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A. de Paor

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Clarification of the Hall effect as an energy transfer mechanism in a theory of the Earth's magnetic field and sunspots

A. de Paor

Department of Electronic and Electrical Engineering, National University of Ireland, Dublin, Belfield, Dublin 4, Ireland

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Abstract. Clarification is offered of the energy transfer role played by the Hall effect in a recent paper: de Paor, A., A theory of the Earth's magnetic field and of sunspots, based on a self-excited dynamo incorporating the Hall effect, Nonlinear Processes in Geophysics, 8, 265–279, 2001.

1 Introduction

In a recent paper on the Earth's magnetic field and sunspots (de Paor, 2001) a key role in deriving equations governing the proposed underlying mechanism was played by the claim that the Hall effect provides "a perfect orthogonal axis energy transfer mechanism." The author has now realised that in his presentation (in Sect. 2) the meaning of this phrase is very obscure and likely to lead to misconceptions. In Eqs. (1) and (3) of de Paor (2001), two voltages e_z and e_x were defined. It was not made clear that these are open-circuit Hall voltages: e_z would be measured if i_z were zero and e_x if i_x were zero. When both currents are flowing, the voltages measured across the faces of the Hall Sample are modified to

$$u_x = e_x + r_x \cdot i_x$$
$$u_z = e_z - r_z \cdot i_z \,, \tag{1}$$

as will be proved below.

The voltages defined by Eq. (1) are indicated on the twoport network representation in Fig. 1 in this note. This shows that power

$$p_e = e_x \cdot i_x \tag{2}$$

is being extracted from the x-loop through the controlled source e_x and power

$$p_i = e_z \cdot i_z \tag{3}$$

is being injected into the z-loop through the controlled source e_z.

Correspondence to: A. de Paor (annraoi.depaor@ucd.ie)

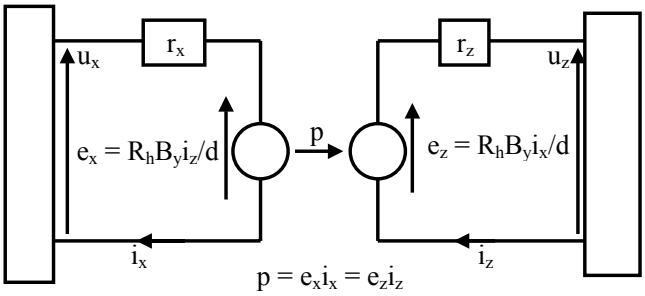

Fig. 1. Hall Sample for derivation of twoport.

Using the expressions given by de Paor (2001), we have

$$p_e = (R_h B_y i_z/d) \cdot i_x \tag{4}$$

and

$$p_i = (R_h B_y i_x/d) \cdot i_z \,. \tag{5}$$

It is clear that

$$p_e = p_i = p \,, \tag{6}$$

as indicated on Fig. 1 of this note. The power being extracted non-resistively from the x-loop is being injected into the z-loop. This, and this only, is the sense in which the Hall effect is claimed to be "a perfect orthogonal axis energy transfer mechanism".

2 Derivation of the twoport representation in Eq. (1)

Equation (1) is actually a macroscopic reflection of the Lorentz expression for the force on a charge carrier moving under orthogonal electric and magnetic fields in a material medium. Just for mathematical simplicity, the derivation is given here for positive charge carriers, but the author will be very happy to supply the derivation for a two-carrier conduction process, positive and negative, to anyone who may be interested.

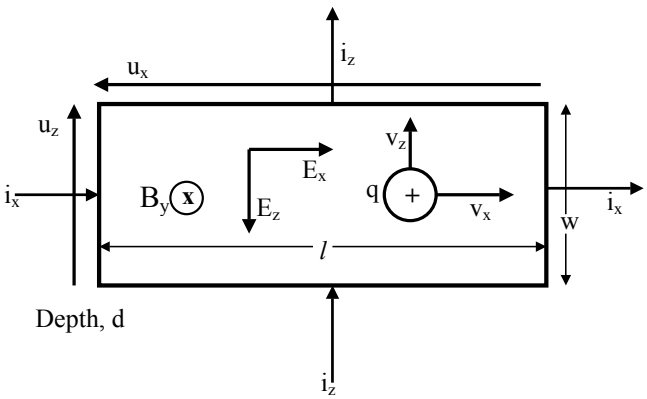

Fig. 2. Hall Sample showing cross-axis power transfer.

