THE IMPULSE RESPONSE FUNCTION ANALYSIS OF PORE PRESSURES MONITORING DATA

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5th International Conference on Dam Engineering
14-16 February 2007
LNEC - LISBON, PORTUGAL

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Abstract

Effective control of dam safety requires that the measured pore-pressure data be interpreted in the shortest possible time following the readings. Direct resolution based on partial differential equations are not appropriate. We present a relevant formalism for analysing pore-pressure monitoring data: the Impulse Response Function Analysis (IRFA) method. The model based on approximations for the impulse response of the dam gives the variations in the pore-pressure measurement resulting from changes in the reservoir and rainfall levels. An expression for the explicit estimation of in situ hydraulic diffusivity is derived. The model were applied to the analysis of monitoring data obtained at a zoned earthdam. Obtained results proved that essential aspects of the observed phenomenon in most cells data can be described in this linear framework, and that they are taking into account.

Keywords: pore pressures, earthdams, monitoring, time series, impulse response function.

1. Introduction

The analysis of time series time series dam monitoring data is an important issue in civil engineering. Auto Regressive Moving Average (ARMA) models are particularly suitable for studying time series. Although they have been used in many fields, they have not been frequently applied for dam monitoring data analysis.

The common tools are the statistical methods of the Hydrostatic-Season-Time (HST) type. They were developed in the 1960s for analysing the displacements resulting from the pendulum effects occurring at arch dams [1]. These methods are used nowadays to analyse measurements of other kinds. The experience acquired on several hundreds of dams has confirmed what an excellent tool this approach can be for interpreting monitoring data.

When dealing with dissipative processes like seepage, it is necessary to look at the loading history responsible for the levels occurring at a given moment, rather than simply taking the values of all the loads measured at the same moment.

Recently a very simple delay model has been designed to simulate the pore pressures measured in and around dams, which are influenced by the reservoir and rainfall levels [2], [3], or to simulate the temperature measured in archdams [4]. This model was based on the approximation of the Impulse Response Function Analysis (IRFA) of the dam works with an exponential decay.

In this paper, we focus on the case on the influence of the water level on pore-pressure measured in the core of an earthdam, and we compare the simple model to a more elaborate one.

2. The necessity of impulse response function analysis

The monitoring data analyses have to be carried out periodically at short time intervals. Direct resolution based on partial differential equations, like the Finite Element Method or the Finite Volume Method, are therefore not appropriate.

In order to quantify the changes occurring under constant conditions(i.e., the ageing, the trends, the drift), it is necessary in the first place to be able to account for any time-independent changes due solely to external constraints, such as variations in the reservoir and precipitation levels.

Impulse response function are an established model in linear systems theory, electrical network theory, signal processing and control theory. They require no previous knowledge of the dam geometry or the properties of the materials. They allow to derive effective algorithms (e.g., fast convolution). The input signal (or loading) $a(t)$ and the output signal (or response) $y(t)$ are connected by the impulse response function of the system $h(t)$ as follows:

$$y(t) = \int_0^t h(t-\tau)a(\tau)d\tau \ .$$

Dams can be assumed to behave in an approximately linear manner in normal operation. The use of impulse response functions in a linear framework makes it possible to check a few properties.

The monitoring data are proportional to the loadings. Any complexity that arises may be described at the result of superposition: the monitoring data are the summation, over all loadings, of the response to each loading, as if it were the only loading to the dam. The response to a sinusoidal signal is a sinusoidal signal (accommodation) with the same period, but with a different phase (the time lag, or delayed effect) and amplitude (damping).

3. IRFA models for pore-pressure analysis.

The pore pressure of water that seeps through the dam can be described by a parabolic equation. The solution can be expressed as a function of the initial condition and the boundary conditions in terms of the Green's function associated with the boundary problem. This representation amounts to an external description in which the impulse responses of the system can be expressed in terms of the Green's function. This approach has a well-established theory [5]. However, the Green’s function is unknown. A classical method in signal processing is to construct models which represent these impulse response functions, at least approximately.

