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Abstract. Many vital insights into the nature of turbuence in fluids have originated from experimental data obtained in geophysical flows. Geophysical data have often helped to stimulate the creation of new turbulence theory, while theory has in many cases motivated the experimental efforts. The present brief review discusses several key examples of this interaction between experiment and theory, citing mainly work which is of particular interest to the author. No attempt is made to provide a complete listing of the extensive and currently rapidly developing literature for some of the problems discussed.

1 Introduction

New experimental results for turbulent flows can stimulate the creation of new theory when 1) the experimental results conflict with accepted existing theory, or 2) the experiments uncover or suggest some effect or simplification not previously considered by the theory. Turbulence theory has a complementary impact on experimental studies . The study of turbulence in fluids is a continuing saga of this interaction between experimental results and theory. Geophysical turbulence data has such a profound influence upon turbulence theory because, in terrestrial flows, Reynolds numbers sufficiently large to provide conclusive tests of many of the most cogent theoretical results are readily obtained only in the geophysical flows found in the earth's oceans and atmosphere. The present account reviews some examples of this process, highly restricted to only a few questions in which the author has had particular interest. The focus here is on the impact of geophysical data on theory, with no discussion of the considerable impact of laboratory experimental results on turbulence theory.

2 Isotropy, Energy Spectra, and Structure Functions

Symmetry breaking associated with the preferred direction of the vertical coordinate z in the atmosphere and oceans, and the presence of rotation and vertical stratification precludes any global large scale isotropy in geophysical flows. However, for the smaller scales of motion the turbulence can be locally isotropic. While making meteorological measurements aboard ships using tethered balloons, Taylor (1927) noticed that, when subjected to buffeting by atmospheric turbulence, the horizontal, lateral, and vertical excursions of his balloons were roughly equal. This led him to the concept of globally isotropic turbulence, for which the mean square fluctuating velocities are equal in all three coordinate directions, the application by Taylor (1935) of the principle of invariance with respect to rotations , and the elegant employment of isotropic tensors in the theory of isotropic turbulence by Karman and Howarth (1938). These developements also led to the very useful concept of local isotropy , in which the exact relations given by local isotropy theory apply only over restricted finite ranges of length scales or wavenumbers, and the global isotropy restriction of equality of the mean square velocities in different directions is not invoked. In the 1940's , the statistical theory was further developed in terms of structure functions, moments of differences in velocity measured at two different spatial locations separated by a distance r, by Kolmogorov (1941) and in spectral terms by Obukhov (1941). These results, which include the inertial subrange $r^{2/3}$ behavior of the second-order structure function and the corresponding $k^{-5/3}$ (k is wavenumber) behavior of the energy spectrum , are sometimes referred to as the K41 theory. Obukhov's spectral formulation was rediscovered by Onsager (1945), von Weizsacker (1948), and Heisenberg (1948). Obukhov also made the first attempt to use geophysical (atmospheric) data for veri-

