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# Yin-Yang balance : a dialectical view of variational principles of physics

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## Abstract

The aim of this work is to provide a possible philosophical motivation to the variational principles of physics and a possible way to unify the axiomatic formulations of classical mechanics and statistical mechanics. The leitmotif of this work is a dialectical view of the world: any smooth motion in nature is the consequence of the interplay and dynamical balance between pairs of opposite, mutually exclusive but complementary and mutually transformable essences or elements of the motion. This dynamical balance is expressed by an invariant variational relationship between all the pairs of opposite elements in the considered phenomenon. Derivation of several principles of physics from this idea is given.

## 1) Introduction

The variational approach in physics, such as the principles of least action [1][2] and of virtual work[3][4] for mechanics as well as the principle of maximum entropy [5][6] for thermodynamics, is widely used for the axiomatic formulation of physical theories. It is also, with a long historical development accompanied by philosophical and mathematical controversies, a philosophy laden subject whose last word is not yet spoken to date.

Variational principles have a strong metaphysical origin[1][2] and were developed under the influence of teleological tendency of the pioneers (Fermat, Leibnitz, Maupertuis, Euler and others)[2][7][8] who used minimum calculus to express the longstanding belief of nature's (or God's) perfection. The essential of their approach was to create virtual world of possible events and then to select only a small number of these by a mathematical extremum believed to express a *purpose*[1][2][7][8]. For this view point, nature is a selective system similar to a biological system. This teleological view was rejected gradually in nineteen and twentieth centuries by the anti-metaphysical trends of empiricism and related philosophical circles[7][8], for which the variational principles are equivalent to differential equations and have only esthetic and economic interest in the adaptation of thoughts to "elementary facts"[7][8], i.e., in axiomatic formulation of physics theory.

Since this moment, variational principles have gradually lost its metaphysical bedrock and become completely devoid of philosophical and physical motivation. The old questions, answered in the past with the belief of God's intelligence, have been left unanswered. Some of these fundamental inquiries are: why the maximum/minimum is a criterion of nature for selecting events; why the time integral of Lagrangian (the difference between kinetic and potential energy) has extremum for real motion; why entropy is maximal for real probability distribution<sup>1</sup>, and why entropy production should be maximal or minimal for nonequilibrium process. To these questions and many others about the origin of the maximum/minimum calculus, we may add a final one: whether or not there is a common cause to so many seemingly different, sometimes mutually exclusive principles in nature? This mutual exclusivity has a very illustrative example. That is the conflict between the least action principle and the maximum entropy (and its production) principle: the former yields motions

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<sup>1</sup> This principle has been more or less justified with non mathematical arguments by Jaynes[5] and Tribus[9] who considered probability in the Bayesian sense as a subjective quantity depending on our incomplete knowledge about given situation. This interpretation of probability encountered enormous resistance from the physicists believing the objective frequency interpretation of probability, for which the principle remains a mysterious hypothesis.

satisfying Liouville's theorem and Poincaré's recurrence theorem, the second uses the notion of entropy, a quantity whose evolution law (second law of thermodynamics) is believed to be at odds with the above mentioned theorems[10].

Since the rejection of teleological view, physicists and philosophers have tried to answer these above questions. For many, the superiority of variational principles with respect to differential equations is obvious[8], but they failed to grasp the matters that would be other than a formal superiority in mathematical description and capable of deepening our understanding of nature. As far as we know, no conclusive statement can be made to date about the very reality behind the variation principles in physics.

Efforts have been made to derive variational principles of physics from other general philosophical hypothesis. For example, the variational principles was attributed to the principle of unique determinacy[7] with the slogan "nature happens only ones"[11]. If this is true in the case of regular motion determined uniquely by maximum/minimum calculus, it is not the case for irregular (random or chaotic) dynamics, since there may be infinite number of possible events taking place at a given moment, or infinite number of different paths (all observable with non zero probability) for a system starting from a sole state. This is the case where the randomness of the motion should be quantitatively tackled in the (statistical) calculus of variation (see below).