The time origin $t = 0$ is the relevant date between the completion of the dam and the first impounding. We propose a model for analysing monitoring data corresponding to dam operation, starting at a date t_0 such as influence of initial pore-pressure can be neglected. The pore-pressure $P(t)$ is assumed to be influenced by the water level, and the time (ageing). The obtained IRFA model is therefore:

$$P(t) = C + H(t) + T(t) \ ,$$

where C is a constant, $H(t)$ is the water level effect, and $T(t)$ is the time effect (trends, drift).

The simplest approximation of the impulse response function is given by the two-parameters (α , η) exponential decay:

$$H(t) = \frac{\alpha}{\eta}\int_0^t \exp\left(-\frac{t-\tau}{\eta}\right)\Delta Z(\tau)d\tau \ ,$$

where $\Delta Z(t) = Z(t) - Z_{\min}$, $Z(t)$ is the reservoir water level, $Z_{\min}$ is the minimum water level (that of the drainage blanket for example), α is the static damping factor (dimensionless), η is the characteristic diffusion time of the located point. The explanatory variable $T(t)$ accounts for the other non stationary effects (as ageing), of which the formulation is beyond the scope of the present paper.

For a ramp description of the water level time series, the discrete expression of this simple model enjoy the following Autoregressive Moving Average (ARMA) relation:

$$H^{n+1} = (1-\theta_1)H^n + \theta_1\left[\theta_2 \Delta Z^{n+1} + (1-\theta_2)\Delta Z^n\right],$$

$$\theta_1 = 1 - \mathrm{e}^{-\Delta t^n/\eta}, \; \theta_2 = \frac{1}{\theta_1} - \frac{\eta}{\Delta t^n}.$$

In order to assess the relevancy of this simple model for analysing dam monitoring data, we compare obtained results with a more elaborare model given by a three parameters (α, η, σ) impulse response function close to the fundamental solution of linear diffusion problems:

$$H(t) = \frac{\alpha}{\sigma\sqrt{2\pi}} \int_0^t \left(\frac{\tau}{\eta}\right)^{-3/2} \exp\left[-\frac{\eta}{2\tau}\left(\frac{\tau-\eta}{\sigma}\right)^2\right] \Delta Z(t-\tau) d\tau,$$

where the third parameter σ is also a characteristic diffusion time, of which the interpretation is beyond the scope of the present paper.

4. Interpretation of the parameters

Coefficient η is a characteristic diffusion time: the system has some memory of the previous values of the loading time series. The role of this parameter is given by the following harmonic analysis: if $\Delta Z(t) = \sin(\omega t)$, then $H(t) \approx \alpha \sin[\omega(t-\eta)]$ under slowly varying loading conditions ($(\omega\eta)^2 << 1$). The characteristic time η quantifies the time elapsing between the onset of the loading and the response, and the dimensionless parameter α characterises the damping.

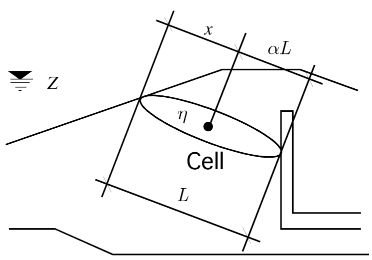
Analysing closed-form solution of a relevant boundary value problem yields the results [3]:

$$\alpha = 1 - \frac{x}{L}, \; \eta = \frac{x}{6L}\left(2 - \frac{x}{L}\right)T, \; T = \frac{L^2}{D}, \quad (1)$$

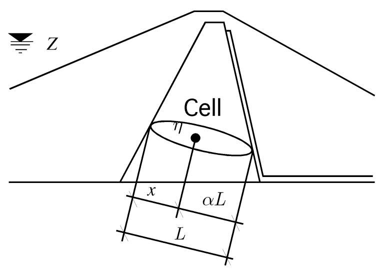
where x is the mean distance between the cell and the loading surface (the upstream face), L is the mean seepage path between the loading point, and the outlet point (chimmey drain, drainage blanket, downstream face) (Fig. 1), T is the diffusion characteristic time of the dam and D is the hydric coefficient of diffusion. A coefficient α around the unit value means either that the instrument is located near the upstream (and x is small) or that the outlet point is located far from the loading point (and L is large). A very large time η will reflect the presence of either a highly impermeable soil or a very long drainage distance L.

