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Nonlinear Processes in Geophysics (1996) 3: 247–261

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Solar wind low-frequency magnetohydrodynamic turbulence: extended self-similarity and scaling laws

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Received 25 March 1996 - Revised 29 November 1996 - Accepted 3 December 1996

Abstract.
In this paper we review some of the work done in investigating the scaling properties of Magnetohydrodynamic turbulence, by using velocity fluctuations measurements performed in the interplanetary space plasma by the Helios spacecraft. The set of scaling exponents ξ_q for the q-th order velocity structure functions, have been determined by using the Extended Self–Similarity hypothesis. We have found that the q–th order velocity structure function, when plotted vs. the 4–th order structure function, displays a range of self–similarity which extends over all the lengths covered by measurements, thus allowing for a very good determination of ξ_q. Moreover the results seem to show that the scaling exponents are the same regardless the various observation periods considered. The obtained scaling exponents have been compared with the results of some intermittency models for Kraichnan's turbulence, derived in the framework of infinitely divisible fragmentation processes, showing the good agreement between these models and our observations. Finally, on the basis of the actually available data sets, we show that scaling laws in Solar Wind turbulence seem to be different from turbulent scaling laws in the ordinary fluid flows. This is true for high–order velocity structure functions, while low–order velocity structure functions show the same scaling laws. Since our measurements involve length scales which extend over many order of magnitude where dissipation is practically absent, our results show that Solar Wind turbulence can be regarded as a testing bench for the investigation of general scaling behaviour in turbulent flows. In particular our results strongly support the point of view which attributes a key role to the inertial range dynamics in determining the intermittency characteristics in fluid flows, in contrast with the point of view which attributes intermittency to a finite Reynolds number effect.

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1 The solar wind laboratory

The Solar Wind is a ionized, magnetized gas, composed mainly of protons, electrons, a small percentage of alpha particles and traces of heavier ions. It continuously flows away from the solar corona in all directions pervading the interplanetary space. The Solar Wind plays an important role in space research for its relevance to many solar, geophysical and astronomical phenomena, as well as for its intrinsic physical interest. General informations are available for example in the monographs by Parker (1963), Hundhausen (1972), and some recent reviews like the two books *Physics of the Inner Heliosphere* (1990, 1991; eds. R. Schwenn and E. Marsch, Springer–Verlag, New York). Among other recent reviews on turbulence in the Solar Wind see Tu and Marsch (1995), Goldstein et al. (1995), Bruno (1992).

Long time ago Parker (1958) conjectured the possibility that the hot solar corona ($T \simeq 10^6$ ^{o}K) out of hydrostatic equilibrium, undergoes a steady expansion where the flow velocity increases from a low value near the Sun to a large supersonic expansion speed and vanishing pressure at large heliocentric distances. Due to the extremely high electrical conductivity of the hot coronal plasma, the magnetic field lines are frozen in it, and the flow transports the solar magnetic field into the interplanetary space. Since the Sun rotates, the resulting magnetic configuration turns out to be the so called Parker's spiral. The universal acceptance of the Parker's model was achieved only after the advent of *in situ* interplanetary observations in the early 1960's. Spacecraft measurements revealed that the interplanetary space was pervaded by a supersonic plasma flow with a "Parker's spiral" average magnetic field configuration. Mariner 2 spacecraft measurements indicated, for the first time, the presence of large amplitude magnetic field fluctuations. The power law observed in the

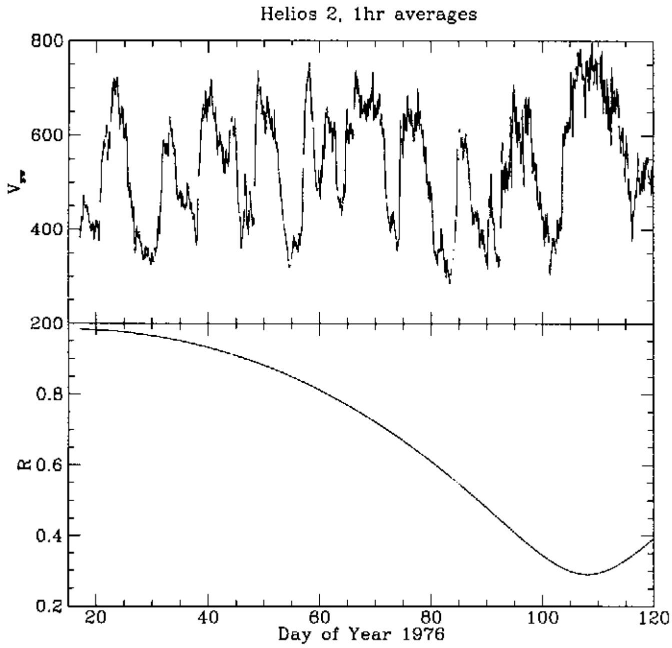

Fig. 1. Hourly averages of wind speed (upper panel) and heliocentric distance (lower panel) are shown versus time. Speed is measured in km/sec and the distance is in AU's.

energy spectrum of these fluctuations leads Coleman (1968) to evoke the use of turbulence concepts.

Magnetic field fluctuations in the interplanetary space are present at all time scales from the ion gyro–period to the solar rotation period, say over scale lengths between few hundred kilometers and some Astronomical Units (1 AU = 1.5×10^8 km). There isn't any Earth laboratory where scientists could access such a wide range of spatial and temporal scales; in this sense the Solar Wind is now considered as the biggest "laboratory" for the investigation of hydromagnetic turbulence. A great number of data have been collected by other probes sent after the Mariner spacecraft. We are now in conditions to study Solar Wind turbulence over a range of heliocentric distances from 0.29 AU perhielion of the Helios probe (in the inner heliosphere) out to the current position of the Voyager spacecrafts (in the outer heliosphere), say at the end of the Solar System. Recently the Ulysses spacecraft has begun to furnish informations about the out–of–ecliptic turbulence flowing from the polar hole of the Sun (Phillips et al., 1994).

As a matter of fact, within 1 AU the interplanetary medium still conserves most of the structure characteristic of the low corona where the wind is born. Beyond 1 AU the increasing bending of the spiral of the magnetic field and the associated onset of shocks largely reprocess the plasma changing its original properties. In Figure 1 we show the wind speed profile vs. time, as recorded by Helios 2 during its primary mission to the Sun in the first four months of 1976 when the spacecraft orbited from 1 AU, on day 17, to 0.29 AU on day 108. The heliocentric distance is also shown. The most impressive feature of this Figure is the alternate presence of high–speed and low–speed regions. The fast wind is produced mostly inside coronal holes which are characterized by open field lines and are mainly located at high latitude, within the least active regions of the photosphere. Slow wind mostly comes from open field line areas located at low latitude, which represent the most active regions of the Sun. Coronal holes topology experiences a strong evolution with solar cycle. Around solar minimum, polar coronal holes extend at very low heliographic latitude showing a quite stable configuration that can last for several solar rotations. At this time an observer confined onto the ecliptic plane would sample regions of space dominated by high–speed streams alternating with regions dominated by low–speed streams.

The characteristics of magnetic and velocity fluctuations at low–frequency, display striking differences when observed in fast and slow speed streams, a comphrensive survey is contained for example in Tu and Marsch (1995). Here we would remind only some distinctive features. In the high–speed streams a high level of correlation between velocity $\delta\vec{v}$ and magnetic field fluctuations $\delta\vec{B}$ occurs. This Alfvénic correlation can be written

$$\delta\vec{v} = \pm\frac{\delta\vec{B}}{(4\pi\rho)^{1/2}}$$

(ρ is the plasma mass density) where the plus or minus sign corresponds to that found in small amplitude Alfvén waves propagating away from the Sun. Moreover mass density and magnetic field intensity remain almost constant, i.e. fluctuations are almost incompressible. In the low–speed streams the Alfvénic correlation is destroyed, while considerable levels of compressible fluctuations are observed (Belcher and Davis, 1971; Veltri, 1994). In all the periods the spectral index μ for magnetic and kinetic energy, as well as for the pseudo–energies obtained through the Elsasser fields $\delta\vec{z}^{\pm} = \delta\vec{v} \pm \delta\vec{B}/(4\pi\rho)^{1/2}$, is strongly variable. An impressive feature can be recovered from the Figure 2a of Marsch (1992) where it is shown that $-2 \leq \mu \leq -1$.

2 Fluid equations for Solar Wind ?

The most reliable description of the Solar Wind plasma can be obtained by using the kinetic collisionless Vlasov equations for ions and electrons (Akhiezer et al., 1975). Indeed, by calculating the collision frequencies $\nu_e = 2.9 \times 10^{-6} n_e \lambda_c T_e^{-3/2}$ and $\nu_i = 4.8 \times 10^{-8} n_i \lambda_c T_i^{-3/2}$ ($n_e \simeq n_i \simeq 1$ and $T_e \simeq T_i \simeq 5$ eV being the mass density and the temperature respectively for the electrons and ions, and $\lambda_c \simeq 20$ is the Coulomb logarithm) it can immediately be seen that the electrons and ions mean free paths $\lambda_e = V_t^{(e)}/\nu_e$ and $\lambda_i = V_t^{(i)}/\nu_i$ (being $V_t^{(i)} \simeq V_t^{(e)} \simeq 40$ km/sec. the thermal speed) are of the order of 1 AU. Nevertheless the simpler two-fluid approximation is often used. This approximation is derived from the kinetic equations, by assuming that the electrons and ions distribution functions are both almost Maxwellian. This hypothesis is satisfied only in a very rough way in the Solar Wind. For this reason the fluid equations obtained are usually considered nothing but balance equations for the low-order moments of the distribution function, while no reliability is given to the values of the transport coefficient obtained from the usual technique (Braginskii, 1965).

