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FAULT DETECTION OBSERVERS FOR SYSTEMS WITH UNKNOWN INPUTS

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Abstract. Many approaches for the design of unknown input observers were developed to estimate the state of a linear time-invariant dynamical system driven by both known and unknown inputs. These observers are often used for component failure detection, for instrument fault detection in systems subject to plant parameter variations or uncertainties and for fault detection and isolation for systems subject to unknown inputs. In this paper, a method for the design of fault detection observers is developed for linear time-invariant systems subject to arbitrary unknown inputs. A constructive numerical algorithm using generalized inverse of matrices is included. Moreover, an example illustrates the applicability of the proposed method.

Key Words. Actuator failures; observers; state estimation; failure detection; linear systems.

1. INTRODUCTION

Many approaches for the design of unknown input observers (UIOs) were developed to estimate the state of a linear time-invariant dynamical system driven by both known and unknown inputs (Kudva *et al.,* 1980), (Kurek *et al.,* 1983), (Guan *et al.*, 1991) and (Hou *et al.,* 1992). Generally, the estimation is established by eliminating the unknown inputs from the system. This technique may be also applied for singular systems (Miller *et al.,* 1982), (Chen *et al*., 1991) and (Ragot *et al.,* 1993). Our principal aim, in this communication, consists to compare the actual behaviour of a system to a reference one in order to detect and localize the faults when they occur.

Generally, the generation of residuals is issued from the process knowledge; for example analytical models, knowledge base, sensor and actuator signals, ... The most popular approach is based on the analytical redundancies which use the mathematical model of the process. At each moment, the residuals represent the inconsistency between the actual plant variables and the model variables; they are ideally zero but become non-zero if the actual system differs from the ideal one (this may be due to sensor or actuator faults, modelling errors, non exact parameters of the model, ...). When only few sensors are available, the analytical redundancies have to be generated on a long time interval. In this case one commonly use system observers to estimate state time histories for processes with incomplete set of state measurements (Gertler, 1991). Then, the fault detection is achieved by testing the magnitude of these residuals.

The detection and the isolation of sensor and actuator failures have received during the last decade, a lot of attention in the literature in the field of automatic control. Moreover, the use of multiple observers has been extended in order to improve the localization of the faulty elements of a process. The first development in this area is probably due to Clark (1978) which uses as many observers as there are outputs, the i^{th} one is driven by the i^{th} output of the process and all the inputs. The same strategy may be applied for inputs (Viswanadham *et al.,* 1987); the method consists in constructing as many observers as there are inputs, the i^{th} one is driven by the i^{th} input and all the outputs. When it is desired to isolate faults on actuators, some algorithms based on unknown input observers have been proposed; such strategies have also been reviewed by several authors and the reader should refer to the papers given in reference (Frank *et al.,* 1989) and (Chen *et al.*, 1991).
Some results dealing with the computational aspects were given by O'reilly (1983) who proposed an approach which consists in reducing the problem of the observer matrices computation by expressing all of them in terms of an arbitrary one; Ge *et al.* (1988) who treated these aspects for component failure detection using robust observers; Saif *et al.* (1993), for instrument fault detection and identification in linear dynamical systems subject to plant parameter variations or uncertainties. Chang *et al.* (1993) gave also, a solution to the problem of designing an observer for fault detection and isolation for linear time-invariant system subject to arbitrary unknown inputs by using the vector projection theory and propose an algorithm for the matrix calculation. The aim of this paper is also to point out the problem of solving the equations of the observer and to propose an appropriate algorithm for linear systems subject to unknown inputs.

This paper is organized as follows: the section 2 deals with the general structure of an unknown input observer giving the constraint conditions that the observer matrices have to satisfy; the section 3 derives an algorithm for the matrix computations from the constraint conditions; the section 4 deals with residual analysis; section 5 presents an application of the design of the unknown input observer in detecting actuator faults and the section 6 illustrates, by an example, the matrix computations and the procedure of actuator failure detection.