The present work is a tentative to fill the void of understanding variational principle by introducing a dialectical view of the duality of nature. This view is: any entity or events of nature contains pairs of opposite, mutually exclusive but nevertheless equally essential and complementary parts. This very ancient and long-standing philosophical point of view is not only a conceptual conviction, but also a belief, common to both occidental and oriental cultures, stemming directly from observation of natural and social movements. In its modern development accomplished mainly by Hegel and Marx, it often stresses conflict, negation and change of the opposite parts. However, one of the inevitable and essential aspects of any motion is a smooth and continuous evolution state with dynamical balance between opposites. This is also an important aspect of dialectics<sup>2</sup>. In fact, in this work, we will concentrate on the smooth and continuous motion of physical system using this notion of harmonic balance to

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<sup>2</sup> This conceptual balance of opposite parts is essential in the description of the harmonic motion of nature in the ancient Chinese mysticism *I Ching* and in the Taoism philosophy [12], according to which the interplay and balance of Yin and Yang completely determine all natural phenomena. In this ancient idea, Yin and Yang are a pair of mystical concepts with completely different attributes which are in general definitely opposing such as femininity and masculinity, passive and active, hidden and open, etc.

develop a variational calculus. By invariance consideration, this balance is mathematically defined by a variational relationship between the opposite quantities. The main idea is that any physical phenomenon in *smooth* motion is the consequence of the *interplay* and *invariant balance* of pairs of opposite parts. To our opinion, this may be the very reality hidden behind the variational principles in physics. Examples of derivation of several variational principles are given. It is hoped that such approach can help us to understand more, at least in a reasonable way, about why things happen that way in physics.

## 2) Yin-Yang variational principle

In this section, we will develop a general variational principle with the name of Yin-Yang, borrowed from an ancient notion of duality indicating two opposite and complementary elements in any entity and event. We prefer this name although its mystical and metaphysical attribute in the ancient religious belief since it reflects an interesting aspect of the opposing parts. Yin means feminine, passive, hidden character; and Yang means masculine, active, open character[12]. In what follows, we will use this attribute as an essential criterion for the choice of opposites.

We are interested only in the smooth continuous motion of mechanical system. The Yin-Yang principle is stated as follows:

*Any event of a system never happens without the occurrence of the invariant balance between all pairs of opposite, mutually exclusive but complementary, mutually rooted and transformable parts of the system that undergo variations due to the event.*

Let us spell out the idea by giving the following definitions.

a) *Yi* is defined as a physical quantity charactering the passive, ordered, inanimate, or hidden aspects and elements of a motion.

b) *Ya* is defined as a physical quantity charactering the active, disordered, animate, or open aspects and elements of the same motion.

It should be stressed here that *Yi* and *Ya* must be empirically measurable quantities or mathematical expressions which can be univocally related to measurable quantities. Conceptual notions or abstract words devoid of physical instance (such as *black* and *white*, *up* and *down*, *large* and *small*, etc) are not considered. *Yi* and *Ya* of mechanical system may be inertial force and driving force (acceleration), or their spatial accumulation (work), potential energy and kinetic energy, or their time accumulation, average energy (representing order and

observability in random case) and heat (or entropy, measuring disorder and randomness), among many others. Examples using these pairs of opposite quantities will be given below.

Sometimes a given quantity can be either related to  $Y_i$  or  $Y_a$  depending on the events containing it. Inertial force, for example, is rather  $Y_i$  in accelerating motion but  $Y_a$  in decelerating dissipative motion since it maintains the motion, an active aspect of the process.

c) The variation  $\delta Y$  of an element  $Y$  is a virtual change without time passage and confined by constraints.

One of the advantages of the use of variation  $\delta Y$  of a quantity  $Y$  is that  $\delta Y$  can be chosen in an independent way from the frame of reference, while this is not possible a priori for the magnitude  $Y$  itself which is in general dependent on its frame. The use of variation makes it possible to establish a universal balance or equilibrium independent of the frame of reference as follows.

d)  $Y_i$ - $Y_a$  (or Yin-Yang) Balance: for a virtual process yielding variation of the opposite elements  $Y_{i_l}$  and  $Y_{a_l}$ , the invariant balance between the pairs of opposites is described by

$$\sum_l \delta Y_{i_l} - \sum_l \delta(\lambda_l Y_{a_l}) = 0 \quad (1)$$

where  $\lambda_l$  for a given pair  $l$  is a coefficient characterizing the interplay and the equilibration of the pair  $l$ . The value of  $\lambda_l$  should depend on the nature of the pair  $l$  as well as on the property expressed in this equation (balance, complementarity, transformability, etc.). The reader will see that this parameter can be determined in each application of the principle.