When considering large scale phenomena, i.e. for typical frequencies lower than ion cyclotron frequency $\omega_B^{(i)}$ and lengths larger than the ion Larmor radius $r_L^{(i)} = V_t^{(i)}/\omega_B^{(i)}$, which in the Solar Wind turn out to be respectively of the order of $\omega_B^{(i)} \simeq 0.1$ Hz and $r_L^{(i)} \simeq 400$ Km, a one-fluid Magnetohydrodynamic (MHD) approximation can also be derived by neglecting the electron inertia and by assuming both zero ion-Larmor radius and charge neutrality, i.e. equal values for the ions and electrons number density. Moreover in this approximation, a fundamental physical effect which is characteristic of the kinetic description of plasmas is completely neglected: the so called Landau damping (Akhiezer et al., 1975). This damping dissipates wave energy by accelerating and/or heating the particles and its efficiency is directly related to the particular form of the particle distribution function. It has been shown (Barnes, 1966) that this damping is particularly efficient for compressible fluctuations while it does not affect at all incompressible fluctuations. This theoretical argument as well as the fact that in fast streams very low levels of compressible fluctuations (i.e. density, magnetic field intensity fluctuations) are observed in the range of time scales 1 min. $\leq \tau \leq$ 1 day, has allowed for the extensive use of MHD incompressible equations when studying fast streams turbulence.

The application of incompressible hypothesis to turbulence in slow streams, is much more questionable, since in these streams compressible fluctuations are in general present. However, as noted by Marsch and Tu (1993) and Tu and Marsch (1994), these fluctuations are much more relevant at daily scales than at hourly scales. For these reasons by limiting ourselves to fluctuations with time scales between 1 min. and 1 day and by taking in mind all the above mentioned caveats, the use of incompressible MHD equations can be considered at best as a rough approximation to describe Solar Wind turbulence behaviour.

It is moreover clear that there is no chance to recover a whatever value for the transport coefficients of the fluid, like resistivity, viscosity, thermal diffusivity etc., and then for the associated dimensionless numbers (Reynolds number, Lundquist number, Prandtl number). The range of lengths where incompressible fluctuations do not suffer any dissipation is nevertheless very large. It extends from 1 AU to the ion-Larmor radius: i.e. about five or six decades. Since, as a very rough approximation, when using the usual $\nu\nabla^2$ dissipative term the ratio between the dissipation length and the injection length, i.e. the usual inertial range extension, behaves as a power of the Reynolds number $R^{1/\mu}$ (being μ the usual spectral index), this means that we can safely assume an effective Reynolds number of the order of 10^8 or 10^9. On the other hand, since in the Solar Wind $\omega_B^{(i)}\tau_A \simeq 10^6$, where $\tau_A = r/c_A$ is the Alfvén time at the scale r and $c_A = |\vec{c}_A|$ ($\vec{c}_A = \vec{B}_0/(4\pi\rho)^{1/2}$ is the Alfvén velocity associated to the large scale magnetic field $\vec{B}_0$), the first-order corrections to the above mentioned ideal and incompressible MHD approximation of the Solar Wind plasma are more probably represented by dispersive effects at frequencies somewhat smaller than $\omega_B^{(i)}$ rather than by dissipative effects which probably occurs at the ion-cyclotron frequency.

3 Scaling laws for MHD

3.1 Scale invariance of MHD equations

The equations describing ideal, incompressible MHD turbulence are

$$\frac{\partial \vec{v}}{\partial t} + \left(\vec{v}\cdot\vec{\nabla}\right)\vec{v} = -\frac{1}{\rho}\vec{\nabla}P + \left(\vec{b}\cdot\vec{\nabla}\right)\vec{b} \tag{1}$$

$$\frac{\partial \vec{b}}{\partial t} + \left(\vec{v}\cdot\vec{\nabla}\right)\vec{b} = \left(\vec{b}\cdot\vec{\nabla}\right)\vec{v} \tag{2}$$

where P is the total (kinetic plus magnetic) pressure and $\vec{b} = \vec{B}/(4\pi\rho)^{1/2}$. By introducing a length scale r and a characteristic value for both the velocity field and the magnetic field, respectively δv_r and δb_r, it can be seen that the MHD equations are invariant under the following scaling transformations (Carbone, 1993)

$$r \to r'\lambda^{-1} \quad , \quad \delta v_r \to \delta v_{r'}\lambda^{-h} \quad , \quad \delta b_r \to \delta b_{r'}\lambda^{-h}$$

($\lambda > 0$, h is a free parameter and the pressure scales as the square of the velocity). As a consequence, since for each value of h the quantities $\delta v_r/r^h$ and $\delta b_r/r^h$ are also invariant, we expect a scaling law where $\delta v_r \sim \delta b_r \sim r^h$. Obviously this scaling cannot fix a value for h. This means that the MHD equations are invariants for all the allowed values of h. By looking at the usual multifractal theory (Benzi et al., 1984) an entire spectrum of values of h must be introduced. In the following, to compare the MHD results with ordinary fluid turbulent scaling laws, we confine ourself to the study of the velocity field.

3.2 Kraichnan's and Kolmogorov's scaling laws

The Richardson's picture is usually introduced to describe the nonlinear energy cascade in ordinary fluid turbulence as the excitation of smaller length scales r in the inertial range. The Kolmogorov's refined similarity hypothesis (Kolmogorov, 1941, 1962; Kraichnan, 1974) relates the velocity fluctuations δv_r at a given scale r, and the energy transfer rate ϵ_r, say

$$\delta v_r \sim \epsilon_r^{1/3} r^{1/3} \tag{3}$$

The energy transfer rate assumes a value independent on r only if $h = 1/3$ (see for example Frisch, 1995).

This picture is qualitatively applicable also to MHD, even if the Kolmogorov's refined similarity hypothesis is no more valid, at least for strong enough magnetic fields. In fact the main difference between MHD and ordinary fluid flows is the presence of the Alfvén effect, that is the decorrelation of the interacting eddies traveling in opposite direction with respect to the large scale magnetic field $\vec{B}_0$ (Iroshnikov, 1963; Kraichnan, 1965; Dobrowolny et al., 1980) which reduces the strength of the nonlinear interactions. This leads to the fact that the nonlinear energy cascade in MHD is realized in a time $T_r \simeq \tau_r^2/\tau_A$, which is lower by a factor (τ_r/τ_A) with respect to the usual eddy–turnover time $\tau_r \sim r/\delta v_r$. Then by defining the energy per unit time transferred towards the smaller scales $\epsilon_r \sim \delta v_r^2/T_r$, it can be found

$$\delta v_r \sim \epsilon_r^{1/4} c_A^{1/4} r^{1/4} \tag{4}$$

which is the relation analogous to (3). By using this last relation it can immediately seen (Carbone, 1993) that the energy transfer rate scales as $\epsilon_r \rightarrow \epsilon_{r'} \lambda^{1-4h}$, so that, if we require that ϵ_r assumes a value independent of the scale r, we must impose $h = 1/4$.

More informations about the consequence of these scaling relations, can be obtained by looking at the q–th velocity structure functions, that is the spatial averages of the longitudinal velocity differences

$$S_r^{(q)} = \langle \delta v_r^q \rangle \tag{5}$$

where

$$\delta v_r = [\vec{v}(\vec{x}+\vec{r}) - \vec{v}(\vec{x})] \cdot \frac{\vec{r}}{r}$$

Using (4) a linear scaling law

$$S_r^{(q)} \sim r^{\xi_q} = r^{q/4}$$

which is the analogous of the linear $q/3$ Kolmogorov scaling (Kolmogorov, 1941) should be obtained. From experiments in fluid flows Anselmet et al. (1984) have shown that the scaling exponents ξ_q are nonlinear functions of q. The corrections to the linear scaling law in fluid flows are attributed to intermittency effects (see for example the monograph by Frisch (1995), or the issue "Kolmogorov ideas 50 years on", *Proc. R. Soc. Lond. A*, *434*, Eds. J. C. R. Hunt and O. M. Phillips, 1995). In fact from measurements we get the information that the Probability Distribution Function (PDF) of δv_r's is not gaussian, with quasi–exponential wings and high values for the kurtosis.

The scaling exponents ξ_q, obtained from Solar Wind measurements (Burlaga, 1991; Marsch and Liu, 1993), show a departure from both the linear scaling laws $\xi_q = q/m$ ($m = 3$ for the Kolmogorov scaling law, and $m = 4$ for the Iroshnikov–Kraichnan scaling law). On the basis of these first observations Carbone (1993) started to introduce intermittency effects in MHD turbulence in the same way as they had been introduced in fluid flows.

In the first multifractal interpretation of intermittency (Parisi and Frisch, 1983; Benzi et al., 1984), it is introduced an entire spectrum of values of h. Then, being $D(h)$ the h–dependent fractal dimension of the set of points where the scaling $\delta v_r \sim r^h$ is verified, by introducing the probability to observe a local scaling (namely $P \sim r^{3-D(h)}$), it can be written immediately

$$S_r^{(q)} \sim \int d\mu(h) r^{hq} r^{3-D(h)} \tag{6}$$

leading to (Benzi et al., 1984)

$$\xi_q = \min_h [hq + 3 - D(h)] \tag{7}$$

By looking at the Kolmogorov (3) or at the Kraichnan scaling law (4) and introducing the scaling exponents for the energy transfer rate

$$< \epsilon_r^q > \sim r^{\tau_q} \tag{8}$$

it can be found

$$\xi_q = q/m + \tau_{q/m} \tag{9}$$

In this way the intermittency corrections to the linear scaling are determined through a cascade model for the energy transfer rate. This opens a "Pandora's box of possibilities" (Kraichnan, 1974), where cascade models are built up in the framework of the general fragmentation processes (Novikov, 1969). When τ_q is a nonlinear function of q, the energy transfer rate displayes a multifractal behaviour (among other see Schertzer and Lovejoy, 1983; Hosokawa and Yamamoto, 1990; Meneveau and Sreenivasan, 1991; Carbone, 1993; Marsch et al., 1996; Frisch, 1995) which can be characterized by introducing the generalized dimensions

$$D_q = 1 + \frac{\tau_q}{(q-1)}$$

(Hentschel and Procaccia, 1983). The scaling exponents of the velocity structure functions is related to D_q by

$$\xi_q = \left(\frac{q}{m} - 1\right) D_{q/m} + 1 \tag{10}$$

The anomalous correction to the usual scaling laws is then given by

$$\xi_q - (q/m) = (1 - D_{q/m})(1 - q/m)$$

It can be immediately seen that when $q/m < 1$ the correction is positive, while when $q/m > 1$ it is negative. Moreover a general fractal behaviour ($D_{q/m} = D =$ const. < 1) gives a linear correction to the scaling exponents. The observed convex nonlinear correction to ξ_q (Anselmet et al., 1984) is obtained through the multifractal approach, say when $D_{q/m}$ is a decreasing function of q. The multifractal theory of the energy transfer rate is obviously related in a simple way (Aurell et al., 1992) to the oldest multifractals introduced by Parisi and Frisch (1983) (see also Benzi et al., 1984).