2. UNKNOWN INPUT OBSERVER APPROACH

Consider the linear time-invariant system:

$$\dot{x}(t) = A\,x(t) + B\,u(t) + D\,d(t) \qquad (1a)$$
$$y(t) = C\,x(t) \qquad (1b)$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^r$ is the input vector, $y(t) \in R^m$ is the output vector and $d(t) \in R^{k+1}$ (the dimension k + 1 will be justified in the section 5) is the disturbance vector whose elements are unknown functions of t. A, B, C and D are constant matrices with appropriate dimensions; without lost of generality, C is assumed to be full row rank and D to be full column rank.

Our aim is to estimate the state vector x(t) in spite of the presence of the unknown perturbation d(t). So, let us use the full order observer described by:

$$\dot{z}(t) = N\,z(t) + L\,y(t) + G\,u(t) \qquad (2a)$$
$$\hat{x}(t) = z(t) - E\,y(t) \qquad (2b)$$

where $z(t) \in R^n$ and $\hat{x}(t) \in R^n$ is an estimation of the system state vector x(t).

It is desired that, at steady state, $\hat{x}$ approaches asymptotically x. Therefore, we define the state estimation error:

$$e(t) = x(t) - \hat{x}(t) \qquad (3)$$

The dynamic equation governing this error can be reduced to $\dot{e}(t) = N\,e(t)$ if one purposely chooses:

$$P = I + E\,C \qquad (4a)$$
$$L\,C = P\,A - N\,P \qquad (4b)$$
$$G = P\,B \qquad (4c)$$
$$P\,D = 0 \qquad (4d)$$
$$N \text{ stable} \qquad (4e)$$

Our aim is to establish a systematic procedure in order to determine the matrices defining the observer (2) and verifying the above conditions (4).

Necessary and sufficient conditions for the existence of this type of observers are very classical and will not be discussed here. They are as follows :

rank(C D) = rank(D) = k+1 with m > k+1

3. THE UNKNOWN INPUT OBSERVER DESIGN

Since C has full row rank, suitable coordinate transformation on the states can be found such that the given system (1) is restricted system equivalent to (O'Reilly, 1983):

$$\dot{x}(t) = A\,x(t) + B\,u(t) + D\,d(t) \qquad (5a)$$
$$y(t) = (I_m \;\; 0_{m(n-m)})\,x(t) \qquad (5b)$$

In order to simplify the presentation, the notation used in equation (1) remain unchanged although the matrix A and the state x are different. According to the reduced form of C, the following equalities can be deduced:

$$E\,C = (E \mid 0_{n(n-m)}) \qquad (6a)$$
$$L\,C = (L \mid 0_{n(n-m)}) \qquad (6b)$$

These expressions (6a) and (6b) will be used in the following to determine respectively the structure of the matrices P and L. Let us determine the matrices characterizing the observer (4).

3.1. *Determination of P*

We remark, taking into account the equations (6a), (4a) and (4d), that $(E \mid 0_{n(n-m)}) = P - I$ and P has to be orthogonal to D. Therefore, we propose the following structure of P which allows us to determine the other matrices:

$$P = \begin{pmatrix} P_1 & 0_{m.(n-m)} \\ P_2 & I_{(n-m).(n-m)} \end{pmatrix} \qquad (7)$$

where $P_1 \in R^{m.m}$, $P_2 \in R^{(n-m).m}$. Taking into account (4d) and (7), D can be partitioned as follows:

$$D = \begin{pmatrix} D_1 \\ D_2 \end{pmatrix} \qquad (8)$$

where $D_1 \in R^{m.(k+1)}$, $D_2 \in R^{(n-m).(k+1)}$, one may write the equation (4d) under the form:

$$P_1\,D_1 = 0_{m.(k+1)} \qquad (9a)$$
$$P_2\,D_1 + D_2 = 0_{(n-m).(k+1)} \qquad (9b)$$