The characteristic time η makes it possible to assess the diffusion coefficient:

$$D = \frac{(1-\alpha^2)L^2}{6\eta}. \quad (2)$$



(a)



(b)

Fig. 1. Interpretation of parameter α as a function of (x, L) in a homogeneous earthdam (a), or in the core of a zoned earthdam (b).

The diffusion coefficient is $D = k / c$ where k is the permeability and c is the hydric capacity of the soil. Unsaturated zones have a non null hydric capacity. In this case, the pore-pressure arises almost entirely from suction forces and capillaries. This is the explanation of delayed responses of the water level effect observed in pore pressure measured with cells located in the drawdown zones of the dam. However, in zones located below the free surface, the hydric capacity can also be non null. In these so-called saturated zones, the pore fluid may be a mixture of incompressible water and compressible gas bubbles. The compressibility of the interstitial fluid is likely to be the most decisive factor involved. St-Arnaud [6] suggested in particular that one should take into account the fact that dam water not only has its own natural air content, but also contains air which was emprisoned when the dam was being filled, which is partly compressed and partly dissolved. Another origin of the presence of air lies in the fact that the water entering the dam is air saturated at a pressure that can be different from the atmospheric pressure [7], [8]. When water pressure increases, air is compressed (Boyle's law) and partly dissolved (Henry's law). When water pressure decreases, air comes out of solution.

5. Application to pore-pressures analysis

The measurements obtained on a zoned earthfill dam 42 meters in height with a horizontal drainage blanket were analysed with the IRFA models. The dam rests on a good rock foundation with a 25 m shallow grout curtain under the central clay core. A plan and three cross section are shown in Fig. 2. Elevations are in meters above normal sea water level. A total of 14 electrical pore-pressure cells were installed on three vertical section of the core.

The water level data cover a heigth years period (3175 days). The level of the reservoir underwent cyclic raising and lowering (Fig 3), with a period of approximately one year ($\omega \approx 2\pi / 365$ in days^{-1}). The analysis period cover a three years period dam operation (1175 days), starting at a date t_0=2000 days sufficiently long after the dam was first filled, so that influence of initial condition can be neglected. Measurements were done approximately every five days and represent 167 numbers for each serie for the analysis period.

The numerical results are summarized in Table 1 for the simple IRFA model (IRFA #1), and in Table 2 for the elaborate IRFA model (IRFA #2). With the exception of CV11, obtained values of α and η were the same for both models. These results show two features : 1) the amplitude of the response α decreases with distance from the upstream face, 2) the time delay η increases in the downstream direction. The characteristic diffusion time T was evaluated by Eq. (1): $T = 6\eta / (1 - \alpha^2)$. This time ranged from 48 days to 506 days. The diffusivity Eq. (2) ranged from 0.3 to 20x10^{-5} m^2/s.

Some typical resulting fits are shown in Figs. 4,5 and 6. Both IRFA models performed well for CV24 and CV10 cells (Figs. 4 and 5), but yielded poor fit for CV11 (Fig. 6). This cell, as well as cells CV05 and CV31, are located far from the loaded surface. In this case, the poor fits are primarly due to the loss of vital pore-pressure versus time information from the material from the static damping of the response. Hence, it appears that the further away the measurement is from excitation, the more difficult the analysis will be.

Obtained results shows a typical hysteresis occurring during increasing/decreasing water level cycles, as well as the response lag, which was of the same order of size as the characteristic time η . Some measurements can be taken to mean that an increase in the interstitial pressure has occurred during a decrease in the reservoir level, and *vice-versa*. This well-know phenomenon has been observed *in situ* [9], [10] as well as being simulated under laboratory conditions [11].

In spite of the advantage of an additionnal fitting parameter σ , the elaborate IRFA model performed not really better than the exponential IRFA model. This fact is primarly due to slowly varying loads of the water level in comparison with the characteristic diffusion time. As the simple IRFA model does not account for high loading harmonics, it correctly reproduces the low frequency behavior (ωT <1 to 10 depending on the location x) [3].