An important point which requires to be stressed is the fact that, by looking at the general infinitely divisible distributions in turbulence (Novikov, 1990, 1994; Hentschel and Procaccia, 1983), under some usual hypotheses (Novikov, 1969) the scaling exponents $\tau_{q'}$ are positive for $q' < 1$ and negative for $q' > 1$, so that $\tau_1 = 0$. Say intermittency corrections are zero for $q' = 1$. In the ordinary fluid flows the 4/5–Kolmogorov's law for the third–order structure functions (Frisch, 1995)

$$S_r^{(3)} = -\frac{4}{5}\,\epsilon r \tag{11}$$

derived from Navier–Stokes equations and valid in the inertial range (Kolmogorov, 1941), assures that the third–order structure function is unaffected by intermittency. Equation (11) is often used as a formal definition of the inertial range. The MHD analogous conjecture of (11) (Carbone, 1994a), say of an inertial range defined by

$$S_r^{(4)} = C\epsilon r \tag{12}$$

(C is a constant) is obtained from the Kraichnan similarity hypothesis (4) by *assuming* that $\tau_1 = 0$ in (8). Unluckly up to now no exact derivation of (12) from MHD equations has been given, and due to the very large uncertainties on the measurements of the scaling exponents (e.g. Burlaga, 1991), we cannot use experimental data neither to confirm that $\tau_1 = 0$, nor at least to be sure that $\tau_q = 0$ for some value of $q \neq 1$. On the other hand independent measurements of the dissipation field $\epsilon(\vec{x})$ in the Solar Wind turbulence do not make sense. In ordinary fluid flows, due to the Taylor's hypothesis and to the existence of a well defined relation between $\epsilon(\vec{x})$ and the velocity field, the dissipation field can be investigated by using its one–dimensional surrogate $\epsilon(t) \sim [dv(t)/dt]^2$ (Meneveau and Sreenivasan, 1991; Aurell et al., 1992). Of course this cannot be true in the Solar Wind turbulence.

3.3 MHD intermittency models

Notwithstanding the lack of a firm theoretical and/or experimental basis for equation (12), cascade models for the energy transfer rate have been obtained in the framework of general fragmentation processes also for MHD turbulent flows. These intermittency models are the analogous of the models built up in the fluid turbulence framework, the only difference being the fact that the Kraichnan's relation (4) is used instead of the Kolmogorov's refined similarity hypothesis (3).

The random–β model has been introduced by Benzi et al. (1984). These authors conjectured that the space–filling factor for the offsprings in the Richardson's energy cascade is given by a random variable β. The probability of occurrence of a given β is assumed to be a bimodal distribution where the eddies fragmentation process generates either space–filling eddies with probability ζ or planar sheets with probability $(1-\zeta)$ $(0 \leq \zeta \leq 1)$. The MHD version of the model (Carbone, 1994a) is introduced in the same way, and both versions give rise to the unique formula

$$\xi_q = \frac{q}{m} - \log_2\left[1 - \zeta + \zeta(1/2)^{1-q/m}\right] \tag{13}$$

The parameter ζ must be fixed through a fit on the experimental data.

The p-model (Meneveau and Sreenivasan, 1987) consists in an eddy fragmentation described by a two–scale Cantor set with equal partition intervals. That is an eddy at the scale r, with a measure ϵ_r, breaks down into two eddies at the scale $r/2$ with measures $p\epsilon_r$ and $(1-p)\epsilon_r$. The parameter $0.5 \leq p < 1$ is not defined by the model, but it is generally fixed by a fit on the experimental data. The MHD version of the p-model has been introduced by Carbone (1993) (see also Carbone, 1994a), the generalized dimensions are given by

$$D_q = \frac{\log_2\left[p^{q/m} + (1-p)^{q/m}\right]}{(1-q)} \tag{14}$$

and the scaling exponents of the structure functions

$$\xi_q = 1 - \log_2\left[p^{q/m} + (1-p)^{q/m}\right] \tag{15}$$

In the She and Leveque (SL) model (She and Leveque, 1994), one assumes an infinite hierarchy for the moments of the energy transfer rate, leading to $\epsilon_r^{(q+1)} \sim [\epsilon_r^{(q)}]^\beta[\epsilon_r^{(\infty)}]^{1-\beta}$, and a divergent scaling law $\epsilon_r^{(\infty)} \sim r^{-x}$ for the most singular dissipative structures in the limit $q \to \infty$. The MHD version of the SL–model has been introduced independently by Grauer et al. (1994) and by Politano and Pouquet (1995), who showed that

$$\xi_q = \frac{q}{m}(1-x) + C\left[1 - \left(1 - \frac{x}{C}\right)^{q/m}\right] \tag{16}$$

In equation (16) the parameter $C = x/(1-\beta)$ is identified as the codimension of the most intermittent structures. In the "standard" MHD case (Grauer et al., 1994; Politano and Pouquet, 1995) $x = \beta = 1/2$, so that $C = 1$, that is the most singular dissipative structures are planar sheets. On the contrary in fluid flows $C = 2$ and the most dissipative structures are filaments. The large q behaviour of the p–model is given by $\xi_q \sim (q/m)\log_2(1/p) + 1$, so that (15) and (16) give the same results provided that $p \simeq 2^{-x}$.

The models we have briefly outlined are based on the hypothesis that the turbulence is fully developed, and the only corrections to the usual scaling laws are due to intermittency through the scaling of ϵ_r. Recently Tu et al. (1996) introduced an extended structure–function model to describe the observations of turbulence in the Solar Wind. As we have already seen the slope μ of the spectral energy is variable, and in some cases (quite in all the cases, see the Figure 2a in Marsch, 1992) is different from both 5/3 and 3/2. Tu et al. (1996) attribute this to the fact that turbulence is not in a fully developed state (mainly in the high speed streams). Then they introduced a model which is based on the assumptions that i) the refined similarity hypothesis (either eq. (3) or (4)) are still valid even in absence of fully developed turbulence, and ii) the energy cascade rate is not constant, but its moments depend both on D_q and μ, say

$$< \epsilon_r^q > \sim \epsilon^q(r,\mu) r^{(q-1)D_q}$$

where the average $\epsilon(r,\mu)$ is assumed to be

$$\epsilon(r,\mu) \sim r^{-(m/2+1)} P_r^{\mu/2}$$

being $P_r \sim r^{\mu}$ the usual energy spectrum (r is the inverse of a wave vector $r \sim k^{-1}$). The model gives rise immediately to the expression

$$\xi_q = \left(\frac{q}{m} - 1\right) D_{q/m} + 1 + \left[\mu\frac{m}{2} - \left(\frac{m}{2}+1\right)\right]\frac{q}{m} \qquad (17)$$

and in the limit of "fully developed turbulence", say when the spectral slopes are given either by the Kolmogorov or the Kraichnan relations $\mu = 2/m + 1$, the usual expression (10) for ξ_q is recovered, where correction is due to multifractals. Apart from the physical arguments used to justify (17), the formula is nothing but a two–parameter model to analyze the intermittency of Solar Wind fluctuations.

To conclude this section we want to stress an important point, say all the above mentioned models suffer from one main limitation: they are set up only for the velocity field, by assuming that, as we have shown in the Section III from ideal MHD equations, the magnetic fluctuations δb_r obeys to the same scaling laws as δv_r.

4 What can be learned from Solar Wind observations ?

We have tried to use the Solar Wind low–frequency fluctuations as a testing bench for scaling laws in MHD turbulence. We examined the q–th order velocity structure function $S_\tau^{(q)} = < |\vec{v}(t+\tau) - \vec{v}(t)|^q >$, where the brackets indicate averages over the different lag–times τ. The knowledge of the structure functions requires the measurements at two different times, and the interpretation in terms of a characteristic scale length is possible by using the usual Taylor's hypothesis, that is the structure function measured at a given scale τ characterizes the velocity of an eddy, frozen in the supersonic Solar Wind with velocity V_0, at the scale length $r = \tau V_0$ (in the following we will use either the symbols r and τ).

Pioneering measurements of the scaling exponents in the Solar Wind plasma have been that of Burlaga (1991) (see also Burlaga, 1993), followed by some other investigations (Marsch and Liu, 1993; Carbone, 1994a; Carbone et al., 1995a,b, 1996; Ruzmaikin et al., 1995). Due to the difficulties encountered in the determination of statistically homogeneous samples (which are even limited to data sets with a small number of points), the high–order structure functions must be handled carefully. Notwithstanding this handicap, the curves ξ_q are reported in the papers just quoted, and these analyses, along with the direct investigation of the non Gaussian PDF's for the velocity differences (Burlaga, 1993; Marsch and Tu, 1994), show convincing evidence for the presence of intermittency in MHD turbulence. Burlaga (1991) showed for the first time the presence of intermittency by using the Voyager data in the outer heliosphere at 8.5 AU. This author found that the velocity structure functions show scaling behaviour in the range of periods from 0.85 hour to 13.6 hours. Then by using measurements at 1 AU, Burlaga (1993) found scaling laws in the range from 8 hours up to 2.7 days. Burlaga (1991) fitted the scaling exponents ξ_q with the fluid random–β model (Benzi et al., 1984) and the fluid p–model (Meneveau and Sreenivasan, 1987). Marsch and Liu (1993) investigated the intermittency in the inner Solar Wind by using the Helios data. They found a remarkable difference between the high–speed and the low–speed streams. In fact the high–speed streams behave like random field with no well defined scaling law, while the scaling exponents in the low–speed streams have been found to be similar to that obtained in ordinary fluid flows (see also Tu et al., 1996; Carbone and Bruno, 1996). Finally Ruzmaikin et al. (1995) investigated the intermittency of fluctuations by using the recent Ulysses data out of ecliptic plane. The scaling exponents obtained in the Solar Wind turbulence are extremely variable. This is also true for the Helios measurements, which in fact show different scaling exponents depending on the streams which have been investigated. As we have already said, the extreme variability of the scaling exponents has been