fication of the theory. In several experimental papers from his group published between 1941 and 1960 the 2/3 and -5/3 power laws were confirmed, but the instrumentation had insufficient resolution to resolve dissipation range behavior. Definitive, quantitative verification of full inertial and dissipative range K41 scaling was found in tidal channel measurements by Grant et al. (1962) using an instrumented ship-towed underwater body , and then in the atmospheric boundary layer by Pond et al. (1966). The tidal channel measurements also showed that the variance of ϵ_r, the rate of dissipation of turbulent kinetic energy ϵ averaged over a spatial region of characteristic length scale r, was relatively large. Universal spectral scaling was obtained employing ϵ_r , thus precluding successful universal scaling when employing the long term average value of ϵ. To address similar observations by Gurvich of strong variability of the dimensional coefficient in the -5/3 law , Kolmogorov (1962), and Obukhov (1962) proposed refined similarity hypotheses based on ϵ_r , instead of simply on the overall value of ϵ. Statistical quantities involving moments of ϵ_r thus acquired an additional r-dependence through the r-dependence of the pdf of ϵ_r. Assuming a lognormal form for the pdf of ϵ_r, with a variance which depended on r, as suggested by Kolmogorov (1962) , Yaglom, A.M. (1966), and Gurvich and Yaglom (1967), yielded small modifications to the -5/3 spectral and +2/3 structure function power laws. The modifications became increasingly larger as the order p of the structure function $< (u(x+r) - u(x))^p >$ increased for $p > 3$, and were strongly dependent on the assumed form of the pdf of ϵ_r for large values of ϵ_r. Experimentally measured large deviations from the K41 scaling were first found , in geophysical data for higher-order structure functions measured in the atmospheric boundary layer over the ocean, by Van Atta and Chen (1970), and by Van Atta and Park (1972). As p increased for $3 < p \leq 9$ their measured values of ζ_p, the power law exponent for the pth order structure functions, increased with p, but at a significantly slower rate than the values of $\zeta_p = p/3$ predicted by a straightforward extension of K41. Qualitatively similar behavior has been found in all successive experiments in both geophysical and laboratory flows, and in direct numerical simulations, as discussed below. Higher-order structure function behavior is naturally strongly dependent on the tails of the pdf's. Gagne et al (1990) compared their data taken in the very large Modane wind tunnel with $R_\lambda = 2720$, where R_λ is the turbulence Reynolds number based on the Taylor microscale λ, with the atmospheric data of Van Atta and Park (1972) for $R_\lambda = 3000$, and with the laboratory data of Anselmet et al. (1984) for R_λ in the range 515-852 . Numerical values of the slopes of exponential fits to the positive and negative tails of the pdf's were very similar for the atmospheric and Modane data sets, defining unique power law functions of r/η, where η is the Kolmogorov scale. The slopes were smaller for the Anselmet et al. (1984) data, increasing with increasing R_λ, and following the same power law dependence on r/η. This comparative behavior suggests a significant R_λ dependence of the ζ_p, and that an R_λ at least as large as several thousand is required to achieve conditions which might be characteristic of the asymptotic limit of infinite R_λ. This suggestion is apparently not supported by the recent atmospheric boundary layer data of Schertzer et al. (1995), who find, for $p \leq 7$, ζ_p that are the same for atmospheric and laboratory flows. However, for $p > 7$ their ζ_p are found to depend on the length of the time series they used. Thus, while they argue that the behavior they observed is compatible with divergence of moments for empirical data, proposed in Schertzer and Lovejoy (1987), the dependence on sample size could be a consequence of nonstationarity . Instead of focussing on the behavior of individual structure functions, Benzi et al. (1993) have examined the comparative behavior of structure functions of different orders . Structure functions obey *scaling* when each structure function has a power law dependence on r. Plotting the structure function of order m versus that of order n will then also yield a power law, and the structure functions then obey *relative scaling* . If two different normalized structure functions, each made dimensionless by dividing by the appropriate powers of the third-order structure function, obey relative scaling , this is called *normalized relative scaling.* Benzi et al. (1993) find that their measured structure functions obey relative scaling, persisting even for small values of r in the viscous range, a behavior to which they give the name *extended self similarity.* Their data suggests that the improvement in scaling behavior is due to the similar dependence of all structure functions in the dissipative range on p-dependent powers of a universal function $f(r/\eta)$, which itself does not depend on p. L'vov and Procaccia (1994) argue that this *extended universality* stems from the structure of the Navier-Stokes equations and from the property of the locality of interactions, and examine the possibility that experimentally observed deviations from K41 behavior are due to the finite values of the Reynolds numbers and anisotropy of the turbulence. Segel et al. (1996) have demonstrated extended self-similarity analytically in the context of the Kraichnan (1968) model of a passive scalar advected by a very rapidly varying velocity field.