If necessary, constraints  $C_k$  on the balance may be added à la Lagrange such that:

$$\sum_l \delta Y_{i_l} - \sum_l \delta(\lambda_l Y_{a_l}) + \sum_k \delta(\alpha_k C_k) = 0 \quad (2)$$

where  $\alpha_k$  is a multiplier related to the constraint  $C_k$ . However, Eq.(1) can be used even when there are constraints, meaning that the constraints can be taken into account in the subsequent treatments.

By using these above definitions, the Yin-Yang variational principle can be stated as follows:

*Any continuous and smooth event of a system never happens without the occurrence of a Yin-Yang balance between all pairs of  $Y_i$  and  $Y_a$  that undergo variation due to the event.*

The application of this dialectical principle consists, first of all, in the identification of pairs of opposite elements whose interplay determines the event, as shown below.

### 3) Principle of least action

We consider a system with Hamiltonian  $H(q, \dot{q}, t) = T(\dot{q}, t) + V(q, t)$  where  $T$  is the kinetic energy depending on time and velocity, and  $V$  the potential energy depending on the configuration of the system as well as time. We suppose, without loss of generality, that in a given frame of reference, the system is in non uniform motion with acceleration  $\bar{a}_i$  of the point  $i$ .  $m_i$  is the mass of the point having the force  $F_i$  ( $i=1,2,\dots,N$ ). Now we have to identify the pairs of opposite elements to put into Eq.(1). Let  $V_i$  be the potential energy associated with  $F_i$ .  $V_i$  represents, at a given moment of time and in a given frame of reference, an immobile and hidden motion (or latent capacity of doing work, *vis mortua*). Let  $T_i$  be the kinetic energy which represents a mobile energy and apparent motion (evident capacity of doing work and of moving against resistance, *Vis viva*). Hence it is evident that  $V = \sum_{i=1}^N V_i$  (total potential energy)

and  $T = \sum_{i=1}^N T_i$  (total kinetic energy) are two essential opposite elements of the motion at any given moment of time. However,  $V$  and  $T$  will not be considered as  $Y_i$  and  $Y_a$ . We choose

instead the time accumulation of them (cumulated energies)  $\int_{t_a}^{t_b} V dt$  and  $\int_{t_a}^{t_b} T dt$  for the case of a

motion between a point  $a$  and point  $b$  in phase space during a given period of time  $\tau = t_b - t_a$ .

This choice has two reasons. The first is to take into account the motion during  $\tau = t_b - t_a$  as a whole. This is, if we want to see the effect of a virtual deformation on the motion, we should calculate the entire effect over the whole motion, not only the instantaneous effect over a temporary state characterized by the instantaneous energy. This first reason coincides with the second one, that is, according to Courant and Hilbert, the object of the calculus of variation “is to find extrema of functionals rather than extrema of functions of a finite number of independent variables. By a functional we mean a quantity or function which depends on entire course of one or more functions” [13]. Now let us define

$$Y_i = \int_{t_a}^{t_b} V dt \quad \text{and} \quad Y_a = \int_{t_a}^{t_b} T dt . \quad (3)$$

Eq.(1) then reads

$$\delta \int_{t_a}^{t_b} V dt - \lambda \delta \int_{t_a}^{t_b} T dt = 0 \quad (4)$$

which is nothing but the usual least action principle (we must let  $\lambda = 1$ ):

$$\delta \int_{t_a}^{t_b} (T - V) dt = \delta A = 0 \quad (5)$$

with the Lagrange action  $A = \int_{t_a}^{t_b} L dt$  where  $L = T - V$  is the Lagrangian. As well known, this

principle yields the Lagrange equation for analytical mechanics :

$$\frac{\partial L}{\partial x_i} - \frac{\partial}{\partial t} \left( \frac{\partial L}{\partial \dot{x}_i} \right) = 0 . \quad (6)$$

