Detection of time-frequency components of signals: Bayes and focus SNR

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Abstract

The Bayesian time-frequency detector operating on spectrogram of non stationary signals is studied. As direct evaluation of the likelihood ratio is impossible, an a priori user parameter called *focus* is introduced. It is defined as a local time-frequency signal to noise ratio at which the detection is tuned to be optimal. Leading to a unique detection threshold, the parameter introduced is equivalent to the probability of false alarm used in the Neyman-Pearson detection strategy. However, we expect the formulation in terms of local signal to noise ratio to be of intuitive and practical interest.

1. Introduction

Detection procedures are used to reveal wether a signal is present in a given observation or not. The detection may be proceeded in the temporal domain [Van Trees (1968)] or in the time-frequency domain as studied by Altes [Altes (1980)] or Flandrin [Flandrin (1988)]. However, the binary hypotheses test is usually formulated in terms of the temporal observation. The time-frequency detection procedure we are interested in is slightly different: it is formulated at each time-frequency location (t, f).

The aim of this detection procedure is to extract the regions of the time-frequency plane where some signal is present. This specific task reveals useful so as to determine whether a signal is present or not but also provides informations about the spectral structure of the signal and its evolution. Interpreted as a time-frequency signature of the signal, this information may be used for further processing such as classification, time-frequency sparse sources separation or estimation of the underlying physical process parameters.

Neyman-Pearson detection strategy has already been employed for this task [Huillery (2006a)]. In this paper we concentrate in the possibility to use the theoretically more optimal Bayesian approach. However due to the lack of information, a focus parameter is introduced to solve the detection problem. It turns out that the resulting procedure is equivalent to a Neyman-Pearson detection strategy, where the focus parameter plays the role of the traditional Probability of False Alarm (PFA).

The paper is organized as follows: in section 2, the Bayesian time-frequency detection procedure is introduced. In section 3, we express the probability density function (pdf) of the time-frequency coefficients. A solution for the detection task and some results on a real-life signal are presented in section 4. In section 5, we finally discuss the link between the method proposed and a Neyman-Pearson detector.

2. Bayesian Time-frequency detection task

The model of signal we are interested in is composed of a deterministic part $d(t)$ embedded in a centered Gaussian perturbation $p(t)$ with autocorrelation function $\Gamma_p(t,\tau)$ associated with the energy time-frequency distribution $\gamma_p(t,f)$. Namely,

$$x(t) = d(t) + p(t). \tag{2.1}$$

The deterministic part contains the information under interest on which no a priori is known. The aim of the proposed approach is to localize the time-frequency support of this information.

The detection task consists in determining whether the energy $S_x(t,f)$ observed at a particular time-frequency location (t,f) originates from the perturbation only (null hypothesis H_0) or is also due to the deterministic part of the signal (signal hypothesis H_1). The two hypotheses test problem is thus formulated at each time-frequency locations as

$$\begin{cases} H_0 : S_x(t,f) = S_p(t,f), \\ H_1 : S_x(t,f) = S_{d+p}(t,f). \end{cases} \tag{2.2}$$

We consider no a priori information about the probability of each hypotheses (or equivalently, we fixe them to $1/2$) and equal costs for erroneous decision. In this particular case, the Bayesian detection strategy results in the maximum likelihood detector [Van Trees (1968)]. It consists in the comparaison of the likelihood ratio $\Lambda_{S_x(t,f)}$ with 1 and writes

$$\Lambda_{S_x(t,f)} = \frac{p_{H_1}(S_x(t,f))}{p_{H_0}(S_x(t,f))} \underset{H_0}{\overset{H_1}{\gtrless}} 1. \tag{2.3}$$

where $p_{H_0}(.)$ and $p_{H_1}(.)$ stand for the pdfs under the null and signal hypotheses respectively. Finally, the time-frequency support of information, noted $\mathcal{I}$, is defined as

$$\mathcal{I} = \left\{(t,f)/S_x(t,f) > S^{th}(t,f)\right\}, \tag{2.4}$$

where $S^{th}(t,f)$ is the detection threshold obtained with the detector 2.3.