The Kolmogorov (1962) lognormal hypothesis yields pathological structure function behavior for large p, and many other pdf's, fractal scaling recipes for the spatial distribution of ϵ, and other scaling scenarios have been hypothesized to model the measured dependence of ζ_p on p . These models include those of Frisch et al (1978), Kida (1991), Saito (1992), Kailasnath et al (1992), She and Leveque (1994), Schmitt et al. (1994), Dubrulle (1994), She and Waymire (1995), Nelkin (1995), and Chen and Cao (1996). Many of the scaling theories are expected to apply strictly only for very large values of

R_λ, and the Reynolds number does not appear in the theoretical predictions for the ζ_p. Another theoretical approach is to calculate the pdf of Δu from its governing equation, as derived from the Navier-Stokes equations by Lundgren (1972) and Monin (1967) , and then to calculate the structure function moments directly using this pdf. However, the integro-differential equation for the pdf of Δu is not closed , since the equation for the n-point pdf (where n=1,2,...) includes also the (n+1)-point pdf (similar to the BBGKY heirarchy of equations for the pdf in the kinetic theory of gases. Lundgren (1972) proposed some model closures of this equation . A great number of other closures were proposed in later years (see, e.g., the surveys by Kollmann and Pope (1990,1991), for some examples) and their applications to higher-order structure functions are a subject of currrent research (see, e.g. Pedrizzetti and Novikov (1994)). A more rigorous approach is to calculate the behavior of higher-order structure functions directly from the Navier-Stokes equations. This has been done using DNS (direct numerical simulations) for modest values of $R_\lambda \leq 200$ by Cao et al. (1996) and Boratov and Pelz (1996). Their computed values of relative scaling exponents are numerically close to those measured by Benzi et al. (1993) and those predicted by She and Leveque (1994).

It thus appears that results from experiments, modelling, and DNS for some aspects of higher-order structure function behavior may be converging. Some questions remain about the behavior of higher-order structure functions for large Reynolds number . Due to the uncontrollability of geophysical flows , lack of sufficient stationarity, etc. , data obtained in geophysical flows may not provide the decisive evidence for the behavior of higher-order moments that was obtained for lower-order structure functions and spectra. It is desirable to continue large Reynolds number laboratory studies of both shear flows and unsheared grid generated turbulence. The latter flow provides an important benchmark as there are no effects of mean shear (which is zero) and the turbulence is locally isotropic over a large range of scales. As discussed by Uberoi (1957) and Durbin and Speziale (1991), mean shear can significantly decrease the degree of local isotropy , even in the limit of infinite Reynolds numbers (for some flows). Systematic, closely coordinated DNS and experimental studies of the influence of varying mean shear and Reynolds number would be valuable.

3 Higher-order spectra

The synergistic interaction of geophysical turbulence experiments and turbulence theory is also evident in studies of higher-order spectra. Initial studies focused on the discrepancy between measured higher-order spectra and K41 dimensional analysis. Successive developments have dealt with other fundamental theoretical issues.

As noted by Dutton and Deaven (1972) , in analogy with K41 inertial subrange scaling, dimensional analyis predicts that spectra of the nth power of the velocity fluctuation will scale as $E_n(k) = C_n \epsilon^{2n/3} k^{-(2n+3)/3}$. However, in their atmospheric turbulence data obtained at four different altitudes with instrumented aircraft, a different behavior was observed. Rather than increasing in slope with increasing order, in the inertial range the higher-order spectra either retained an approximately $k^{-5/3}$ power law behavior or decreased somewhat in slope as n increased. This puzzle stimulated Van Atta and Wyngaard (1975) to propose an alternative extension of Kolmogorov's ideas. The key factor in their argument is an order-dependent dissipation term ϵ_n, which appears in the dynamical equation for $< u^n >$. In a separate analytical and numerical study , they also derived expressions for higher-order inertial range spectra based on the assumption of a Gaussian velocity distribution . Results of their dimensional arguments and Gaussian analyses showed good agreement with the measured spectral slopes and energy levels of Dutton and Deaven (1972) and with those of the higher-order spectra computed from the same open ocean atmospheric boundary later data that was employed earlier by Van Atta and Chen (1970) and Van Atta and Park (1972) in their studies of higher-order structure functions. In studies of higher-order spectra of a different sort, bispectral analysis of velocity fluctuations in the atmospheric boundary layer by Lii et al (1976) and Van Atta (1979) indicated a k^{-3} behavior in the inertial subrange, as predicted by extension of K41 scaling. More recently, Zhou et al (1993) extended ideas of Kraichnan (1965) to predict that higher-order spectral moments can scale as $k^{-5/3}$ for any order without the assumption of Gaussianity, in agreement with the results of the ϵ_n-based dimensional analysis of Van Atta and Wyngaard (1975), which also did not assume Gaussianity . In a different theoretical context Nelkin and Tabor (1989) noted that a consequence of the random sweeping hypothesis is that the spectrum of the kinetic energy u^2 scales as $k^{-5/3}$. If, on the other hand, the u^2 spectrum satisfies the $E_2 = C_2 \epsilon^{4/3} k^{-7/3}$ scaling predicted by the expression for E_n derived in the spirit of K41 , then the renormalization group theory prediction of no sweeping is recovered. Since the experiments clearly indicate that the spectrum of u^2 goes as $k^{-5/3}$ at high Reynolds numbers, they provide positive support for the sweeping hypothesis.