(with only one degree of freedom  $x_i$  of the point  $i$ ). As well known, the principle in Eq.(5) has been used for the axiomatic formulation of almost all the physics theories including relativity theory, electromagnetism theory, geometrical and wave optics. However, we would like to mention it in passing, the situation concerning quantum mechanics is somewhat ambiguous since in Feynman's formalism, the least action principle does exist but only in the limit  $\hbar \rightarrow 0$ . The formulation based on the factor  $e^{iA/\hbar}$  cannot be derived, as far as we know, directly from variational calculus using Lagrange action.

#### 4) Principle of virtual work

That the above analysis is based on the cumulative energy of the motion during a period is natural since a motion should be in principle tackled according to its global effect. As a matter fact, this global treatment finally yields Eq.(6) which is an instantaneous description of the motion in differential equation. From this observation, basic principles regarding only instantaneous effect should be possible. One of these is the so called Lagrange-d'Alembert principle of virtual work. To see this, we write Eq.(5) as

$$\delta \int_{t_a}^{t_b} L dt = \int_{t_a}^{t_b} \sum_{i=1}^N \left( \frac{\partial L}{\partial x_i} - \frac{\partial}{\partial t} \left( \frac{\partial L}{\partial \dot{x}_i} \right) \right) \cdot \delta \vec{r}_i dt = 0 . \quad (7)$$



where  $\delta\vec{r}_i$  is the virtual displacement of a point  $i$ . Let us introduce a virtual velocity  $\vec{v}_i = \frac{\delta\vec{r}_i}{\delta\tau_i}$ ,

the above equation reads

$$\int_a^b \sum_{i=1}^N \left( \frac{\partial L}{\partial x_i} - \frac{\partial}{\partial t} \left( \frac{\partial L}{\partial \dot{x}_i} \right) \right) \cdot \vec{v}_i \delta\tau_i dt = 0. \quad (8)$$

where  $\delta\tau_i|_a = \delta\tau_i|_b = 0$ . According to the calculus of variation[8], it follows that

$$\sum_{i=1}^N \left( \frac{\partial L}{\partial x_i} - \frac{\partial}{\partial t} \left( \frac{\partial L}{\partial \dot{x}_i} \right) \right) \cdot \vec{v}_i = \sum_{i=1}^N (\vec{F}_i - m_i \vec{a}_i) \cdot \vec{v}_i = 0. \quad (9)$$

This is the principle of virtual velocity equivalent to the Lagrange-d'Alembert principle of virtual work[4]

$$\delta W = \sum_{i=1}^N (\vec{F}_i - m_i \vec{a}_i) \cdot \delta\vec{r}_i = 0. \quad (10)$$

As a matter of fact, this equation can be derived directly from Eq.(7) if we consider only the motion in the infinitesimal time interval  $t \rightarrow t + dt$ .

When the motion is uniform without acceleration:  $\vec{a}_i = 0$ , Eq.(10) reads

$$\delta W = \sum_{i=1}^N \vec{F}_i \cdot \delta\vec{r}_i = 0 \quad (11)$$

which characterizes static equilibrium. We can extend the term *equilibrium* characteristic of Eq.(11) to the dynamical state characterized by Eq.(10) and refer to it as *dynamical equilibrium*, a term which will be useful for the thermodynamic equilibrium discussed below.

On the other hand, Eq.(10) can be obtained directly from the *Yi-Ya* principle if we consider *Yi* as the work (spatial accumulation) of the initial forces, i.e.,  $Yi = \int_a^b m_i \vec{a}_i \cdot d\vec{s}_i$  and *Ya*

that of the driving forces  $Ya = \int_a^b \vec{F}_i \cdot d\vec{s}_i$  from point  $a$  to point  $b$ , where  $d\vec{s}_i$  is the elementary

displacement of the point  $i$ . Then the variation of *Yi* due to the virtual displacements  $\delta\vec{r}_i(t)$  at a given moment  $t$  must be

$$\delta Y_i = \sum_{i=1}^N m_i \bar{a}_i(t) \cdot \delta \bar{r}_i(t) \quad (12)$$

and similarly

$$\delta Y_a = \sum_{i=1}^N F_i(t) \cdot \delta \bar{r}_i(t). \quad (13)$$

Hence the  $Y_i$ - $Y_a$  principle directly yields Eq.(10) with  $\lambda = 1$ .