Figure 2 shows a Hall sample, with indicated dimensions, carrying currents i_x and i_z along two orthogonal axes, with a magnetic field of flux density B_y along an axis orthogonal to them. It is assumed that the associated current densities are uniform:

$$j_x = i_x/(wd)$$
$$j_z = i_z/(ld) \ . \tag{7}$$

If the carrier concentration is p, if each has charge q (q > 0) and is drifting along the x-axis with speed v_x and along the z-axis with speed v_z, the current densities in Eq. (7) are given by

$$j_x = qpv_x$$
$$j_z = qpv_z \ . \tag{8}$$

If the mobility of a charge carrier is μ and if it experiences a force vector $\boldsymbol{F}$, then its vector drift velocity is

$$\boldsymbol{v} = (\mu/q)\,\boldsymbol{F} \ . \tag{9}$$

The force on a charge carrier along the x-axis is qE_x due to the x-axis electric field and $-qv_zB_y$ due to the z-axis magnetic field. Thus, applied along the x-axis, Eq. (9) gives

$$v_x = (\mu/q)\{qE_x - qv_zB_y\} \ . \tag{10}$$

Due to the vector relationship between v_z and B_y, the magnetic force is in the negative x-direction.

Multiplying Eq. (10) through by the sample length l, rearranging and noting that the voltage u_x is given by $u_x = l\,E_x$, gives the expression

$$u_x = r_xi_x + (B_yi_z)/(qpd) \ , \tag{11}$$

where

$$r_x = l/(\mu qpdw) \ , \tag{12}$$

is the resistance of the sample in the x-direction.

Noting that for a single species of carrier, the Hall coefficient is given by

$$R_h = 1/(qp) \ . \tag{13}$$

and that

$$e_x = R_hB_yi_z/d \tag{14}$$

Equation (11) immediately becomes identical with the first member of Eq. (1). This establishes the validity of the input loop of the twoport.

If the same type of analysis is applied along the z-axis, the only essential difference, due to the vector relationship between i_x and B_y, is that the magnetic force is in the positive y-direction. This is reflected in the equation

$$u_z = -r_zi_z + (B_yi_x)/(qpd) \ , \tag{15}$$

where

$$r_z = w/(\mu qpdl) \tag{16}$$

is the resistance in the z-direction. Equation (14) immediately gives the second member of Eq. (1), which describes the output loop of the twoport.

3 Application to geomagnetic and sunspot theory

The twoport network in Fig. 1 here is implicit in Eqs. (11) and (12) of de Paor (2001).
In Eq. (11) are made the substitutions $i_z \rightarrow i_a, r_z \rightarrow R_a, e_z \rightarrow e_{ha} = k_hi_ai_d$. The right hand box in Fig. 1 now contains the inductor L_a in series with the Johnson-Nyquist noise voltage source of $emfn_a(t)$.
In Eq. (12) are made the substitutions $i_x \rightarrow i_d$ (the dynamo current), $r_x \rightarrow R_{da}$ (as defined by Eq. (48) of de Paor (2001)), $e_x = e_{hd} = k_hi_a^2$. The left hand box in Fig. 1 now contains the series combination of the dynamo emf, $k_di_a^2$, the resistances R_{dr} and R_{dm} defined by Eqs. (49) and (50) of de Paor (2001), the inductor L_d and the Johnson-Nyquist source of $emfn_d(t)$.
Equations (11) and (12) of de Paor (2001) are the only places in the theory in which Eq. (6) of this note is invoked.

The author wishes to apologise to anybody who was confused by the obscurity of his original exposition. He wishes to thank Professor J.E. Allen of the University of Oxford for pointing out the obscurity.

In conclusion, a few typographical errors in de Paor (2001) are pointed out. In the second last line of page 269, the conductivity estimate should read $\sigma \approx 10^7\,\mathrm{S\,m^{-1}}$. In Eq. (50), the argument of the arcsin function – which must, of course, have magnitude ≤ 1 – should read r_2/r_3. In Eq. (56), in the second term within the parentheses, the square root sign should not extend over the quantity $-l/2$. There is a multiplicative factor of π omitted from the expression for L_a in Eq. (57).

References

de Paor, A.: A theory of the Earth's magnetic field and of sunspots, based on a self-excited dynamo incorporating the Hall effect, Nonlinear Processes in Geophysics, 8, 265–279, 2001.