4 Skewness and flatness of velocity derivatives

Geophysical data for moments of velocity gradients have stimulated a great deal of theoretical work , as the numerical values of the normalized moments, such as skewness and flatness factors (denoted by S and F) measured

for large R_λ differ greatly from those obtained for lower R_λ in laboratory experiments.

The K41 equilibrium hypothesis predicts that for sufficiently large R_λ, dimensionless ratios of mean powers of velocity derivatives, like skewness and flatness factors, should be absolute constants independent of Reynolds number (see, e.g., Batchelor (1953)). Except for the very low Reynolds numbers associated with the final period of decay , measured skewness and flatness factors of velocity derivatives increase with increasing R_λ (see the data compiled by Van Atta and Antonia (1980)). Intermittency corrections in the spirit of K62 predict that S and K will increase with increasing R_λ, as seen in the experimental data (for scalar variables , see, e.g. Van Atta (1973)). Many other models , e.g., scaling intermittency models, would also predict such a dependence. Another approach to the problem is the use of vortex-based physical models of turbulence fine scales using spatial ensembles of small-scale structures represented by local solutions of the Navier-Stokes equations. Pullin and Saffman (1993) have used the statistical methodology of Townsend (1951) combined with the Lundgren (1982) spiral vortex model to calculate higher-order moments of velocity derivatives for homogeneous turbulence. Lundgren (1982) showed that an ensemble of such spiral vortex structures produces a $k^{-5/3}$ range in the energy spectrum of velocity, so its use lends dynamical credentials to the calculation. Numerical values of two dimensionless groups in the model are fixed by requiring agreement with experimental estimates of the Kolmogorov constant and with the skewness S. When the vortex lateral scale R is assumed to be the geometric mean of the Tayor microscale λ and the smallest physical scale of the model $(\nu/a)^{1/2}$, where ν is the kinematic viscosity of the fluid and a is the rate of strain, the flatness factor F is found to be proportional to ${R_\lambda}^{1/4}$. From figure 4 of Pullin and Saffman (1993) , one notes that there is effectively a discontinuous upward jump in the F data at $R_\lambda = 700$. The ${R_\lambda}^{1/4}$ dependence is in good agreement with the experimental data and that of DNS (for the presently available DNS results the maximum R_λ is about $2x10^2$). For $R_\lambda > 300$ the experimental data lie in a " scatter" band lying significantly above a continuation of the ${R_\lambda}^{1/4}$ fit to the data for the lower values of R_λ, but a separate ${R_\lambda}^{1/4}$ fit to this data also looks quite reasonable. The model thus furnishes an interesting predictive physical theory for intermittency effects based on the essential mechanism of balance between vorticity production through amplification of the vorticity by local rate-of-strain and vorticity dissipation by viscosity. In contrast with the above findings, recent data of Tabeling et al (1996) show a decrease in S and K with increasing R_λ for their largest values of R_λ, so this question cannot yet be considered to be closed. The recent review of Sreenivasan and Antonia (1997) contains an updated discussion and list of references on this subject.

5 Final Comments

I hope that the above brief account gives the reader some useful insight into the impact of geophysical data on turbulence theory, and that it may encourage others to continue the interactive process by devising new experiments and complementary theories .

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