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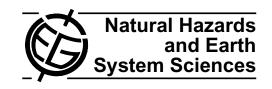
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Flicker-noise spectroscopy in earthquake prediction research

A. V. Descherevsky¹, A. A. Lukk¹, A. Y. Sidorin¹, G. V. Vstovsky², and S. F. Timashev²

¹O. Yu. Schmidt United Institute of Physics of the Earth, Russian Academy of Sciences, Bolshaya Gruzinskaya 10, D-242, GSP-5, Moscow 123995, Russia

²L. Ya. Karpov Research Institute of Physical Chemistry Russian State Scientific Center, Vorontsovo pole 10, Moscow 103064, Russia

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Abstract. The problem of earthquake prediction and the methods of identification of geophysical precursory signals are discussed. To get information on the dynamics of earthquake preparation processes, fluctuations in geophysical time series are analyzed with the method of flicker-noise spectroscopy. Integral indices – power spectra and various moments ("structural functions") – are used as information relations. We demonstrate that the method allows us to reveal earthquake precursors.

1 Introduction

The present state of the art of earthquake prediction research evidences that deterministic concept has exhausted its capabilities. This concept assumes that observed geophysical temporal realizations are determined by the medium passive response to its deformation due to certain external forces. An adequate description of the observations by means of this concept is not possible.

The crisis of the deterministic paradigm of earthquake prediction has allowed the growth of other opinions. In particular, a model of the medium in a form of an active discrete hierarchically structural geophysical system has received wide recognition (Scholz, 1991; Turcotte, 1994; Lukk et al., 1996; Descherevsky et al., 2000). Its principal difference from the classical model of passive continuum is that the medium can actively redistribute and release the energy. At the same time, its elements could be sated to a different extent with heat, elastic and "structural" energy (Timashev, 2001a, b). Due to additional input of energy from outside (for example due to tidal motions, tectonic shifts etc.) some elements of the medium could reach an unstable state and throw off the energy surplus, which is absorbed by neighboring fragments or separate pieces. Such processes of accumulation and redistribution of this energy could gradually lead the whole system to instability and ultimately seismic catastrophe (Sadovsky and Pissarenko, 1989).

Following this model, we have to change the paradigm of earthquake prediction research. An analysis of preparation of each specific large seismic event as reflected in non-stationarities of geophysical signals needs an application of stochastic models which could be fundamental to search for large earthquake precursors.

A degree of non-stationarity of geophysical medium can be evaluated analyzing temporal variations in certain characteristics of studied signals. The Hurst exponent obtained from the analysis of temporal fluctuations in the inter-event intervals of the earthquakes and exponent of the power spectrum, which behaves as a power-law function of the frequency, was considered as an example of such characteristics (Telesca et al., 2001). The time evolution of these parameters was studied using overlapping time windows in order to have a sufficient number of points to estimate the scaling exponents. The authors demonstrated that a tendency of both parameters to converge toward a unity, which is typical for self-organized critical dynamics, was evident before the occurrence of the major earthquake event recorded in the area during the observation period. A subionospheric VLF/LF (very low frequency/low frequency) propagation was also investigated to detect the seismo-ionospheric perturbations (Hayakawa, 2000). The author studied day-to-day sequence of diurnal variation in the transmitted signal phase, and a significant change in the terminator times before several earthquakes was discovered. The terminator time was defined while diurnal phase variations exhibit a minimum around sunrise and sunset.

In accordance with the ideas presented we should expect the occurrence of precursors, associated in particular with changes in the fluctuation behavior of the monitored parameters with respect to their enrichment with a high frequency component instead of traditional, for example bay-like, precursors. The problem we run into is how to extract the required information a priori, knowing that analyzed time series have a boundless volume of information, because the number of freedom degrees in natural systems is infinite.

This leads us to a conclusion that a noise component of the variations in geophysical parameters for the majority of cases could not be related to the measurement errors. The analysis shows that these chaotic variations can often be identified as such a natural phenomenon as flicker noise or "l/f noise". On the basis of these results we propose a new approach to earthquake prediction research, the flicker noise spectroscopy (FNS) (Timashev, 2001a, b).