## 5) Maximum path entropy principle for nonequilibrium thermodynamics

We still consider non dissipative system. The system is now perturbed by noises and has a random (irregular) dynamics. An essential difference between this irregular dynamics and the regular one is that the path from a point  $a$  to a point  $b$  for a given period of time is no more unique, as discussed in our previous work[14]. At any moment of time, a system in irregular dynamics does not have only one state, contrary to the case of regular dynamics. In the case of thermodynamic system containing a large  $N$  particles, the different states at any given moment are called microstates ( $j=1,2, \dots$ ) each having a likelihood to be visited by the system. In this case, in addition to the pairs of opposite parts related to regular forces and kinetic energy, the aspects such as disorder and order, represented by some measurable quantities, of the motion are expected to play a role in the variational balance.

This role of order and disorder in the variational calculus can be shown as follows. Let  $p_j$  be the probability of occurrence of  $j$  microstate at a given moment of time. The expectation of a quantity  $Y$  at this moment is given by  $\bar{Y} = \sum_j p_j Y_j$  where  $Y_j$  is the value of  $Y$  at the state  $j$ .

Suppose now a variation  $\delta Y_j$  of  $Y_j$  in the state  $j$ . The regular or observable quantity of  $Y$  is the expectation  $\bar{Y}$ . It is obvious that the variation  $\delta \bar{Y}$  is not only determined by  $\delta Y_j$  since  $\delta \bar{Y} = \sum_j p_j \delta Y_j + \sum_j Y_j \delta p_j$ . So between the regular variation  $\delta \bar{Y}$  and the expectation of microstate variation  $\bar{\delta Y} = \sum_j p_j \delta Y_j$ , there is the variation of a quantity  $\Omega$  given by

$$\delta \Omega = \sum_j Y_j \delta p_j = \delta \bar{Y} - \bar{\delta Y} \quad (14)$$

It has been proven[15] that this quantity  $\Omega$  is a measure of uncertainty associated with a random variable  $Y$  and referred to as 'varentropy' in order to distinguish it from the usual information or entropy defined with given formula. It is straightforward to show that[16], in

the case of equilibrium thermodynamic system,  $\delta\Omega$  is the heat transfer directly related to the thermodynamic entropy of the second law. This example shows well the role of disorder (irregularity represented by  $\Omega$ ) and order (regularity and observability represented by the statistical expectation  $\bar{Y}$ ).

Now let us extend the Yi-Ya principle to the irregular dynamics in the case of a motion between two given points (microstates)  $a$  and  $b$  in phase space. Suppose a large number of possible paths  $h$  ( $h=1,2, \dots$ ) for a given period of motion., each having a probability  $p_h(a,b)$  of occurrence. Due to the perturbation of noise, the Yi-Ya balance over each path is in general lost. So we do not have Eq.(4) for every path. Since, as stated in the principle, the essential element for the balance is the variation but not the magnitude of the opposite quantities, we calculate the expectation of the variations  $\delta Y_i$  and  $\delta Y_a$  over all the paths such that

$$\overline{\delta Y_i} = \sum_h p_h(a,b) \int_{t_a}^{t_b} \delta V_h dt \quad \text{and} \quad \overline{\delta Y_a} = \sum_h p_h(a,b) \int_{t_a}^{t_b} \delta T_h dt . \quad (15)$$

Then the application of Yi-Ya principle to these expectation  $\overline{\delta Y_i} - \lambda \overline{\delta Y_a} = 0$  yields ( $\lambda = 1$ )

$$\sum_h p_h(a,b) \int_{t_a}^{t_b} \delta V_h dt - \sum_h p_h(a,b) \int_{t_a}^{t_b} \delta T_h dt = 0 . \quad (16)$$

This can be straightforwardly written as

$$\sum_h p_h(a,b) \delta A_h = \overline{\delta A_{ab}} = 0 . \quad (17)$$

Here  $A_h(a,b) = \int_{t_a}^{t_b} L_h dt$  and  $L_h = T_h - V_h$  is respectively the action and the Lagrangian of a path  $h$ . Eq.(17) is the stochastic least action principle proposed in [14].

