be: $\theta = \arctg \frac{\omega}{\alpha}$ and $\rho = \frac{1}{\sqrt{\omega^2 + \alpha^2}}$,

then

$$F_r(j\omega) = \sqrt{2\alpha} \rho [a_0 e^{-j\theta} + a_1 e^{-j3\theta} + \dots + a_{r-1} e^{-j(2r-1)\theta}] \quad (21b)$$

which relation leads to

$$\begin{cases} \text{Re}[F_r(j\omega)] = \sqrt{2\alpha} \rho [a_0 \cos(\theta) + \dots + a_{r-1} \cos((2r-1)\theta)] \\ \text{Im}[F_r(j\omega)] = -\sqrt{2\alpha} \rho [a_0 \sin(\theta) + \dots + a_{r-1} \sin((2r-1)\theta)] \end{cases}$$

where $\text{Re}(Z)$ and $\text{Im}(Z)$ represent the real and the imaginary parts of Z .

For a given pulsation ω_1 , the imposed constraint is

$$F_r(j\omega_1) = F(j\omega_1) = u_1 + jv_1 \quad \text{where } u_1 \text{ and } v_1 \text{ are reals} \quad (22a)$$

it may be summarised into a two rows constraint matrix as

$$\begin{cases} L A - y = 0 \\ L = \sqrt{2\alpha} \rho \begin{bmatrix} \cos(\theta) \ \dots \ \cos((2r-1)\theta) \\ \sin(\theta) \ \dots \ \sin((2r-1)\theta) \end{bmatrix} \text{ and } y = \begin{bmatrix} u_1 \\ -v_1 \end{bmatrix} \end{cases} \quad (22b)$$

4 Example

This example comes from a publication by Lin and Han [20], dealing with model simplification by continued fraction expansions. Consider the system shown in figure 2

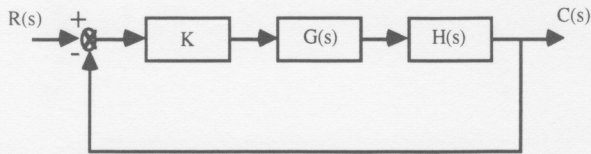


Fig. 2: Bloc diagram of control system

with $K = 1.2$, $H(s) = \frac{1}{s}$ and

$$G(s) = \frac{(s+0.5)(s+10)(s+14)(s^2+8s+32)}{(s+1)(s^2+4s+8)(s^2+4s+20)(s^2+6s+18)} \quad (23)$$

The most accurate approximation of $G(s)$ is sought based on Laguerre filters and considering the different types of constraints. This is followed by a study of the obtained reduced or approximate model performance, according to the system configuration (open-loop or closed-loop). On the basis of the first four Laguerre filters and from $G(s)$ impulse response, an approximate model for $G(s)$ is obtained from non-constrained optimisation (with also optimal search of the Laguerre pole) as follows

$$G_{L4}(s) = \frac{0.542s^3 - 8.452s^2 + 94.884s + 43.078}{s^4 + 11.210s^3 + 47.122s^2 + 88.037s + 61.680} \quad (24)$$

with $\alpha = 2.802$ and $J_4(\alpha) = 0.97$.

The following three figures illustrate the impulse and unit-step responses as well as the magnitude frequency responses of $G(s)$ and its approximate model $G_{L4}(s)$.