As D_1 and D_2 are known and rank(D) = k+1, we remark respectively from the equations (9a) and (9b) that m.(m-(k+1)) and (n-m).(m-(k+1)) parameters in P_1 and P_2 can be chosen and that the others can be deduced. More precisely, from the equations (9a) and (9b), the matrices P_1 and P_2 may be computed as:

$$P_1 = K_1\,(I - D_1\,D_1^+) \qquad (10a)$$
$$P_2 = -\,D_2\,D_1^+ + K_2\,(I - D_1\,D_1^+) \qquad (10b)$$

where D_1^+ is a generalized inverse matrix of D_1, K_1 and K_2 are arbitrary matrices (we will define this choice later) with $K_1 \in R^{m.m}$ and $K_2 \in R^{(n-m).m}$. If D_1 is a non singular matrix then P_1 is null and $P_2 = -\,D_2\,D_1^{-1}$.

3.2 *Determination of E*

From (4a) and using (7), one may write:

$$(E \mid 0) = \begin{pmatrix} P_1 - I_{m.m} & 0_{m.(n-m)} \\ P_2 & 0_{(n-m).(n-m)} \end{pmatrix} \qquad (11)$$

From which we deduce:

$$E = \begin{pmatrix} P_1 - I_{m.m} \\ P_2 \end{pmatrix} \qquad (12)$$

3.3. *Determination of N*

The matrices A and N can be respectively partitioned as:

$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \qquad N = \begin{pmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{pmatrix}$$

with $A_{11} \in R^{m.m}$, $A_{21} \in R^{(n-m).m}$, $A_{12} \in R^{m.(n-m)}$, $A_{22} \in R^{(n-m).(n-m)}$, $N_{11} \in R^{m.m}$, $N_{21} \in R^{(n-m).m}$, $N_{12} \in R^{m.(n-m)}$ and $N_{22} \in R^{(n-m).(n-m)}$.
The matrices P A and N P are then expressed as:

$$P\,A = \begin{pmatrix} P_1 A_{11} & P_1 A_{12} \\ P_2 A_{11} + A_{21} & P_2 A_{12} + A_{22} \end{pmatrix} \tag{13}$$

$$N\,P = \begin{pmatrix} N_{11} P_1 + N_{12} P_2 & N_{12} \\ N_{21} P_1 + N_{22} P_2 & N_{22} \end{pmatrix} \tag{14}$$

From (13) and (14) and taking into account the expression of C, the following equations have to be satisfied:

$$P_1 A_{12} = N_{12} \tag{15a}$$
$$P_2 A_{12} + A_{22} = N_{22} \tag{15b}$$

Taking into account (10a) and (10b), equations (15a) and (15b) can be written:

$$K_1 (I - D_1 D_1^+) A_{12} = N_{12} \tag{16a}$$
$$A_{22} + \left(-D_2 D_1^+ + K_2 (I - D_1 D_1^+)\right) A_{12} = N_{22} \tag{16b}$$

So, with the above expression (16) of N_{12} and N_{22}, the matrix N can be written:

$$N = \begin{pmatrix} N_{11} & K_1 (I - D_1 D_1^+) A_{12} \\ N_{21} & K_2 (I - D_1 D_1^+) A_{12} + A_{22} - D_2 D_1^+ A_{12} \end{pmatrix} \tag{17}$$

Otherwise:

$$N = R + (\bar{N} \quad K)\, Q = R + F\, Q \tag{18}$$

where:

$$R = \begin{pmatrix} 0 & 0 \\ 0 & A_{22} - D_2 D_1^+ A_{12} \end{pmatrix} \qquad \bar{N} = \begin{pmatrix} N_{11} \\ N_{21} \end{pmatrix}$$

$$Q = \begin{pmatrix} I & 0 \\ 0 & (I - D_1 D_1^+) A_{12} \end{pmatrix} \qquad K = \begin{pmatrix} K_1 \\ K_2 \end{pmatrix}$$

The matrix F is chosen such that N, which fix the dynamic of the observer, has a desired set of stable eigenvalues. Consequently, a simple partition of F gives the matrices $\bar{N}$ and K.