As shown in our previous work, Eq.(17) is equivalent to another expression:

$$\delta S_{ab} - \eta \delta \overline{A_{ab}} = 0 \quad (18)$$

where  $\overline{A_{ab}} = \sum_h p_h(a,b) A_h$  is the expectation of action over all possible paths and  $S_{ab}$  is the uncertainty in the choice of paths by the system and referred to as path entropy between  $a$  and  $b$ . It is defined by

$$\delta S_{ab} = \eta (\delta \overline{A_{ab}} - \overline{\delta A_{ab}}) = \sum_h A_h \delta p_h(a,b) . \quad (19)$$

Hence, obviously Eq.(18) is an expression equivalent to Eq.(16) for random motion.  $\eta$  is a characteristic constant of the process. For example, in the case of a Brownian motion,  $\eta$  is inversely proportional to the diffusion constant[14].

As a matter of fact,  $S_{ab}$  and  $\bar{A}_{ab}$  can be considered as another pair of opposite aspects of the motion and represent, respectively, the disorder (uncertainty, randomness) and order (certainty, observability). This consideration makes it possible to yield Eq.(18) from *Yi-Ya* Principle Eq.(1) with  $Yi = \bar{A}_{ab}$  and  $Ya = S_a$  and  $\lambda = \eta$ .

Since the variation of path entropy is due to the variation of action, the functional property of the entropy can be discussed with respect to the action. The concavity of  $S_{ab}$  was proved in [14]. So Eq.(18) is a maximum path entropy algorithm. This maximum calculus was an assumption in our previous work. But here it is a consequence of the *Yi-Ya* variation principle or of the stochastic least action principle Eq.(17). Note that the functional form of path entropy is not necessary in this principle for the maximum to be valid. But if the entropy functional is given, this maximum calculus leads to corresponding path probability distribution as well known.

Appendix I shows an example with exponential action distributions stemming from this principle.

## 6) Maximum entropy principle for equilibrium thermodynamics

In this section, we are interested in the variational calculus relative to the instantaneous dynamical uncertainty associated with equilibrium probability distribution. The measure of this uncertainty is still defined by Eq.(14) with one or more random variables [16] of equilibrium states. According to the above application of *Yi-Ya* principle to disorder and order, we can already write a variational algorithm such as

$$\delta\Omega - \sum_m \lambda_m \delta\bar{A}_m = 0 \quad (20)$$

where  $m$  is the index of the involved random variables and  $\delta\Omega = \sum_m (\delta\bar{A}_m - \bar{\delta A}_m)$ .  $A_m$  can be determined or inferred through the nature of the system. For example, for a canonical ensemble in equilibrium state with  $w$  microstates  $j$  (1,2, ...  $w$ ) each having the probability  $p_j$ , the sole  $A_1$  ( $m=1$ ) should be the system energy  $E_j$  according to the state principle[18].

However, in what follows, we will derive this result from the fundamental consideration of the  $Y_i$  and  $Y_a$  defined in Eqs.(12) and (13). Using the same mathematics as in Eq.(15) with the probability  $p_j$  of microstates, we obtain the expectation of  $(\delta Y_i)_j$  and  $(\delta Y_a)_j$  depending on the microstates  $j$  of the system :

$$\overline{\delta Y_i} = \sum_j p_j \left( \sum_{i=1}^N m_i \bar{a}_i \cdot \delta \bar{r}_i \right)_j \quad \text{and} \quad \overline{\delta Y_a} = \sum_j p_j \left( \sum_{i=1}^N \bar{F}_i \cdot \delta \bar{r}_i \right)_j . \quad (21)$$

Then the application of  $Y_i$ - $Y_a$  principle to these expectation  $\overline{\delta Y_i} - \lambda \overline{\delta Y_a} = 0$  yields:

$$\sum_j p_j \left( \sum_{i=1}^N (\bar{F}_i - m_i \bar{a}_i) \cdot \delta \bar{r}_i \right)_j = 0 \quad \text{with} \quad (\lambda = 1) \quad (22)$$

By using this equation, a rather tedious calculation has been carried out in reference [16] for equilibrium thermodynamic of Grand-canonical, canonical and microcanonical ensembles. In what follows, we only briefly review the result for canonical ensemble for a illustration.