3. Time-frequency probability distributions

We use the spectrogram as the time-frequency representation of the signal. As a main advantage, it does not display any interference terms and remains interpretable as a physical representation of signal energy. However, these features are counterbalanced by a poor energy concentration.

The pdf of spectrogram coefficients under the null hypothesis H_0 is a central χ^2 laws with $\delta = 2$ degrees of freedom and proportionality coefficient $\alpha = \gamma_p(t,f)/2$ [Huillery (2006b)]. It writes

$$p_{H_0}(s) = \frac{1}{\gamma_p(t,f)} \exp\left(-\frac{s}{\gamma_p(t,f)}\right). \tag{3.1}$$

Under signal hypothesis H_1, the pdf of a spectrogram coefficient is a non central χ^2 law. The noncentrality parameter is equal to the spectrogram coefficient $S_d(t,f)$ of the deterministic part $d(t)$ of the signal. At time-frequency location (t,f), the pdf p_{H_1} writes

$$p_{H_1}(s) = \frac{1}{\gamma_p(t,f)} \exp\left(-\frac{s + S_d(t,f)}{\gamma_p(t,f)}\right) I_0\left(\frac{2\sqrt{s.S_d(t,f)}}{\gamma_p(t,f)}\right), \tag{3.2}$$

where $I_0(.)$ stands for the zero order modified Bessel function of the first kind.

4. Solution to the detection problem

From the formulation given in section 2, the Maximum Likelihood (ML) detection threshold S_{ML}^{th} is defined as the quantile that equates the two probability distributions p_{H_0} and p_{H_1}. Using the expressions of probability distributions given in section 3, S_{ML}^{th} is solution of the equation

$$I_0\left(\frac{2\sqrt{S_d(t,f).S_{ML}^{th}}}{\gamma_p(t,f)}\right) = \exp\left(\frac{S_d(t,f)}{\gamma_p(t,f)}\right). \tag{4.1}$$

We solved this equation numerically and after linear regression, the ML detection threshold S_{ML}^{th} can be obtained as

$$S_{ML}^{th}(t,f) = 0.26 \times S_d(t,f) + \gamma_p(t,f). \tag{4.2}$$

Without further developments, the detection threshold S_{ML}^{th} depends on two parameters, namely $S_d(t,f)$ and $\gamma_p(t,f)$. The deterministic contribution $S_d(t,f)$ being precisely what we are looking for, an ill problem is faced.

Let us define the Local Time-Frequency Signal to Noise Ratio (local-SNR), noted $\rho(t,f)$, as the ratio between the signal and perturbation energies observed at a single time-frequency location (t,f). In linear scale, it writes

$$\rho(t,f) = \frac{S_d(t,f)}{\gamma_p(t,f)}. \tag{4.3}$$

To determine the detection threshold $S^{th}(t,f)$ of eq. 2.4, we propose to introduce a *focus* parameter, noted ρ_0, defined as a local-SNR for which the detector will be tuned optimally in the ML sense. This parameter has to be fixed before the detection procedure and leads to a detection threshold $S^{th}(t,f)$ calculated as

$$S^{th}(t,f) = (0.26 \times \rho_0 + 1)\,\gamma_p(t,f). \tag{4.4}$$

When ρ_0 is different from the local-SNR $\rho(t,f)$ (presumably most of the time), the detection threshold $S^{th}(t,f)$ is not optimal. In practice, ρ_0 approximately corresponds to the minimal local-SNR on the "information support" $\mathcal{I}$ that is detected. We also note the dependence of the detection threshold $S^{th}(t,f)$ on the perturbation spectral power $\gamma_p(t,f)$ that needs to be estimated when unknown. An estimation procedure dedicated to central χ^2 laws can be found in [Hory (2002)].