3.4. Determination of L

The expression of L can be deduced from (4b). Indeed, taking into account (6b), this equation is reduced to:

$$(L \mid 0) = P\,A - N\,P \tag{19}$$

Summarizing the different matrix computations the design of the observer is achieved by the following algorithm:

Algorithm

System equations

$\dot{x}(t) = A\,x(t) + B\,u(t) + D\,d(t)$
$y(t) = C\,x(t)$

Observer equations

$\dot{z}(t) = N\,z(t) + L\,y(t) + G\,u(t)$
$\hat{x}(t) = z(t) - E\,y(t)$

- Partition D and A (according to the dimension of y and d)

- Compute $R = \begin{pmatrix} 0 & 0 \\ 0 & A_{22} - D_2 D_1^+ A_{12} \end{pmatrix}$ and $Q = \begin{pmatrix} I & 0 \\ 0 & (I - D_1 D_1^+) A_{12} \end{pmatrix}$.

- Determine F by the method of eigenvalues assignment such that N = R + F Q has a desired set of stable eigenvalues.

- Deduce K (K_1 and K_2) and $\bar{N}$ (N_{11} and N_{21}) .

- Compute $P_1 = K_1 (I - D_1 D_1^+)$
$P_2 = - D_2 D_1^+ + K_2 (I - D_1 D_1^+)$.

- Deduce P, N_{12} and N_{22}.

- Compute G = P B and $E = \begin{pmatrix} P_1 - I_{m.m} \\ P_2 \end{pmatrix}$.

- Deduce L from $(L \mid 0) = P\,A - N\,P$.

Remark : according to all the previous partitions of matrices, it is easy to verify that if K_1 is a null matrix, the first m states of z do not depend on u and y. That implies that output estimations are exactly equal to the measurements.

4. ANALYSIS OF THE RESIDUALS

Let us analyse the aptitude of the observer to reconstruct the state of the process and its ability to detect faults on actuators and sensors. In the presence of sensor faults p(t) and actuator faults a(t), the system (1) is equivalent to the one shown by the fig. 1.

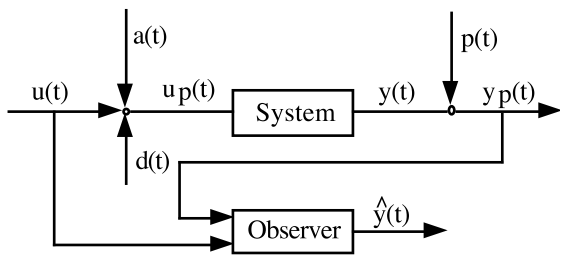

Fig. 1. UIO in the presence of faults

The signals $u_p(t)$ and $y_p(t)$ are respectively the perturbed input and output. In this condition, the system (1) and the observer (2) become respectively:

$$\dot{x}(t) = A\,x(t) + B\,u(t) + D\,d(t) + F_a\,a(t) \quad (20a)$$
$$y(t) = C\,x(t) + F_p\,p(t) \quad (20b)$$

$$\dot{z}(t) = N\,z(t) + L\,y(t) + G\,u(t) \quad (21a)$$
$$\hat{x}(t) = z(t) - E\,y(t) \quad (21b)$$

where F_p and F_a are matrices with appropriate dimensions. The output estimation error is defined by:

$$\varepsilon(t) = y(t) - \hat{y}(t) \quad (22)$$

Taking into account the remark formulated in the previous section, it is obvious that, if K_1 is a null matrix, the output estimation error will not be very useful for detecting actuator faults. So, in order to design efficient fault detection observers it is necessary to introduce a supplementary constraint: the computation of the matrix F using eigenvalues assignment must be done taking into account that the matrix K_1 must be different from a null matrix. Therefore, using Laplace transform, equation (22) can be expressed as:

$$\varepsilon(s) = T_p\,p(s) + T_a\,a(s) \quad (23)$$

where:

$$T_p = \left(I - C\,(s\,I - N)^{-1}\,L\right) F_p \quad \text{and} \quad T_a = C\,(s\,I - N)^{-1}\,F_a$$