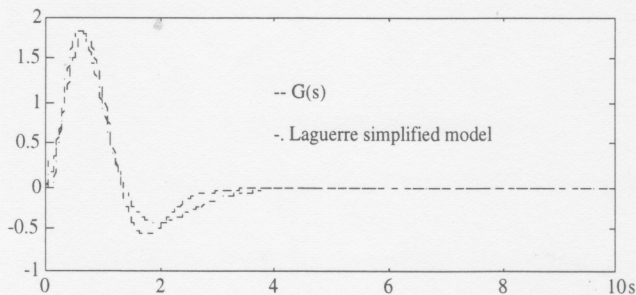


Fig. 3a: Impulse responses of $G(s)$ and its reduced model $G_{L4}(s)$

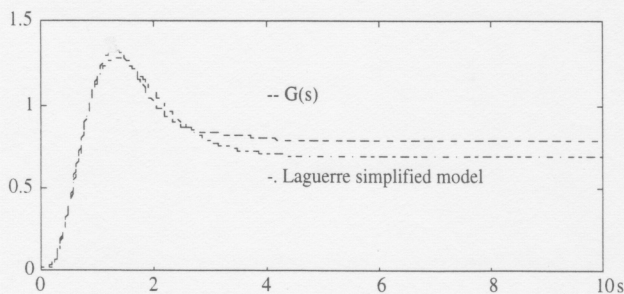


Fig. 3b: Unit step responses of $G(s)$ and its reduced model $G_{L4}(s)$

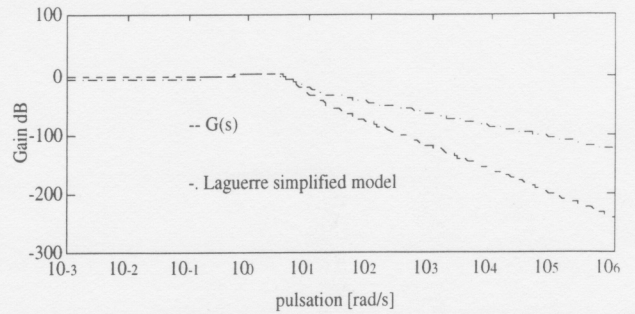


Fig. 3c: Magnitude Bode plots of $G(s)$ and its reduced model $G_{L4}(s)$

It is noted that despite a high value of $J_4(\alpha)$, and relatively accurate impulse response, the so-obtained approximate model $G_{L4}(s)$, merely gives a rough estimation of the $G(s)$ steady-state gain. Application of the previously described modelling procedure, based on the first four Laguerre filters and under constraint (here steady-state gain of $G(s)$ and initial value of the impulse response are taken into account), for $\alpha = 2.802$, leads to:

$$G_{Lc4}(s) = \frac{-7.829s^2 + 90.630s + 47.973}{s^4 + 11.210s^3 + 47.122s^2 + 88.037s + 61.680} \quad (25)$$

$J_4(\alpha)$ thus increases to **0.98** and as illustrated in figures 4a and 4b, the behaviour of $G_{Lc4}(s)$ is more similar to the open-loop response of $G(s)$ compared to that of $G_{L4}(s)$ obtained without constraint.

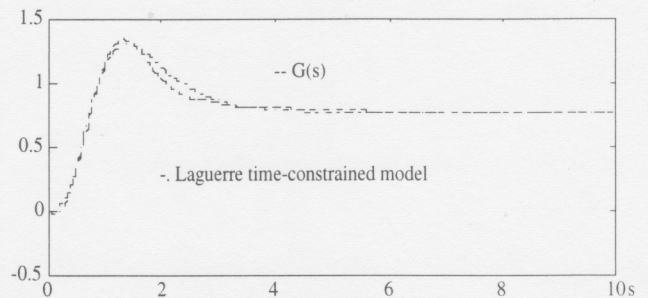


Fig. 4a: Unit step responses of $G(s)$ and its reduced model $G_{Lc4}(s)$

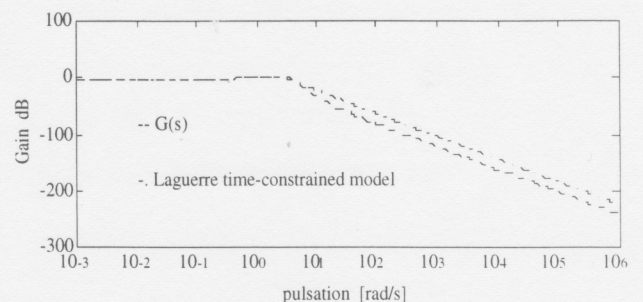


Fig. 4b: Magnitude Bode plots of $G(s)$ and its reduced model $G_{Lc4}(s)$

Behaviour of $G(s)$ may now be simulated within the global closed-loop system represented in figure 2. Comparison is further made to that of $G_{L4}(s)$ and $G_{Lc4}(s)$ approximate models using a similar configuration. Figure 5 represents the unit-step response of the closed-loop system as well as its

simulated behaviour when $G(s)$ is replaced by $G_{L4}(s)$ and $G_{Lc4}(s)$.

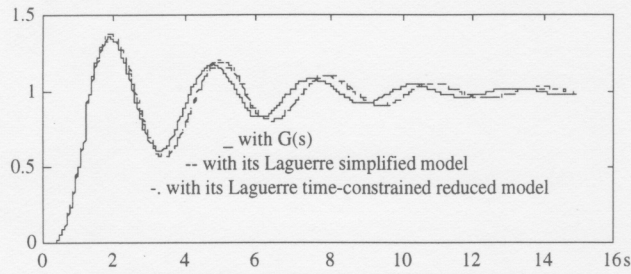


Fig. 5: Closed-loop unit step responses for system with $G(s)$ and with its models $G_{L4}(s)$ and $G_{Lc4}(s)$

We remark that, in the light of utilising an approximate closed-loop model, when identification is solely performed in the time domain, considerations of constraints on time response of the system is not sufficient to guarantee optimal dynamic model behaviour with respect to the studied system. Indeed, as shown in figures 3c and 4b, a slight uncertainty remains on the frequency response of obtained models. Figure 4 perfectly illustrates the consequence in the closed-loop context.

As a solution, we thus propose the use of modelling by Laguerre filters with frequency constraints. In particular, if access to the phase and gain margin of the closed-loop system is possible (which is the case in model simplification), the modelling method with linear frequency constraints, developed here, yields satisfactory results. In the previous system, the gain crossover pulsation of the loop transmission function $L(s) = K G(s) H(s)$ is $\omega_g = 1.74$ rad/s. Similarly, its phase crossover pulsation is $\omega_p = 2.396$ rad/s.