The main range of frequencies encountered in the reservoir level variation of earthdams are low as compared to the characteristic diffusion time. We conclude that the exponential decay IRFA model is sufficient for engineering applications, in order to obtain an accurate estimate of the evolution of the pore-pressure with time as a function of the water level.

Table 1. IRFA results of cells data analysis with the exponential IRFA model.

Data	R^2 (%)	α	η (days)	T (days)	D (10^{-5} m^2/s)
Left side					
CV01	82	0.15	26	163	0.4
CV04	99	0.78	8	128	3.1
CV05	95	0.30	32	210	2.0
Middle					
CV26	90	0.51	6	48	1.0
CV14	92	0.37	22	156	0.7
CV17	99	0.89	3	83	19.4
CV18	99	0.72	9	116	10.8
CV19	98	0.43	21	156	3.9
CV24	99	0.85	7	150	18.9
CV10	98	0.64	27	273	6.3
CV11	87	0.37	73	506	2.7
Right side					
CV27	84	0.26	8	56	1.4
CV30	99	0.84	8	175	4.3
CV31	97	0.50	32	258	3.2

Table 2. IRFA results of cells data analysis with the elaborate IRFA model.

Data	R^2 (%)	α	η (days)	σ (days)	T (days)	D (10^{-5} m^2/s)
Left side						
CV01	91	0.15	29	20	179	0.3
CV04	99	0.77	8	5	121	3.3
CV05	99	0.28	30	16	200	2.1
Middle						
CV26	90	0.51	6	2	50	0.9
CV14	93	0.37	23	20	163	0.7
CV17	99	0.89	2	2	81	19.9
CV18	99	0.73	9	12	127	9.9
CV19	99	0.42	20	19	151	4.0
CV24	99	0.85	7	7	153	18.5
CV10	99	0.60	26	17	249	6.9
CV11	97	0.28	55	28	362	3.7
Right side						
CV27	84	0.26	9	4	59	1.4
CV30	99	0.84	8	5	164	4.6
CV31	99	0.47	30	18	234	3.5