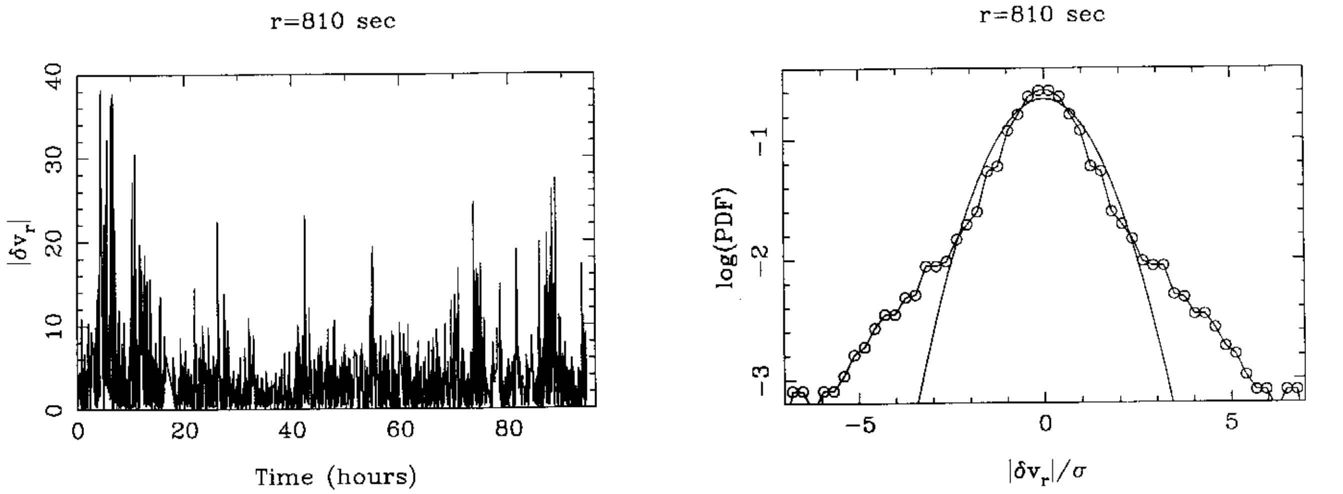

Fig. 2. We report the time evolution of the velocity differences $|\delta v_r|$ **(left–hand panel) and the PDF of the corresponding variables** $|\delta v_r|/ < \delta v_r^2 >$ (right–hand panel), for the data set D (see **Table 1) at scale** $\tau = 810$ **sec.**

attributed by Tu et al. (1996) to the absence of a fully developed turbulent state in the Solar Wind turbulence.

In order to verify the universality of scaling laws in the interplanetary MHD turbulence, we analyzed the plasma measurements of the velocity field $\vec{v}(t) = \vec{V}(t)/V_0$ as recorded by the German plasma instrument (P. I. H. Rosenbauer) on board Helios 2 during its primary mission in the inner heliosphere. The original data were collected in 81 sec. bins and the following five subintervals of 2 days each were chosen: A) day 100:00 to 102:00 at 0.33 AU, B) day 81:00 to 83:00 at 0.58 AU, C) day 54:00 to 56:00 at 0.85 AU, D) day 46:00 to 48:00 at 0.89 AU and, finally, E) day 36:00 to 38:00 at 0.94 AU. All the intervals were selected within low–speed regions were, as we have see in the Section I, the Solar Wind turbulence appears in a state which is not Alfvénic (Grappin et al., 1991). We investigate the scaling behaviour in the range between 81 sec and 1 day, that is, by using a typical value $V_0 \simeq 350$ km/s, the scales length under investigation range between 10^4 km $\leq r \leq 10^7$ km. In Figure 2 we report the time evolution of the differences $|\delta v_\tau|$, along with the PDF, at the scale $\tau = 810$ sec, for the data set D. As it can be seen the intermittency appears as localized spikes of high activity leading to the non Gaussian PDF with wings which evidence the fact that strong events have a probability of occurrence grater than a Gaussian event.

The scaling exponents of the low–frequency fluctuations in the Solar Wind are extremely variable, this is due to a difficulty in identifying the inertial range. As an example of this difficulty in Figure 3 we show $\log S_\tau^{(3)}$ and $\log S_\tau^{(4)}$ vs. $\log \tau$ for all our data sets. As it can be seen an inertial range is not defined, even if the Reynolds number is very high. Both the scaling relations (11) and (12) are reported in the Figure as a thin line.

4.1 The Extended Self–Similarity hypothesis

To override the difficulties in determining the scaling exponents, we have used the so called Extended Self–Similarity (ESS), which in general allows for a better determination of the scaling exponents (Benzi et al., 1993a). When the refined similarity hypothesis (3) holds, that is in the inertial range, due to the relation (11) the structure functions are not independent, rather the q–th structure function is related to the p–th one through

$$S_r^{(q)} = C_{p,q} \left[S_r^{(p)} \right]^{\alpha_p(q)} \tag{18}$$

with $\alpha_p(q) = \xi_q/\xi_p$. In fluid flows both $S_r^{(q)} \sim r^{\xi_q}$ and the usual models described in the last section furnish $\xi_3 = 1$, so that in the inertial domain the relation $\alpha_3(q) = \xi_q$ holds. On the other hand, from the point of view of the Navier–Stokes equations, the 4/5–law (11) assures this result. Benzi et al. (1993a) verified that in fluid flows, almost unexpectedly, a linear relation between $\log S_r^{(q)}$ and $\log S_r^{(3)}$ extends well outside the inertial range within the dissipative range, showing that equation (18) has a more general validity than the usual scaling equation valid in the inertial range (see Dubrulle, 1994). This feature allows for a very good experimental determination of $\alpha_3^F(q)$ (here F is for fluid). The presence of ESS has been shown and the scaling laws have been derived in a lot of different situations, say ordinary fluid flows Benzi et al. (1993b, 1994, 1995, 1996a,b), simple models which mimic either fluid or MHD turbulence (Carbone, 1994b; Frick et al., 1995), numerical simulations (Briscolini et al., 1994; Grauer and Marliani, 1995) and MHD turbulence (Carbone et al., 1995a,b, 1996).

Taking in mind the relations (12) and (18), in Figure (4) we have reported the plots of $\log S_\tau^{(q)}$ vs. $\log S_\tau^{(4)}$ for seven different values of q. As it can be seen from

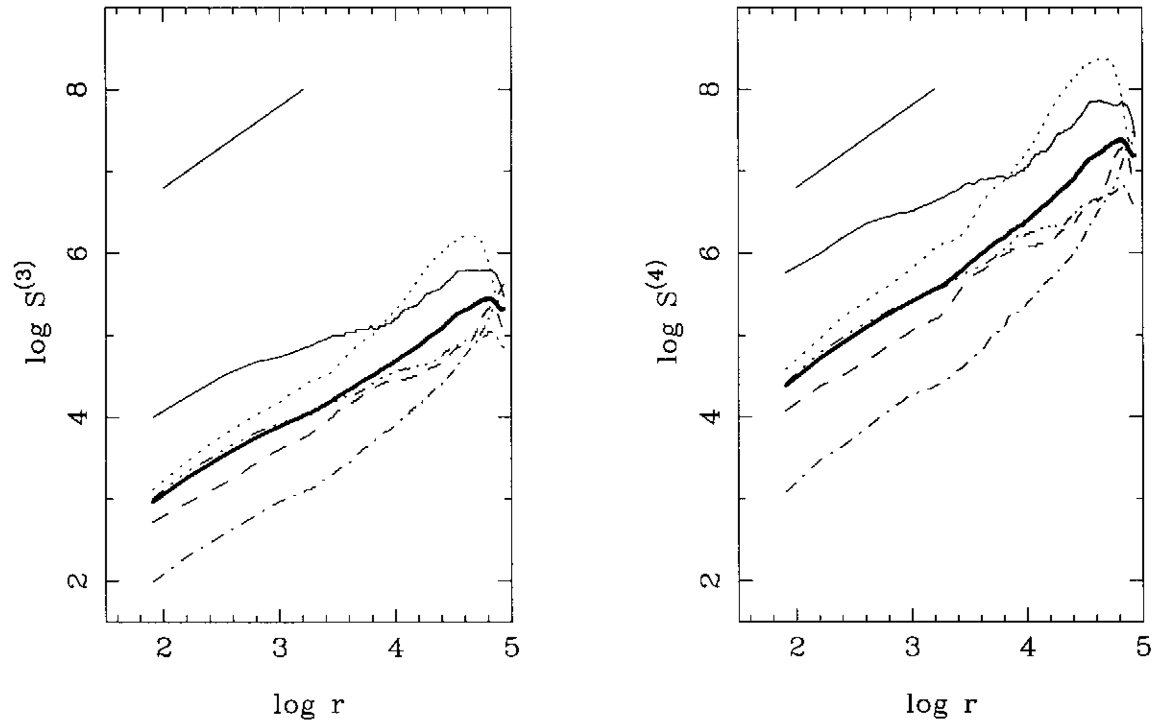

Fig. 3. The behaviour of both $\log S_r^{(3)}$ (left–hand panel) and $\log S_r^{(4)}$ (right–hand panel) vs. $\log r$ for the different data sets we used (represented as different curves). The thin line refers to the scaling laws $S_r^{(3)} \simeq r$ in the left–hand panel and $S_r^{(4)} \simeq r$ in the right–hand panel.