Using the first five Laguerre filters (with $\alpha = 3.762$) and imposing that the approximate model provides an identical value as that of $G(s)$ firstly for $s = j\omega_g$ (which results in $G_{Lcg}(s)$), secondly for $s = j\omega_p$ (which results in $G_{Lcp}(s)$) and finally for both frequency points ($G_{Lcgp}(s)$), the following approximate models are derived:

$$G_{Lcg}(s) = \frac{-0.18s^4 + 7.62s^3 - 39.55s^2 + 974.25s + 519.23}{s^5 + 18.8s^4 + 141.5s^3 + 532.4s^2 + 1001.4s + 753.5}$$

$$G_{Lcp}(s) = \frac{-s^4 + 9s^3 - 32.8s^2 + 1035.1s + 561.7}{s^5 + 18.8s^4 + 141.5s^3 + 532.4s^2 + 1001.4s + 753.5}$$

$$G_{Lcgp}(s) = \frac{-0.19s^4 - 12s^3 - 41.35s^2 + 914.83s + 513.89}{s^5 + 18.8s^4 + 141.5s^3 + 532.4s^2 + 1001.4s + 753.5}$$

All three models possess $J_5(\alpha)$ values of **0.93**, **0.96** and **0.98**, respectively. The quality of these models is also assessed in figures 6. Illustrated are the step response provided by the studied closed-loop system and its simulated behaviour when $G(s)$ is replaced by $G_{Lcg}(s)$, $G_{Lcp}(s)$ and $G_{Lcgp}(s)$.

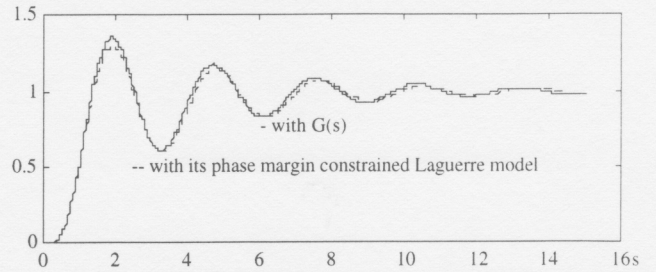


Fig. 6a: Closed-loop unit step responses for system with $G(s)$ and with its constrained model $G_{Lcg}(s)$

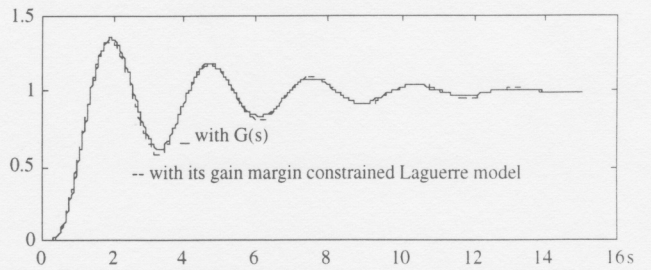


Fig. 6b: Closed-loop unit step responses for system with $G(s)$ and with its constrained model $G_{Lcp}(s)$

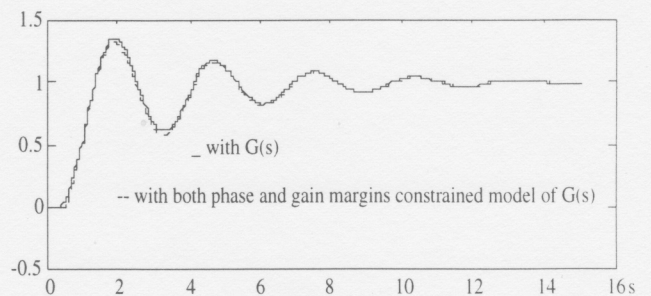


Fig. 6a: Closed-loop unit step responses for system with $G(s)$ and with its constrained model $G_{Lcgp}(s)$

As observed, the obtained results are satisfactory. Furthermore, considering the objectives of this work, time and frequency constraints may be combined as long as the number of rows of the resulting constraint matrix remains smaller than the number of filters used in the approximation.

5 Conclusion

Although non-linear optimisation techniques are often required in the determination of the Laguerre pole, the order of the reduced system may be a priori fixed and the proposed model simplification methods may nevertheless ensure a quick convergence of the Laguerre-series expansion coefficients.

Moreover, as the Laguerre pole is chosen with a negative real part, the obtained reduced model is necessarily stable. Although our presentation is focused on Laguerre filters or Laguerre models, it is straightforward to extend the results to other filter basis like the Kautz ones, that have been successfully used to approximate highly oscillating systems [21]-[23].

Finally, the possibility of approximation using Laguerre models and user-defined constraints demonstrates the flexibility and adaptability of Laguerre models for system modelling and model simplification.

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