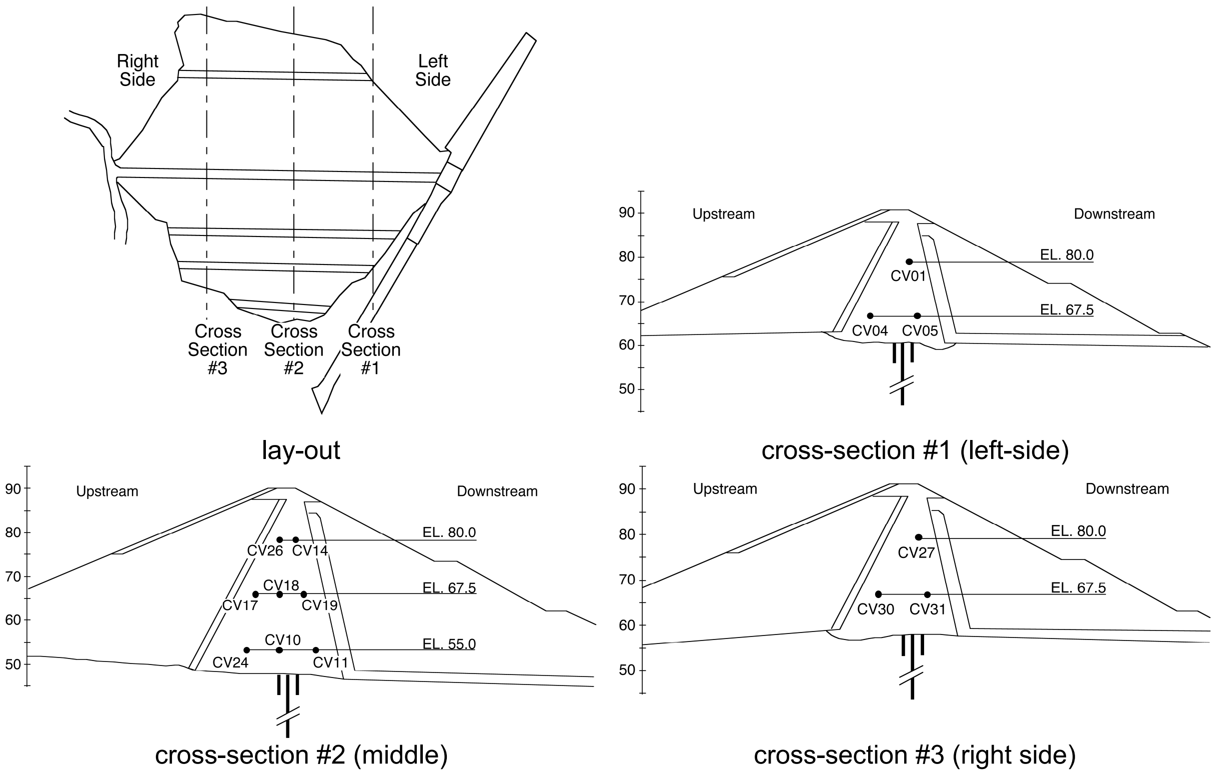


Fig. 2. Lay-out, cross-sections locations and pore-pressure cells locations.

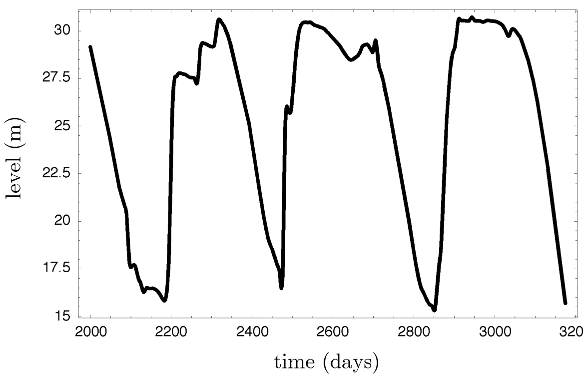


Fig. 3. Water level vs. time.

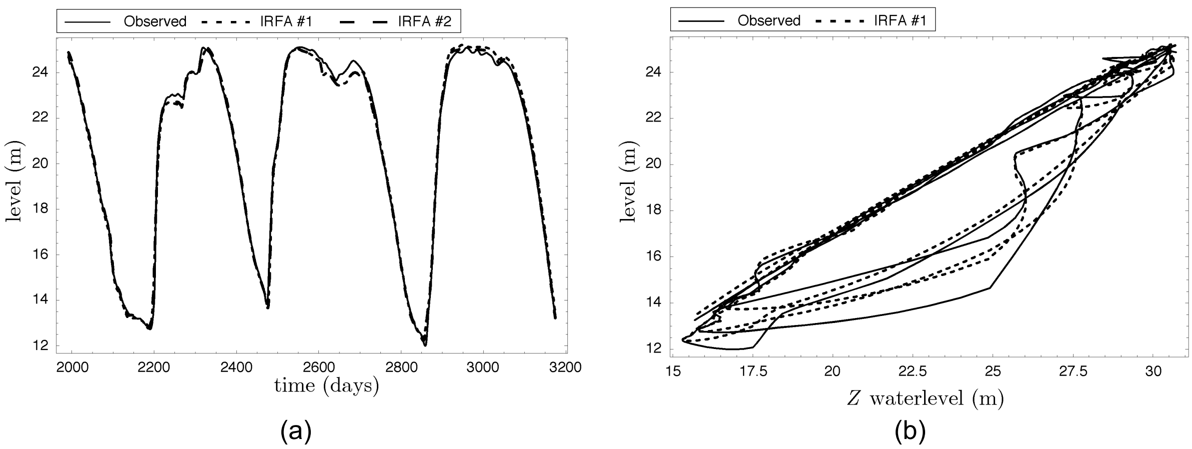
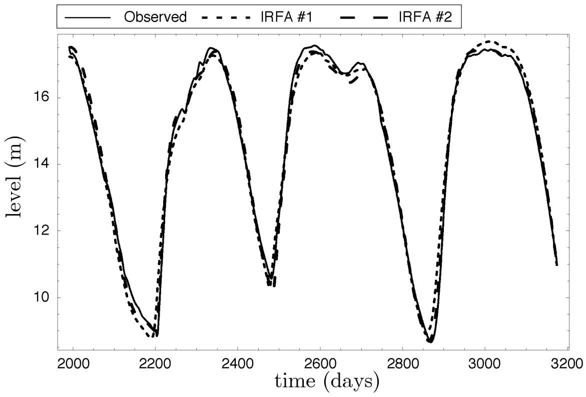
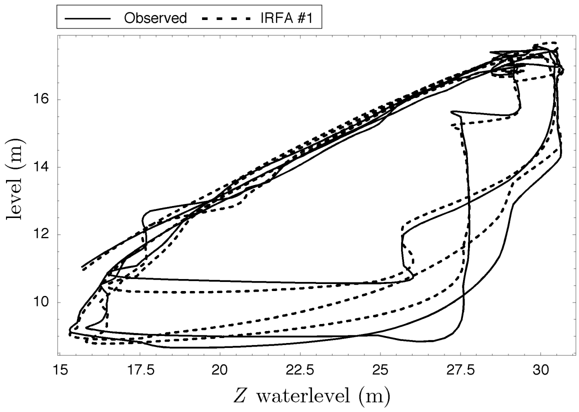