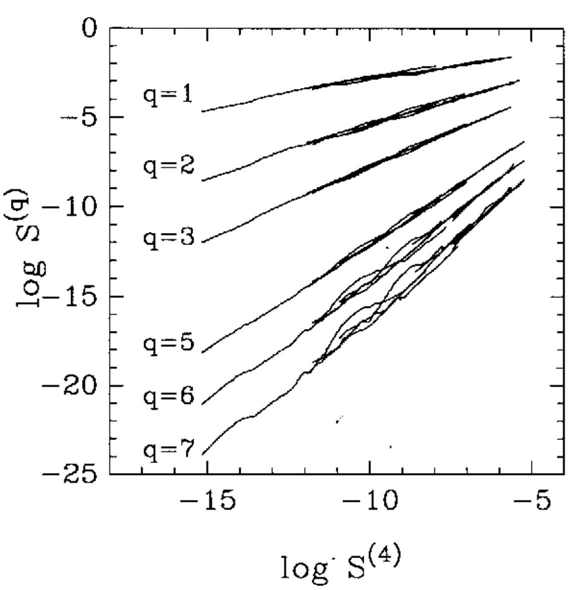

Fig. 4. We show the behaviour of $\log S_r^{(q)}$ vs. $\log S_r^{(4)}$ for some values of q ranging from $q = 1$ up to $q = 7$. The different data sets are represented by different curves (from Carbone, 1996).

this Figure the ESS exists for all the values of q we have examined, that is the range of self–similarity extends almost over all the length range covered by the measurements. From the same data sets the values of $\alpha_4(q)$ have also been calculated and are reported in Table 1. In the same Table we report the values of the scaling exponents obtained by using the velocity differences merging from all the data sets. As a first result let us note that the turbulence, observed in different no–corotating streams at different heliocentric distances, presents a universal behaviour, in the sense that the scaling exponents $\alpha_4(q)$ obtained in the different data sets are quite the same and no remarkable difference has been found between the column All Data and the other columns. Finally let us stress that, using a suitable normalization for the velocity differences, say the average velocity V_0 within each sample set, the relation (18) turns out to be exact, with $C_{4,q} = 1$.

Before discussing how the experimental values obtained for the scaling exponents of the velocity structure functions compare with those derived from intermittency models, let us discuss the direct consequences of the observed ESS. First of all we would to stress that the relation (18) is not a trivial property of fluid flows. This can be seen by looking at the expression for the structure function

$$S_r^{(q)} = \int dP(\delta v_r)\delta v_r^q$$

Table 1. The values of the scaling exponents $\alpha_4(q)$, for $1 \le q \le 7$, calculated through ESS. The first column refers to the order q of the structure functions. The next five columns refer to the different five data intervals used in the analysis and list the values of the scaling exponents obtained for the different q's. The last column reports the scaling exponents computed from the merged data sets.

q	A	B	C	D	E	All data
1	0.299	0.272	0.302	0.284	0.303	0.292
2	0.554	0.535	0.564	0.546	0.573	0.553
3	0.786	0.778	0.793	0.782	0.799	0.788
4	1.000	1.000	1.000	1.000	1.000	1.000
5	1.200	1.205	1.191	1.199	1.194	1.198
6	1.384	1.399	1.370	1.388	1.362	1.382
7	1.556	1.585	1.543	1.566	1.551	1.561

A simple dimensional analysis of (18) gives the relation $\alpha_p(q) = q/p$. Since from the data analysis the scaling exponents $\alpha_p(q)$ show intermittency correction, we are forced to consider a nontrivial behaviour for the probability measure (Benzi et al., 1995; Stolovitzky and Sreenivasan, 1993). The triviality should come from the great variety of situations where ESS is recovered, i.e. since ESS is found almost everywhere, it is a trivial property of turbulent fluid flows. With respect to this argument, the non triviality is assured by the simple observations that a strong velocity shear destroys ESS, that is in presence of velocity shear the extended scaling law breaks down at a given length scale of the order of the shear length (Stolovitzky and Sreenivasan, 1993; Benzi et al., 1995).

Notwithstanding the fact that no clear physical motivation for the occurrence of ESS has yet been given (Benzi et al., 1995), two direct consequences of ESS can be pointed out:

1) From relation (18) the velocity structure function $S_r^{(q)}$ can be written on the basis of a function $f(r/\eta)$ ($\eta = \nu^3/\epsilon$ is the usual Kolmogorov dissipation scale and ϵ is the mean energy dissipation rate)

$$S_r^{(q)} = C_q U_0^q \left[\frac{r}{L} f(r/\eta)\right]^{\zeta(q)} \qquad (19)$$

where C_q are dimensionless constants, $U_0^3 = S_L^{(3)}$ and L is the integral scale. This last relation has been carefully checked on laboratory data (Benzi et al., 1995, 1996b).

2) From relation (18) a well defined PDF for the velocity differences can be derived (Carbone et al., 1995b, 1996)

$$P(\delta v_r) = \int_{-\infty}^{\infty} dk\, e^{-ik\delta v_r} \sum_{q=0}^{\infty} \frac{(ik)^{2q}}{2\pi(2q)!} \left[S_r^{(p)}\right]^{\alpha_p(2q)} \qquad (20)$$

(we have neglected contributions from odd moments). This implies that, for each scale length r, the knowledge of the scaling exponents $\alpha_p(q)$ determines the probability distribution as function of a single parameter $S_r^{(p)}$.

Table 2. The best-fit values of the parameters ζ, p and C respectively for the random–β–model, the p–model and the SL–model, obtained through a fir on the data reported in Table 1.

Data sets	ζ	p	C
A	0.16 ± 0.05	0.734 ± 0.006	0.89 ± 0.04
B	0.13 ± 0.05	0.714 ± 0.007	1.04 ± 0.06
C	0.16 ± 0.06	0.743 ± 0.006	0.84 ± 0.04
D	0.18 ± 0.06	0.727 ± 0.006	0.94 ± 0.05
E	0.17 ± 0.07	0.739 ± 0.006	0.86 ± 0.04
All data	0.16 ± 0.05	0.731 ± 0.006	0.91 ± 0.04

This is true if the velocity structure functions are carefully normalized, so that $C_{p,q} = 1$. As we have noted (Carbone et al., 1996) this is verified in the Solar Wind turbulence if the velocity is normalized to the average velocity within each stream.

4.2 Comparison with intermittency models

The scaling exponents $\alpha_4(q)$ reported in Table 1 have been fitted, through a minimum–χ^2 method, to the model for intermittency which we have introduced in the Section 3. We used the MHD version of the model, with $m = 4$. The fit on the SL–model has been made by using $x = 1/2$ leaving C as free parameter. The best–fit parameters ζ, p, and C are reported in Table 2.

Both models (15) and (16) are in good agreement with the values of $\alpha_4(q)$ which we have measured, since $\chi^2 \simeq 0.02$ for all the data sets. On the contrary the χ^2 obtained when we use the random–β model is quite higher $\chi^2 \simeq 0.4$, even if it is yet acceptable. In Figure 5 we report the values of the scaling exponents along with the curves obtained from the intermittency models (13), (15), and (16).

We want to remark that the parameter p close to 0.7, obtained firstly by Carbone (1993) in analyzing the intermittency in the Solar Wind turbulence, is close to what has been found to describe the energy cascade in the fluid turbulence Meneveau and Sreenivasan (1987, 1991). Marsch et al. (1996) have found higher values, of the order of $p \simeq 0.87$ by analyzing the intermittency in the variables $\epsilon_r = |\delta v_r|^3/r$. Indeed this variable, when averaged, represents nothing but the third–order structure function. The results obtained by Marsch et al. (1996) are somewhat affected by the fact that analyzing structure functions of the type $S_r^{(3q)}$, they were obliged to go up to $q = 5$ to obtain the convergence of D_q, which means the 15–th order structure function where, if not calculated through ESS, uncertainties are very large.

The result $C \simeq 1$ is particularly significant from an experimental point of view, as just discussed by Grauer et al. (1994) and by Politano and Pouquet (1995). In fluid turbulence the value of C turns out to be $C \simeq 2$, showing that the most singular dissipative structures are filaments (vortices, as conjectured by She and Leveque,

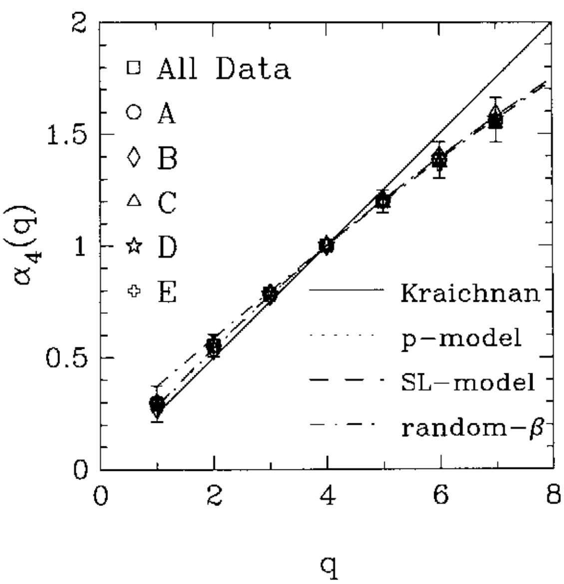

Fig. 5. The values of $\alpha_4(q)$ for all the data sets we have examined. The various symbols refer to: set A (diamonds), set B (circles), set C (triangles), set D (upturned triangles), set E (crosees), and the values obtained by using all the merged data sets (squares). The uncertainties refer to the values of the scaling exponents obtained by using all the merged data sets. Superimposed we show the MHD random–β–model (dotted line) with $x = 0.12$, the MHD p-model (full line) with $p = 0.731$, and the MHD SL model (dashed line) with $C = 0.91$. Shown also is the linear Kraichnan relation $\alpha_4(q) = q/4$.

1994). On the contrary in MHD turbulent flows, the most intermittent structures should be two–dimensional, i.e. they should be planar (current) sheets ($C \simeq 1$).

Actually the model (16) strongly depends on the value chosen for x. Using a different choice for x, for example $x \simeq -\log_2 p$, with the values of p reported in Table 2, gives rise to a different value for the parameter C which is no more close to 1. Also in this case the reduced χ^2 of the fit is of the order of $\chi^2 \simeq 0.02$. Finally note that imposing the value for x through the equality $x = \beta$ determines the value $C = x/(1-x)$. A fit on the data in this case gives the best-fit value $x \simeq 0.48$ (thus $C \simeq 0.92$) for all the data set, but with a value of the reduced χ^2 which is somewhat higher, that is $\chi^2 \simeq 3$.