It is straightforward to calculate that  $(\bar{F}_i - m_i \bar{a}_i) \cdot \delta \bar{r}_i = -\delta e_i$  where  $e_i$  is the energy of the particle  $i$ . Eq.(22) reads  $-\sum_j p_j \left( \sum_{i=1}^N \delta e_i \right)_j = -\sum_j p_j \delta E_j = -\overline{\delta E} = 0$  for canonical ensemble.

By the consideration of the first and second laws of thermodynamics of equilibrium system, it was proved that  $\overline{\delta E} = 0$  led to

$$\delta(S - \beta \overline{E}) = 0 \quad (23)$$

where  $S$  is the thermodynamic entropy defined by  $\delta S / \beta = \overline{\delta E} - \overline{\delta E}$  and  $\beta$  is the inverse temperature[16]. This is the famous principle of maximum entropy for therostatistics. When derived in this way, the definition of the entropy functional is not necessary for its validity. It may take if necessary other forms[15] different from the Gibbs-Shannon formula used in the formulation of statistical mechanics founded by Jaynes[5] and Tribus[9]. In their formulism, statistical mechanics is a inference theory instead of a theory of physics, and the most fundamental gradient of the formulism, the maximum entropy principle, is only a inductive method but not a deductive law of physics. In our formulation, this principle, due to its tight kinship with the most fundamental law of mechanics such as virtual work principle, is no doubt a law of physics.

## 7) Concluding remarks

From the above examples of derivation of several principles of physics, the Yin-Yang variational principle seems to be the reality behind the apparently different principles. The ontological basis of this approach, the dialectical view of the world, is not only a long-standing philosophical belief but also a very elementary observed fact of the nature in both occidental and oriental culture. The only new gradient introduced in this work in order to formulate the dialectical variational principle is the *Yi-Ya* variational balance. This balance is different from the conventional notion of equilibrium defined by the vanishing difference between the magnitudes of the opposing quantities. It seems to be a more general criterion for equilibrium especially for dynamical equilibrium. The use of variational relationship in this principle makes it possible for the balance to be independent of frame of reference chosen for the description. It is expected this axiomatic approach improves our understanding of physical world and constitutes a possible way of answering the questions asked in the introduction around the foundation of axiomatic formulation of physics theory.

The choice of the involved pairs of opposite quantities in a motion is primordial for the use of the Yin-Yang variation. From the given examples, it seems that the cumulative energetic definition of *Yi* and *Ya*, i.e., the time integral of energy and the spatial integral of forces, is the most fundamental choice. But this needs further confirmation with more examples in the future. A point to be noticed is that, for a given process, there may be several pairs of opposite quantities. But they do not enter into the Yin-Yang principle all together in a same application. Example is the time integral of potential and kinetic energy, the spatial integral of driving and initial forces, and the entropy and expectation. All three pairs play important and essential role in a random dynamics. But they should be used in three separate applications which are different only in more or less complicated mathematical formulation but all equivalent in importance and in effect for the description the phenomena.

One of the attainments of this work is the unification of the axiomatic formulations of classical mechanics and statistical mechanics which have since long time their own different fundamental principles. However, the confirmation of this approach needs further application to electromagnetism theory, relativity theory, quantum physics and other physical theories with or without axiomatic formulation with variational principle. It should also be mentioned that the systems considered in this work are all Hamiltonian systems having well defined Hamiltonian (potential and kinetic energies) at each moment. Consideration of dissipative system and more complex systems, as well as the associated principles for them (least

dissipation, extremum of entropy production due to dissipation, etc.) would be necessary and useful for further understanding of the approach.