Figure 1 displays some detection results obtained with the French speech signal "Joyeux noël" embedded in a white gaussian perturbation of known variance. In each cases b), c), d) and e), the local-SNR of the time-frequency locations detected as "information support" are around or greater than the focus parameter ρ_0. We note the increasing amount of false alarms as the focus local-SNR ρ_0 decreases.

5. Discussion

Starting from a Bayesian formulation of the detection procedure, a lack of information imposes the introduction of a detection condition so as to determine a detection threshold. We choose to formulate this condition as a local signal to noise ratio for which the detection procedure is set to be optimal. The focus parameter introduced now imposes a single value to the detection threshold (see eq. 4.4). Considering a binary hypothesis test where the probability distribution under H_1 is unknown, the Neyman-Pearson detection strategy [Van Trees (1968)] consists in fixing *a priori* a Probability of False Alarm (PFA). The choice of this PFA also imposes a single value to the detection threshold. The focus

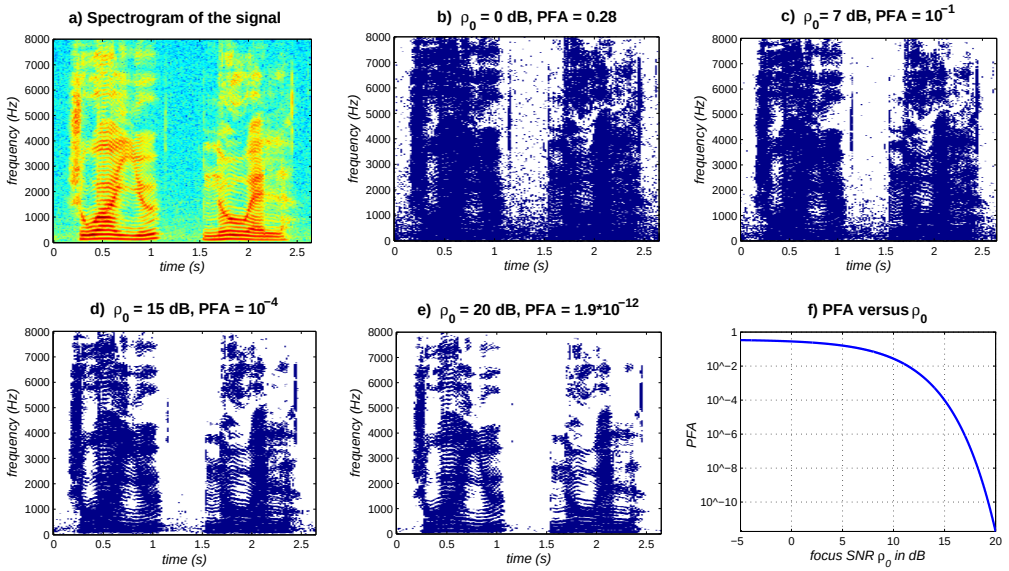

FIGURE 1. Detection results for different focus parameter ρ_0. Spectrogram is constructed with 50% overlapping 512 points Hanning windows, with a zero-padding of 2.

parameter ρ_0 and the PFA thus play a similar role in the detection procedure and can be expressed one as a function of the other. Starting from the definition of the PFA and using equation 4.2 and 3.1, we obtain

$$PFA = \int_{S^{th}}^{+\infty} p_{H_0}(x)dx = \exp\left(-\left(0.26 \times \rho_0 + 1\right)\right), \tag{5.1}$$

with ρ_0 expressed in linear scale. In figure 1, the PFA corresponding to each focus parameter ρ_0 is also noted. Figure 1-f) depicts the relation between ρ_0 and the PFA.

However, as the PFA is a noise dedicated detection parameter, the focus parameter introduces a signal condition in the detection procedure. It is an answer to the problem: "I want to detect the signal time-frequency components with energy at least ρ_0-time higher than the background perturbation.". We expect this formulation of detection condition to be of practical interest.

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