4.3 A generalized extended self–similarity

Up to now the physical motivations of the fact that ESS seems to work remarkably well in a great variety of situations has not been captured. ESS suggests (Dubrulle, 1994) the existence of a generalized scale length

$$\zeta_r^{(m)} \sim \frac{S_r^{(m)}}{< \epsilon_r >}$$

so that $\epsilon_r \sim \delta v_r^m < \epsilon_r > / S_r^{(m)}$ and a generalized refined similarity hypothesis can be recovered

$$S_r^{(q)} \sim [S_r^{(m)}]^{q/m} \frac{\left\langle \epsilon_r^{q/m} \right\rangle}{[\langle \epsilon_r \rangle]^{q/m}} \tag{21}$$

From this equation a new form of ESS, which in its essence has been introduced some time ago (see Ref.(1) of Bershadskii, 1996), can be derived. In fact by looking at equation (21) we can introduce a set of dimensionless moments

$$G_m^{(q)}(r) = \frac{S_r^{(q)}}{[S_r^{(m)}]^{q/m}}$$

which contains entirely the intermittent correction due to the anomalous scaling laws of the energy transfer rate. This suggests to look for a generalized form of ESS through the scaling relations

$$G_m^{(q)}(r) \sim [G_m^{(p)}(r)]^{\rho_m(q,p)} \tag{22}$$

where the scaling exponents $\rho_m(q,p)$ account for the energy transfer rate. Benzi et al. (1996a) (see also Benzi et al., 1996b) found that the generalized ESS (22) is verified also in the cases where ESS (18) fails. On the other hand the generalized ESS is implied by a conjecture which should be a key to understand the occurrence of ESS. In fact Benzi et al. (1996a) (see also Benzi et al., 1996b) conjectured that a sharp dissipative cutoff does not exist in fluid flows, but the energy transfer continues to hold also at very small scales. In this case the PDF's of δv_r depends on the ratio (r/η), where $\eta \simeq (\nu^m / < \epsilon_r >)^{1/(1+m)}$ the usual length where the local Reynolds number $R_\eta \simeq 1$. Following this conjecture Benzi et al. (1996a) argued that, if h_0 represents the scaling of the most singular structures and d_0 is the corresponding codimension, the viscosity should reduce the amplitude of these singularities, as r goes down. Then the probability to observe these structures, proportional to r^{d_0}, should decreases, which implies that d_0 is an increasing function of r. ESS states that there exists a balance between h_0 and d_0 such that d_0/h_0 is constant, and in this picture ESS is broken when the balance does not exist. On the contrary, since the scaling exponents $\rho_m(q,p)$ do not depend on h_0, the generalized ESS should continues to hold. Benzi et al. (1996a) (also Benzi et al., 1996b) found that (22) is verified also when ESS fails. Obviously relation (22) is verified in our case, and the slopes are given by

$$\rho_m(q,p) = \frac{\alpha_m(q) - q/m}{\alpha_m(p) - p/m}$$

obtained from Eq.s (22) and (18).

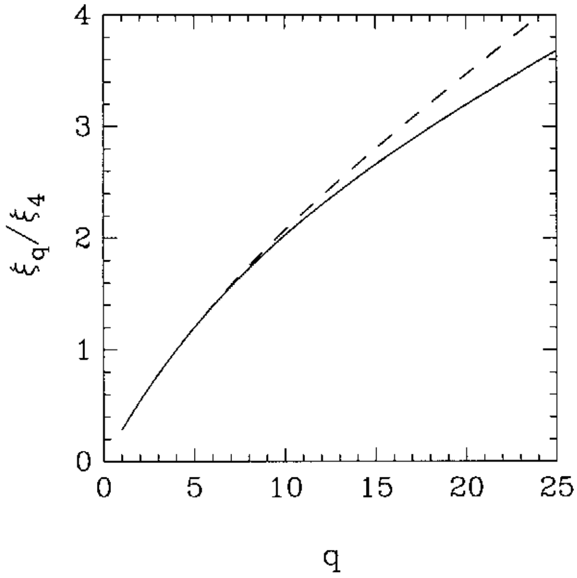

Fig. 6. We show the normalized scaling exponents ξ_q/ξ_4 vs. q obtained from both the SL fluid model (full line) and the MHD version of the same model (dashed line).

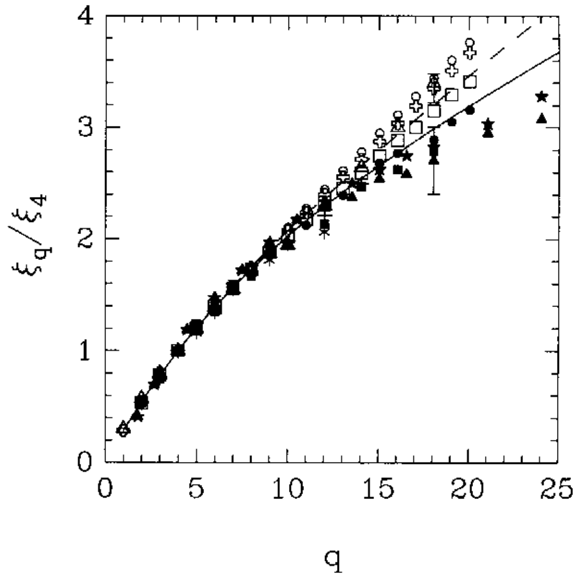

Fig. 7. In the plot are represented the scaling exponents $\alpha_4^F(q)$ and $\alpha_4^{MHD}(q)$ collected from different measurements both in ordinary fluid flows and in the Solar Wind turbulence. White symbols refer to the Solar Wind measurements, while black symbols refer to laboratory measurements. White squares, diamonds and circles refer to the analysis in the inner Solar Wind on the Helios data respectively during the periods B, D, and E. White triangles refer to the data by Burlaga (1991) obtained in the Solar Wind turbulence from the Voyager satellite measurements at 8.5 AU. Black squares, diamonds and circles refer to the measurements by Anselmet et al. (1984) on a turbulent jet and on a turbulent duct flow. Black triangles refer to the measurements by L. Zubair (private communication) in a wind tunnel. Black crosses and reversed triangles refer to the measurements by Meneveau and Sreenivasan (1991) by hot-wire measurements in wind tunnel respectively in the boundary layer and in the wake of the cylinder. Finally black stars refer to the measurements by Benzi et al. (1993) in wind tunnel. Superimposed we reported the SL fluid model (full line) and the MHD model (dashed line) (from Carbone et al., 1995b).

4.4 Are the scaling laws derived from ESS really different for fluid and MHD turbulence ?

Even if usually one determines the scaling exponents $\alpha_3^F(q)$ in the fluid case and $\alpha_4^{MHD}(q)$ in the MHD case, equation (18) shows that it should be possible to compare the scaling laws obtained in the two cases by calculating for example $\alpha_4^F(q)$ also from data obtained in fluid flows, simply through

$$\alpha_4^F(q) = \alpha_3^F(q)/\alpha_3^F(4) \tag{23}$$

So an interesting question can be posed: are the scaling exponents derived from experimental data through equation (18) really different for fluid and MHD turbulent flows, or rather the relation (18) represents a universal law valid in both cases with the same set of values for $\alpha_p(q)$?

The knowledge of the scaling exponents is equivalent to the knowledge of the PDF for the velocity differences. If $\alpha_p(q)$ for the fluid and MHD turbulent flows were indistinguishable, through equation (20) this could be an indication for the existence of a universal non Gaussian PDF valid for both fluid and MHD velocity differences δv_r. Theoretical models provide different scalings. But when we plot $\alpha_4(q) = \xi_q/\xi_4$, obtained from the SL-model (16) respectively for $m = 3$ (fluid flows) and $m = 4$ (MHD flows), we can see that the curves are practically superposed as far as $q \leq 8$ and only for $q \geq 10$ they separate out (see Figure 6). From a physical point of view, according to the She and Leveque model (She and Leveque, 1994; Grauer et al., 1994; Politano and Pouquet, 1995), a difference between ordinary fluid flows and MHD flows is due to the different topological properties of the most singular structures. This difference is contained in the different values for the parameter C in the models (16), and cannot be found in other intermittency models. Obviously the difference becomes visible only when we look at the most singular structures, that is when we examine the high-order scaling exponents, because higher values of q enhance the more singular structures.

In order to give an answer to the above mentioned question, we have collected scaling exponents from both laboratory measurements (fluid flows) and space data (MHD flows). The scaling exponents ξ_q^F have been obtained in laboratory flows, say turbulent jets, duct flows and wind tunnel. From these exponents the values of $\alpha_4^F(q)$ can be derived, and are reported in Figure 7. The scaling exponents we used can be found in literature (see the caption of Figure 7). As concern the MHD flows we used the scaling exponents ξ_q^{MHD} obtained by Burlaga (1991). Even in this case the values of $\alpha_4^{MHD}(q)$ are plotted in Figure 7. Moreover we used the scaling exponents $\alpha_4^{MHD}(q)$ obtained from Helios spacecraft.

Looking at Figure 7 it can be seen for smaller values of q, the scaling exponents $\alpha_4^F(q)$ and $\alpha_4^{MHD}(q)$ follow the same curve, while as q increases it can be noted that the scaling exponents seem to belong to two distinct populations. To show that this behaviour is statistically meaningful, we divide our data in two different samples. The first sample is built up with the scaling exponents $\alpha_4^{MHD}(q)$ coming from the Solar Wind measurements, and the second sample is built up with the data $\alpha_4^F(q)$ coming from laboratory measurements on ordinary fluid flows. For each value of q we calculate the average values for both the samples, say $\mu^{MHD}(q)$ and $\mu^F(q)$. Through a t–test we make inferences about the means of the two populations, for each value of q, by testing the hypothesis H_0 that the two populations have the same mean

$$H_0(q) := \{\mu^{MHD}(q) = \mu^F(q)\} \tag{24}$$

By using a Satterthwaite's procedure, we have calculated the probabilities $P[H_0(q)]$ that the hypothesis (24) is true. This probability is shown in Figure 8. As it can be seen $P[H_0(q)]$ is about 0.7 for $q \leq 7$. This is in agreement with what we expect, because the scaling exponents are almost the same for low values of q and the difference between the most intermittent structures in fluid and MHD flows are not achieved for low values of q. For $8 \leq q \leq 10$ the probability is about $P[H_0(q)] = 0.1$, while for $q \geq 12$ the probability of accepting H_0 falls down to $P[H_0(q)] = 10^{-3}$. This indicates that for high values of q the two populations are well separated, that is the complementary hypothesis $H_1(q) := \{\mu^{MHD}(q) \neq \mu^F(q)\}$ can be accepted with the very high probability $1 - P[H_0(q)]$.