Our final remark is about the linear annihilation of the opposite variations in the Yin-Yang balance defined in this work. It possibly corresponds to a simplest case of mechanical dynamics, and is perhaps not the only criterion of dynamical balance. Nonlinear annihilation, with integer or fractional powers of the *Yi-Ya* variations for example, or other functional relations between the variations, is not excluded *a priori* and might be useful for more complex phenomena and non smooth dynamics. In view of the fact that dialectical aspects and dualities of nature is a widely observed fact in either simple or complex systems, applications of the present approach, possibly with more sophisticated Yin-Yang balance, to other fields, especially to the fields of complexity such as social, biological and network related sciences, are welcome.

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## *Appenix I*

### *A path probability distribution*

The maximum entropy approach in Eq.(14) does not require any given functional form of  $S_{ab}$ . Hence different functionals of it are possible[15]. The first possible form which comes to one's mind is the Shannon entropy  $S_{ab} = -\sum_h p_h(a,b) \ln p_h(a,b)$ . If it is the case, the entropy maximum will yield an exponential path probability distribution  $p_h(a,b) \propto e^{-\eta A_h(a,b)}$  [14].

Hence the probability  $p_h(a,b)$  for the system to go from a fixed point  $a$  to a fixed point  $b$  through a certain path  $h$  is given by

$$p_h(a,b) = \frac{1}{Z_a} e^{-\eta A_h(a,b)}. \quad (24)$$

where  $Z_{ab} = \sum_h e^{-\eta A_h(a,b)}$ . This probability distribution has been verified by numerical simulation of diffusion with certain noises[14]. It is also shown that this probability distribution satisfies a Fokker-Planck equation for normal diffusion and is just the transition probability of free Brown motion[14]. Finally the path entropy between two fixed points  $a$  et  $b$  can be calculated by [14]

$$S_{ab} = \ln Z_{ab} + \eta \bar{A}_{ab}. \quad (25)$$

This result can be extended to the case where  $b$  is an unfixed point in the final phase volume  $B$  of the system at time  $t_b$ . In this case, the probability  $p_h(a,B)$  for the system to go from a fixed point  $a$  to a certain point  $b$  (unfixed in the final phase volume  $B$  of the system at time  $t_b$ ) through a certain path  $h$  (depending on  $a$  and  $b$ ) is given by

$$p_h(a,B) = \frac{1}{Z_a} e^{-\eta A_h(a,b)}. \quad (26)$$

where  $Z_a = \sum_{b,h} e^{-\eta A_h(a,b)} = \sum_b Z_{ab}$ . Hence the path entropy is given by

$$S_{aB} = \ln Z_a + \eta \bar{A}_a. \quad (27)$$



Here  $\bar{A}_a = \sum_b \sum_h p_h(a, B) A_h(a, b)$ . If we extend still this to the case where neither  $a$  nor  $b$  is fixed, we can study the path entropy (the total path uncertainty) between the initial phase volume  $A$  of the system at time  $t_a$  and the final phase volume  $B$  at time  $t_b$ . Obviously this is

$$p_h(A, B) = \frac{1}{Z} e^{-\eta A_h(a, b)}. \quad (28)$$

where  $Z = \sum_{a, b, h} e^{-\eta A_h(a, b)} = \sum_a Z_a$ . The total path entropy between A and B will be

$$S_{AB} = \ln Z + \eta \bar{A}. \quad (29)$$

Here  $\bar{A} = \sum_a \sum_b \sum_h p_h(A, B) A_h(a, b)$ .

If we are interested in the total transition probability  $p_{ab}$  between two fixed points  $a$  and  $b$ ,  $p_{ab}$  can be calculated from Eq.(28) by  $p_{ab} = \frac{1}{Z} \sum_h e^{-\eta A_h(a, b)} = \frac{Z_{ab}}{Z}$ . By using Eq.(25), it can be straightforwardly written

$$p_{ab} = \frac{1}{Z} \exp(S_{ab} - \eta \bar{A}_{ab}). \quad (30)$$

Using a Legendre transformation  $F_{ab} = \bar{A}_{ab} - S_{ab} / \eta = \frac{1}{\eta} \ln Z_{ab}$  which can be called 'free action' which mimics the free energy of thermodynamics, we can write  $p_{ab} = \frac{1}{Z} \exp(-\eta F_{ab})$ .

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