Fig. 4. CV24 piezometric head as a function of time (a) and water level (b). Comparison of IRFA models (a).

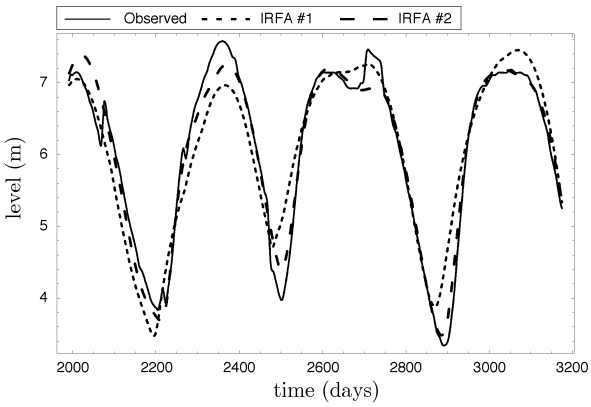


(a)

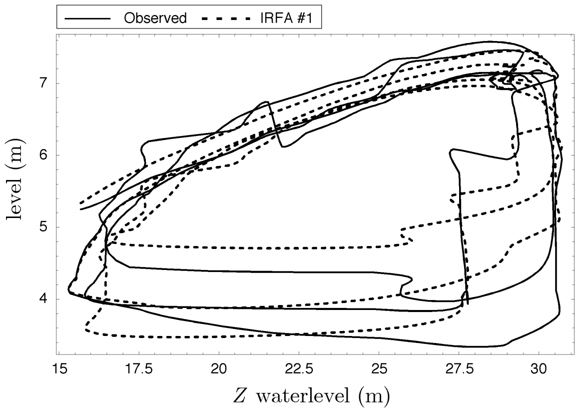


(b)

Fig. 5. CV10 piezometric head as a function of time (a) and water level (b). Comparison of IRFA models (a).



(a)



(b)

Fig. 6. CV11 piezometric head as a function of time (a) and water level (b). Comparison of IRFA models (a).

6. Conclusion.

We propose an Impulse Response Function Analysis (IRFA) method for interpreting pore pressures measurements in earthdams. The model accounts for some of the main aspects of delayed effects: dissipation, accomodation (delay and damping) under cyclic loading, and influence of the previous loading history.

The characteristic diffusion times were found to range between a few days and a few months. An expression for the explicit estimation of in situ hydraulic diffusivity was proposed. This diffusivity ranged in the present case from 10^{-5} to 10^{-4} m^2/s.

The simplest IRFA model with a exponential decay performed surprisingly well. This model involves a recurrence equation with which it is possible to apply convolution products using a simple numerical method. The discrete time formulation used is similar to that of a model of the ARMA(1,2) type. This study opens new perspectives as regards the potential use of modern methods of this kind for analysing dam monitoring data.

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