We want to stress that, as it is well known, from an experimental point of view higher order velocity structure functions are in general calculated with a progressively lower reliability. This is due to the fact that the averages in (5) imply a statistics on events more and more rare with increasing order q and thus require very long data sets which are not yet at our disposal. On the other hand, due to the occurrence of very strong events ("wild" singularity in Schertzer and Lovejoy, 1992), moments $< \delta v_r^q >$ with q greater than the exponent corresponding to the algebraic fall–off of the PDF's of δv_r diverge (Schertzer and Lovejoy, 1987, 1992). This should also affects our results, by making unreliable the high–order velocity structure functions. For these reasons the result which we found must be considered only as an indication that differences exist between scaling exponents in fluid flows and hydromagnetic flows.

5 Conclusions

In this paper we have reviewed some of the work done by using the satellite observations of the velocity and magnetic field in the interplanetary medium, to recover the scaling laws of the associated MHD turbulence. Spacecraft observations are particularly useful in this regard because the Solar Wind represents the almost unique laboratory were hydromagnetic turbulence can be investigated. Owing to the lot of observations from different spacecrafts at heliocentric distances from 0.29 AU up to the end of the Solar System, turbulence in the Solar Wind can be investigated in detail for a wide range of scale lengths. The most interesting features obtained are summarized in the following.

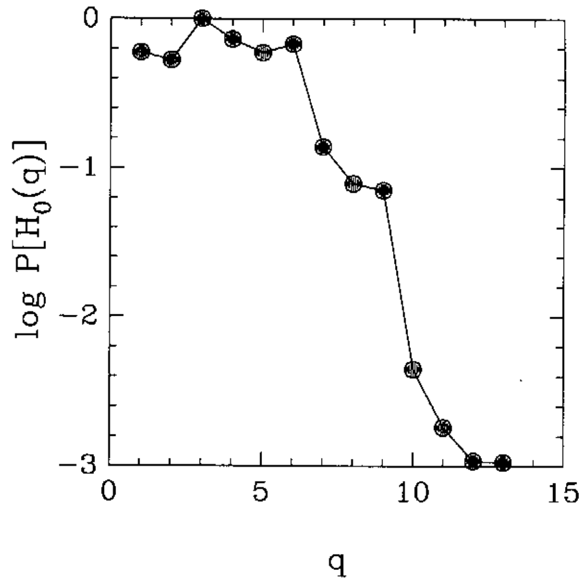

Fig. 8. We show the probability $P[H_0(q)]$ of the hypothesis $H_0(q)$ that the two populations in figure 7, made with the laboratory measurements and the Solar Wind measurements, belong to a single sample with the same mean (from Carbone et al., 1995b).

1) The good agreement found between scaling exponents derived from general fragmentation models and those measured in Solar Wind, indicates that intermittency should be rather independent of the dissipation mechanism. In fact, also if we think that the dynamics of nonlinear energy transport process is sufficiently well described by ideal MHD equations, we are absolutely sure that the physical dissipation mechanism in the Solar Wind plasma is related to kinetic effects and has nothing to do with the usual assumed MHD dissipation ∇^2–terms. This supports the point of view which attributes intermittency in the inertial range (Kraichnan, 1974, 1995; Chen et al., 1995; Stolovitzky and Sreenivasan, 1994) much more to the dynamics than to the characteristics of dissipation mechanism (see also Borne and Orszag, 1996; L'vov and Procaccia, 1995).

2) Kraichnan scaling law (4), say the analogous of the Kolmogorov refined similarity hypothesis, has been conjectured to be the true scaling for hydromagnetic turbulence. Second–order scaling exponent (the usual spectral index) measured in the Solar Wind, shows ex-

tremely variable value $1 \leq \mu \leq 2$ (Marsch, 1992; Tu and Marsch, 1995), thus not always allowing for a clear distinction between Kolmogorov and Kraichnan scaling laws.

3) Scaling exponents of the velocity structure functions has been found to depend strongly on the local state of plasma. This has been attributed to the fact that fluctuations are not in a fully developed turbulent state (Marsch et al., 1996). Indeed by using ESS we find that the plots of $\log S_r^{(q)}$ vs. $\log S_r^{(4)}$ are linear over all the range of the measurements, thus allowing for a very good determination of the scaling exponents $\alpha_4(q)$. On the other hand these plots show a unexpected universality, that is the values of $\alpha_4(q)$ are the same within the low–speed streams observations of Helios 2 spacecraft. These scaling exponents are in a very good agreement with the MHD models for intermittency. This universality is at variance with the results obtained by Tu et al. (1996) using the two–parameters extended structure–function model. Indeed these authors find values of p which for low–speed streams range from $p \simeq 0.73$ to $p \simeq 0.81$. We think that the difference can be attributed to our use of the ESS hypothesis which has allowed for a much better determination of the scaling laws from the experimental data.

4) On the basis of our analysis we are lead to the conclusion that there is a strong probability that the low–frequency MHD turbulence in the Solar Wind turbulence is physically different from turbulence in the ordinary fluid flows. The physical difference seems to be due to the topological properties of the most intermittent structures, which appear to be filaments in fluid flows and planar sheets in MHD flows. Our results are in agreement with the different versions of the SL–model for intermittency in fluid flows, which is the only model which takes into account the physical difference. As concerns the Solar Wind turbulence our results show that measurements indicate a strong tendency to follow the MHD scaling laws, even if (Tu and Marsch, 1995) the small–order scalings (including the usual spectral index), do not allow for a meaningful distinction between the Kolmogorov and the Kraichnan scaling law.

5) The measurements of scaling exponents require the definition of the usually called "inertial range". In MHD the inertial range is not defined, the problem arising from the fact that the analogue of the Kolmogorov 4/5–law does not exists in this framework. We are then forced to define the inertial range as the range of scales where (4) is verified. From an observational point of view we measure the structure functions and the scaling exponents, which are affected by intermittency. Then the problem arises on what is the order of the structure function (if any exists) which is not affected by intermittency (in the usual multifractal framework). Looking at the Kraichnan scaling, one can conjecture that the fourth–order velocity structure function should have unitary slope in the inertial range. When we try to investigate this conjecture on the data, we found that only a very small fraction of scales exists where $S_r^{(4)} \sim r$. A different way to face this question consists in the analysis of high–resolution numerical simulations. The first results of this analysis (Pouquet, Politano and Carbone, 1997 paper in preparation), seems to strongly corroborate the conjecture.

Acknowledgements. We thank H. Rosenbauer and R. Schwenn for making the Helios plasma data available to us. We are also grateful to C. Meneveau and L. Zubair which kindly supplies us with their experimental data. We acknowledge useful discussions with A. Pouquet, R. Grappin, E. Leveque, H. Politano, S. Ciliberto and T. Zurbruken. Finally we are also grateful to Referees whose comments have improved the final version of the paper.

References

Akhiezer, A. I., Akhiezer, A., Polovin, R. V., Sitenko, A. G., and Stepanov, K. N., *Plasma Electrodynamics, Vol. 2 Linear Theory*, Pergamon, New York, 1975.

Anselmet, F., Gagne, Y., Hopfinger, E. J., and Antonia, R. A., High–order structure functions in turbulent shear flows, *J. Fluid Mech.*, *140*, 63–89, 1984.

Aurell, E., Frisch, U., Lutsko, J., and Vergassola, M., On the multifractal properties of the energy dissipation derived from turbulence data, *J. Fluid Mech.*, *238*, 467–486, 1992.

Barnes, A., Collisionless damping of hydromagnetic waves, *Phys. Fluids*, *9*, 1483, 1966.

Benzi, R., Paladin, G., Parisi, G., and Vulpiani, A., On the multifractal nature of fully developed turbulence and chaotic systems, *J. Phys.*, *A17*, 3521–3531, 1984.

Benzi, R., Ciliberto, S., Tripiccione, R., Baudet, C., Massaioli, F., and Succi, S., Extended self–similarity in turbulent flows, *Phys. Rev. E*, *48*, R29–R23, 1993a.

Benzi, R., Ciliberto, S., Baudet, C., Ruiz Chavarria, G., and Tripiccione, R., Extended self–similarity in the dissipation range of fully developed turbulence, *Europhys. Lett.*, *24*, 275–279, 1993b.

Benzi, R., Tripiccione, R., Massaioli, F., Succi, S., and Ciliberto, S., On the scaling of the velocity and temperature structure functions in Rayleigh–Bénard convection, *Europhys. Lett.*, *25*, 341–346, 1994.

Benzi, R., Ciliberto, S., Baudet, C., and Ruiz Chavarria, G., On the scaling of three–dimensional homogeneous and isotropic turbulence, *Physica D*, *80*, 385–398, 1995.

Benzi, R., Biferale, L., Ciliberto, S., Struglia, M. V., and Tripiccione, R., Scaling property of turbulent flows, *Phy. Rev. E*, *53*, R3025–R3027, 1996a.

Benzi, R., Biferale, L., Ciliberto, S., Struglia, M. V., and Tripiccione, R., Generalized scaling in fully developed turbulence, *Physica D*, in press 1996b.

Bershadskii, A., Pseuso–scaling in laboratory and in numerical simulations of turbulence, *J. Phys. A*, *29*, L453–L458, 1996.

Borne, V. and Orszag, S. A., Kolmogorov's refined similarity hypothesis for hyperviscous turbulence, *Phys. Rev. E*, *53*, R21–R24, 1996.

Braginskii, S. I., Transport processes in a Plasma, *Reviews of Plasma Physics, Vol. 1*, ed. M.A. Leontovich Consultant's Bureau New York, 1965.

Briscolini, M., Santangelo, P., Succi, S., and Benzi, R., Extended self-similarity in the numerical simulations of three-dimensional homogeneous flows, *Phys. Rev. E*, *50*, R1745–R1747, 1994.

Bruno, R., Inner heliosphere, observations of MHD turbulence in the solar wind: challenges to theory, in *Solar Wind Seven*, eds. E. Marsch and R. Schwenn, Pergamon Press Oxford, 423–428, 1992.