We remark that this estimation error is insensitive to the unmeasured perturbation d(t) and depends on the two types of faults which can be correctly detected if the elements of the matrices T_a and T_p are not null and isolated if the columns of the matrices T_a and T_p are independent. Thus, once the observer has been designed, it is then easy to test the isolability condition of faults. In fact, the failure signatures are closely related to the structure of the matrices T_a and T_p As suggested by Gertler *et al*. (1992), equation (23) may be characterized by an occurrence matrix which contains 0 and 1, at any given position, whether the matrices T_a and T_p contain zero or non zero elements. The examination of the columns of the occurrence matrix allows us to define which fault can be detected and isolated.

5. APPLICATION TO THE ISOLATION OF ACTUATOR FAULTS

Our aim here is to detect and identify actuator faults using a bank of UIOs. The detection and identification scheme is constructed to be sensitive to these faults and to isolate them (Chen *et al*., 1991). The scheme is not affected by uncertainties (parameter variations, process noises, ...): their influences are eliminated from the state estimation error (Viswanadham *et al*., 1987). So, the idea is to associate a part of the input u to the uncertainties (these inputs will be considered as unknown inputs) and to construct estimators which are insensitive to failures in the actuators corresponding to the unknown inputs. We can formulate this proposition by writing the system (1) as follows:

$$\dot{x}(t) = A\,x(t) + B^i\,u_p^i(t) + B_i\,u_{pi}(t) + H\,v(t) \quad i = 1, ..., r \quad (26a)$$
$$y(t) = C\,x(t) \quad (26b)$$

where $v \in R^k$ is an unknown input (the dimension of v justifies that of the vector d(t) given in (1a)), H is its distribution matrix, B_i is the i^{th} column of B and B^i is the n(r-1) matrix obtained from B by deleting B_i, u_i is the i^{th} entry of u and u^i is the input obtained from u by deleting u_i.

Denoting $d^i = (u_{pi} \quad v^T)^T$ and $D^i = (B_i \quad H)$, the system (26) becomes:

$$\dot{x}(t) = A\,x(t) + B^i\,u_p^i(t) + D^i\,d^i(t) \quad (27a)$$
$$y(t) = C\,x(t) \quad (27b)$$

where $D^i \in R^{n.(k+1)}$

As the estimation of the system state is not affected by failures in the unknown inputs, we assume that $\text{rank}(D^i) = k + 1$ and that $\text{rank}(C\,D^i) = k + 1$ (see section 3) to construct r UIOs given by:

$$\dot{z}^i(t) = N^i\,z^i(t) + L^i\,y(t) + G^i\,u^i(t) \quad (28a)$$
$$\hat{x}^i(t) = z^i(t) - E^i\,y(t) \quad (28b)$$

where $N^i \in R^{n.n}$, $L^i \in R^{n.m}$, $G^i \in R^{n.(r-1)}$, $E^i \in R^{n.m}$ for i = 1, ..., r. The matrices of each observer that might satisfy the following conditions

$$P^i = I + E^i\,C \quad (29a)$$
$$L^i\,C = P^i\,A - N^i\,P^i \quad (29b)$$
$$G^i = P^i\,B^i \quad (29c)$$
$$P^i\,D^i = 0 \quad (29d)$$
$$N^i \text{ stable} \quad (29e)$$

are computed according to the algorithm given in the third section.

Then, we construct a set of residuals ($\varepsilon^i(t) = y(t) - \hat{y}^i(t)$) using the proposed elimination strategy by considering u_i as an unknown input. These residuals are used to detect and identify the actuator faults. By nature of construction, the residuals are insensitive to u_i whereas variations and failures in u_i will affect the outputs y(k). By monitoring these sets of residuals, the actuator failures can be isolated. Indeed, if all sensors are well functioning and all residuals are statistically null except the i^{th} one, then the i^{th} actuator is faulty (fig. 2).

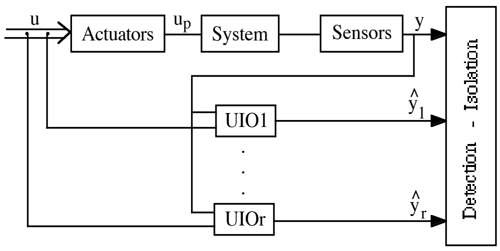

Fig. 2. The decision logic

The residual $\varepsilon^i(t)$ may be used to detect and identify the faulty actuators by comparing its magnitude with a threshold. Let us denote s^i the signature of the residual $\varepsilon^i(t)$, we can write:

$$\|\varepsilon^i(t)\| < \text{threshold: no failure} \Rightarrow s^i = 0. \quad (30a)$$
$$\|\varepsilon^i(t)\| > \text{threshold: the } i^{th} \text{ actuator has failed} \Rightarrow s^i = 1. \quad (30b)$$

If the failure affects the known input u^i then the residual is sensitive to this anomaly; on the other hand, when the failure affects the unknown input u_i, the residual remains close to zero. If two or more actuators fail simultaneously, the above decision logic is not able to isolate faults; for such situation, Chen *and al.* (1991) have proposed a scheme in which each UIO is driven by r-t inputs if t actuators have simultaneously failed.

6. NUMERICAL RESULTS

In this section, we give a numerical example to illustrate the technique of the matrix computation and the detection and isolation of actuator faults. The system is driven by two known inputs u_1 and u_2 (figures 3a and 3b) and one perturbation; its state has four components and the dimension of the measurement vector is equal to three (figure 4). The matrices of the system (27) are:

$$A = \begin{pmatrix} 1.0223 & 0.0952 & 0.0385 & -0.0704 \\ 0.0384 & 0.8822 & 0.1715 & -1.3474 \\ -0.0006 & -0.0033 & 0.974 & -0.0736 \\ 0.0141 & 0.0285 & -0.0912 & 0.9732 \end{pmatrix}$$

$$B = \begin{pmatrix} -0.0349 & -0.0066 \\ -0.7320 & -0.1114 \\ 0.1783 & -0.0648 \\ -0.0052 & 0.0031 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \quad H = \begin{pmatrix} 1 \\ 0.5 \\ 0 \\ 0 \end{pmatrix}$$

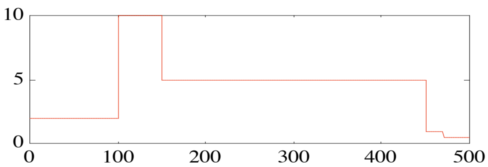

Fig. 3a. First input u_1

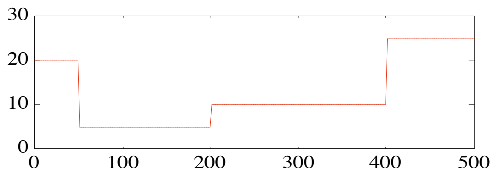

Fig. 3b. Second input u_2

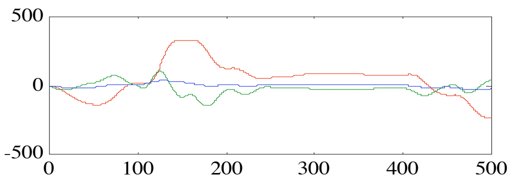

Fig. 4. System outputs

A fault with constant magnitude has been added to the first input between the instants 100 and 150. For this example, the eigenvalue assignment problem has enough degrees of freedom. So, the matrix K may entirely be imposed. In the following, we have chosen:

$$K = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}.$$

One of the two inputs is alternately considered as unknown. In both cases, the observer matrices and the output error estimation ε^i have been determined and computed.