Belcher, J. W. and Davis, L., Large-amplitude Alfvén waves in the interplanetary medium, *J. Geophys. Res.*, *76*, 3534–3563, 1971.

Burlaga, L. F., Intermittent turbulence in the Solar Wind, *J. Geophys. Res.*, *96*, 5847–5851, 1991.

Burlaga, L. F., Intermittent turbulence in large-scale velocity fluctuations at 1 AU near solar maximum, *J. Geophys. Res.*, *98*, 17467–17473, 1993.

Carbone, V., Cascade model for intermittency in fully developed magnetohydrodynamic turbulence, *Phys. Rev. Lett.*, *71*, 1546–1548, 1993.

Carbone, V., Scaling exponents of the velocity structure functions in the interplanetary medium, *Ann. Geophys.*, *12*, 585–590, 1994a.

Carbone, V., Scale similarity of the velocity structure functions in fully developed magnetohydrodynamic turbulence, *Phys. Rev. E*, *50*, R671–R674, 1994b.

Carbone, V., Bruno, R., and Veltri, P., Scaling laws in the Solar Wind turbulence, *Lecture Notes in Physics*, *462*, 153–158, 1995a.

Carbone, V., Veltri, P., and Bruno, R., Experimental evidence for differences in the extended self-similarity scaling laws between fluid and magnetohydrodynamic turbulent flows, *Phys. Rev. Lett.*, *75*, 3110–3113, 1995b.

Carbone, V., Bruno, R., and Veltri, P., Evidences for extended self-similarity in hydromagnetic turbulence, *Geophys. Res. Lett.*, *23*, 121–124, 1996.

Carbone, V and Bruno, R., Cancellation exponents and multifractal scaling laws in the solar wind magnetohydrodynamic turbulence, *Ann. Geophys.*, *14*, 777–785, 1996.

Chen, S., Doolen, G. D., Kraichnan, R. H., and Wang, L.-P., Is the Kolmogorov Refined Similarity relation dynamic or kinematic?, *Phys. Rev. Lett.*, *74*, 1755–1758, 1995.

Coleman, P. J., Turbulence, viscosity, and dissipation in the solar wind plasma, *Astrophys. J.*, *153*, 371, 1968.

Dobrowolny, M., Mangeney, A., and Veltri, P., Fully developed anisotropic hydromagnetic turbulence in the interplanetary space, *Phys. Rev. Lett.*, *45*, 144, 1980.

Dubrulle, B., Intermittency in fully developed turbulence: log-Poisson statistics and generalized scale covariance, *Phys. Rev. Lett.*, *73*, 959–962, 1994.

Frick, P., Dubrulle, B., and Babiano, A., Scaling properties of a class of shell models, *Phys. Rev. E*, *51*, 5582–5593, 1995.

Frisch, U., *Turbulence: the legacy of A.N. Kolmogorov*, Cambridge Univ. Press., 1995.

Goldstein, M. L., Roberts, D. A., and Matthaeus, W. H., Magnetohydrodynamic turbulence in cosmic winds, *Ann. Rev. Astron. Astrophys.*, *33*, 283–325, 1995.

Grappin, R., Velli, M., and Mangeney, A., "Alfvénic" versus "standard" turbulence in the solar wind, *Ann. Geophys.*, *9*, 416–426, 1991.

Grauer, R., Krug, J., and Marliani, C., Scaling of high-order structure functions in magnetohydrodynamic turbulence, *Phys. Lett. A*, *195*, 335–338, 1994.

Grauer, R. and Marliani, C., Numerical and analytical estimates for the structure functions in two-dimensional magnetohydrodynamic flows, *Phys. Plasmas*, *2*, 41–47, 1995.

Hentschel, H. G. E. and Procaccia, P., The infinite number of generalized dimensions of fractals and strange attractors, *Physica D*, *8*, 435–444, 1983.

Hosokawa, I. and Yamamoto, K., Intermittency of dissipation in a directly simulated fully-developed turbulence, *J. Phys. Soc. Jap.*, *59*, 401–404, 1990.

Hundhausen, A. J., *Coronal expansion and Solar Wind*, Springer Verlag, New York, 1972.

Kolmogorov, A. N., Local structure of turbulence in incompressible fluid, *Dokl. Akad. Nauk. SSSR*, *30*, 9–13, 1941, (in Russian) translated in "Kolmogorov ideas 50 years on", *Proc. R. Soc. Lond. A*, *434*, 9–13, Eds. J. C. R. Hunt and O. M. Phillips, 1995.

Kolmogorov, A. N., A refinement of previous hypotheses concerning the local structure of turbulence in a viscous incompressible fluid at high Reynolds number, *J. Fluid Mech.*, *13*, 82–85, 1962.

Kraichnan, R. H., Inertial-range spectrum of hydromagnetic turbulence, *Phys. Fluids*, *8*, 1385, 1965.

Kraichnan, R. H., On Kolmogorov's inertial-range theories, *J. Fluid Mech.*, *62*, 305–330, 1974.

Kraichnan, R. H., Inertial-range intermittency, *Lecture Notes in Physics*, *462*, 115–122, 1995.

Iroshnikov, P. S., Turbulence of a conducting fluid in a strong magnetic field, *Soviet Astron.*, *7*, 566, 1963.

L'vov, V. S. and Procaccia, I., "Intermittency" in hydrodynamic turbulence as intermediate asymptotics to Kolmogorov scaling, *Phys. Rev. Lett.*, *74*, 2690–2693, 1995.

Marsch, E., Introduction to kinetic physics, waves and turbulence in the solar wind, in *Solar Wind Seven*, eds. E. Marsch and R. Schwenn, Pergamon Press Oxford, 499–504, 1992.

Marsch, E. and Liu, S., Structure functions and intermittency of velocity fluctuations in the inner solar wind, *Ann. Geophys.*, *11*, 227–238, 1993.

Marsch, E. and Tu, C.-Y., Correlations between the fluctuations of pressure, density, temperature and magnetic field in the solar wind, *Ann. Geophys.*, *11*, 659–677, 1993.

Marsch, E. and Tu, C.-Y., Non-gaussian probability distributions of solar wind fluctuations, *Ann. Geophys.*, *12*, 1127–1138, 1994.

Marsch, E., Tu, C.–Y., and Rosenbauer, H., Multifractal scaling of the kinetic energy flux in solar wind turbulence, *Ann. Geophys.*, *14*, 259–269, 1996.

Meneveau, C. and Sreenivasan, K. R., Simple multifractal cascade model for fully developed turbulence, *Phys. Rev. Lett.*, *59*, 1424–1427, 1987.

Meneveau, C. and Sreenivasan, K. R., The multifractal nature of turbulent energy dissipation, *J. Fluid Mech.*, *224*, 429–484, 1991.

Novikov, E. A., Scale similarity for random fields, *Sov. Phys. Dokl.*, *14*, 104–107, 1969.

Novikov, E. A., The effects of intermittency on statistical characteristics of turbulence and scale similarity of breakdown coefficients, *Phys. Fluids A*, *2*, 814–820, 1990.

Novikov, E. A., Infinitely divisible distributions in turbulence, *Phys. Rev. E*, *50*, R3303–R3305, 1994.

Parisi, G. and Frisch, U., On the singularity structure of fully developed turbulence, in *Turbulence and Predictability in Geophysical Fluid Dynamics, Proceed. Intern. School of Physics "E. Fermi", 1983, Varenna Italy*, 84–87, eds. M. Ghil, R. Benzi, & G. Parisi, North–Holland, Amsterdam.

Parker, E. N., Dynamics of the interplanetary gas and magnetic field, *Astrophys. J.*, *128*, 664, 1958.

Parker, E. N., *Interplanetary dynamical processes*, Interscience, New York, 1963.

Phillips, J. L., Balogh, A., Bame, S. J., Crooker, N. U., Goldstein, B. E., Gosling, J. T., Hoeksema, J. T., McComas, D. J., Neugebauer, M., Sheeley Jr., N. R., and Wang, Y.-M., Ulysses at 50° South: Constant immersion in the high–speed solar wind, *Geophys. Res. Lett.*, *21*, 1105–1109, 1994.

Politano, H. and Pouquet, A., A scale–invariant model of intermittency in MHD, *Phys. Rev. E*, *52*, 636–641, 1995.

Ruzmaikin, A. A., Feynman, J., Goldstein, B. E., and Smith, E. J., Intermittent turbulence in solar wind from the south polar hole, *J. Geophys. Res.*, *100*, 3395–3403, 1995.

Schertzer, D. and Lovejoy, S., Elliptical turbulence in the atmosphere, Proceedings of the 4–*th symposium on turbulent shear flows*, Karlsrhule, West Germany, 111, 1983.

Schertzer, D. and Lovejoy, S., Singularités anisotropes, et divergence de moments en cascade multiplicatifs, *Annales Math. du Que.*, *11*, 139, 1987.

Schertzer, D. and Lovejoy, S., Hard and soft multifractal processes, *Physica A*, *185*, 187–194, 1992.

She, Z.–S. and Leveque, E., Universal scaling laws in fully developed turbulence, *Phys. Rev. Lett.*, *72*, 336–339, 1994.

Stolovitzky, G. and Sreenivasan, K. R., Scaling of structure functions, *Phys. Rev. E*, *48*, R33–R36, 1993.

Stolovitzky, G. and Sreenivasan, K. R., Kolmogorov's refined similarity hypothesis for turbulence and general stochastic processes, *Rev. Mod. Phys.*, *66*, 229–240, 1994.

Tu, C.–Y. and Marsch, E., On the nature of compressive fluctuations in the solar wind, *J. Geophys. Res.*, *99*, 21481–21509, 1994.

Tu, C.–Y. and Marsch, E., MHD structures, waves and turbulence in the solar wind, *Space Sci. Rev.*, *73*, 1–210, 1995.

Tu, C.–Y., Marsch, E., and Rosenbauer, H., An extended structure–function model and its application to the analysis of solar wind intermittency properties, *Ann. Geophys.*, *14*, 270–285, 1996.

Veltri, P., Turbulence and energy dissipation in the solar wind, *Space Sci. Rev.*, *68*, 63, 1994.