- Case 1: $u_p^i = u_p^1$

$$N^1 = \begin{pmatrix} 0.5968 & 0.2083 & -0.1171 & 0.6396 \\ 0.0359 & 0.6731 & 0.0261 & -0.1475 \\ -0.1088 & 0.1170 & 0.7370 & 0.3445 \\ -0.1010 & 0.1132 & -0.0588 & 1.1431 \end{pmatrix}$$

$$P^1 = \begin{pmatrix} 0.2689 & -0.5377 & 0.8968 & 0 \\ -0.0620 & 0.1240 & -0.2068 & 0 \\ 0.1448 & -0.2897 & 0.4831 & 0 \\ 0.0729 & -0.1458 & 0.2911 & 1 \end{pmatrix}$$

$$L^1 = \begin{pmatrix} 0.0765 & -0.0974 & 0.1699 \\ -0.0195 & 0.0261 & -0.0452 \\ 0.0413 & -0.0526 & 0.0918 \\ 0.0422 & -0.0128 & -0.0203 \end{pmatrix}$$

$$E^1 = \begin{pmatrix} -0.7311 & -0.5377 & 0.8968 \\ -0.0620 & -0.8760 & -0.2068 \\ 0.1448 & -0.2897 & -0.5169 \\ 0.0729 & -0.1458 & 0.2911 \end{pmatrix}$$

$$G^1 = \begin{pmatrix} 0.5441 \\ -0.1255 \\ 0.2931 \\ 0.1508 \end{pmatrix}$$

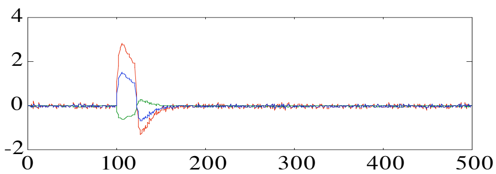

Fig. 5. Output error estimation ε^1 with $u_p^i = u_p^1$

We remark from the figure 5 that an anomaly occurs between the moments 100 and 150. The fault has affected all the components of the residual. This is due to the fact that P_1 and P_2 are not null; so, to the choice of K.

- Case 2: $u_p^i = u_p^2$

$$N^2=\begin{pmatrix} 0.9246 & 0.0483 & 0.1560 & -0.3257 \\ 0.0277 & 0.7077 & 0.0283 & -0.0464 \\ 0.1606 & 0.0555 & 1.0006 & -0.4185 \\ 0.1032 & 0.0349 & 0.1283 & 0.5170 \end{pmatrix}$$

$$P^2 = \begin{pmatrix} -0.1013 & 0.2026 & 0.8120 & 0 \\ -0.0144 & 0.0289 & 0.1158 & 0 \\ -0.1302 & 0.2604 & 1.0435 & 0 \\ -0.1413 & 0.2825 & 1.1614 & 1 \end{pmatrix}$$

$$L^2 = \begin{pmatrix} -0.0276 & 0.0290 & 0.2808 \\ -0.0036 & 0.0034 & 0.0372 \\ -0.0355 & 0.0374 & 0.3611 \\ -0.0194 & 0.0591 & 0.2608 \end{pmatrix}$$

$$E^2 = \begin{pmatrix} -1.1013 & 0.2026 & 0.8120 \\ -0.0144 & -0.9711 & 0.1158 \\ -0.1302 & 0.2604 & 0.0435 \\ -0.1413 & 0.2825 & 1.1614 \end{pmatrix}$$

$$G^2 = \begin{pmatrix} -0.0746 \\ -0.0106 \\ -0.0958 \\ -0.1027 \end{pmatrix}$$

In this case the residual is null. It can be interpreted by the fact that u_2 is not affected by faults and that the anomaly is associated to the unknown input. So, we can conclude that the first actuator has failed between the moments 100 and 150.

7. CONCLUSION

We have developed a method which allows one to determine systematically the matrices describing a fault detection observer in the presence of unknown inputs. The detection and isolation of faulty actuators have been done using a classical extension, taking into account unknown inputs, of the generalized observer scheme. This method has been experimented on various different systems and seems to give interesting results.

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