The FNS methodology allows the total information contained in chaotic series to be classified discernibly in the most general phenomenological form. In this case, an arbitrary desirable number of parameters with clear physical meaning can be extracted from arbitrary chaotic series of dynamic variables. The problem is formulated in such a way as to identify unambiguously the state of the investigated complex system during its evolution or its structural peculiarities from the totality of parameters, whose number should be determined by a special analysis. The method makes it possible to reveal different qualities and to distinguish between the peculiarities of different levels of the system structural hierarchy.

In this paper, the fundamentals of FNS are briefly discussed and principal capabilities of the approach to reveal earthquake precursors are demonstrated.

2 Flicker noise structure of geophysical temporal variations and consequences

In our previous research we established that a twocomponent model including a seasonal and a flicker-noise components, seems to be more adequate to model statistical structure of time series of long-term geophysical observation data. After the variations are filtered from regular seasonal component, their relation to a flicker-noise class is beyond question (Descherevsky et al., 1997, 2000).

The examples of geophysical time series, characterized by a flicker-noise structure, are given in Fig. 1a, and their spectra – in Fig. 1b.

For all the considered realizations the amplitude spectrum can be approximated by a power function $A \sim f^{-k}$, where A is the spectrum amplitude; f is the frequency and the spectrum parameter k is fluctuating within the range $40.5 \le k \le 1.0$. In this case, we consider the amplitude spectrum as a square root of the power spectrum, i.e. a value proportional to the amplitude but not to initial signal dispersion. We should bear in mind that the spectrum slope in bilogarithmic coordinates would be twice less then the slope of the power spectrum.

The established statistical structure of geophysical field variations is significant not only from the point of view of selection of adequate techniques for geophysical data analysis, but these techniques could also lead to understanding of processes taking place in the geophysical medium. Thus, the values and techniques directly or indirectly assuming the signal stationarity, for example application of the correlation

coefficient, appear to be incorrect. Contrary to white noise, which means that the values of the signal absence are totally uncorrelated, asymptotics of power spectrum reveal that such a correlation exists in the system. The observed dynamic processes exhibit properties of a scaling invariance: at any time scales the series' properties remain the same.

It seems natural to relate the self-similarity of statistical properties of geophysical signals to the self-similarity, fractality of the geophysical medium on different scales. In this case self-similar geophysical parameter time variations could evidence the presence of deterministic chaos in the geophysical system evolution.

The application of stochastic (not deterministic) models of preparation of a specific large seismic event, based on identification of non-stationarities signals, could be a very effective tool in searching for large earthquake precursors. We expect the presence of precursors associated with changes in the temporal fluctuations of the monitored parameters due to their enrichment with the high frequency component.

However, high frequency fluctuations of the monitored values are only one of a number of possible manifestations of nonstationary processes occurring in the active geophysical medium prior to a seismic event. A more general approach to the problem is a formalized search in the geophysical monitoring time series for a wide spectrum of non-stationarities and collective effects of different sorts.

We offer a promising approach which, however, has not been virtually tested yet in geophysical practice. The FNS approach consists of giving an informational significance to sets of different discernible irregularities – bursts, jumps, and derivative discontinuities of different order – occurring on all space-time hierarchical levels of the system. The capacity spectra and various moments (transient structural functions) of different orders are used as information relations of integral indices of the analyzed signals.

3 Fundamentals of flicker-noise spectroscopy

- (1) The hierarchy of space-time organization levels of complex open dynamical dissipative systems is introduced. For the sake of clarity, we consider chaotic temporal dynamics of such systems, which is expressed in terms of the measured dynamic variable V(t), where t is the time.
- (2) Information carriers in the measured chaotic series, specifically in time series V(t), are sets of different discernible irregularities bursts, jumps, and derivative discontinuities of different orders occurring at all space-time hierarchical levels of the system. The "discernibility" of the irregularities means that parameters characterizing the properties of irregularities are discernibly extracted from power spectra S(f) and difference moments $\Phi^{(p)}(\tau)$ of the order $p(p=1,2,3,\ldots)$:

$$S(f)\overline{T \to \infty} \left| \int_{-T/2}^{T/2} \langle V(t)V(t+t_1) \rangle \cdot \exp(2\pi i f t_1) dt_1 \right|$$
 (1)

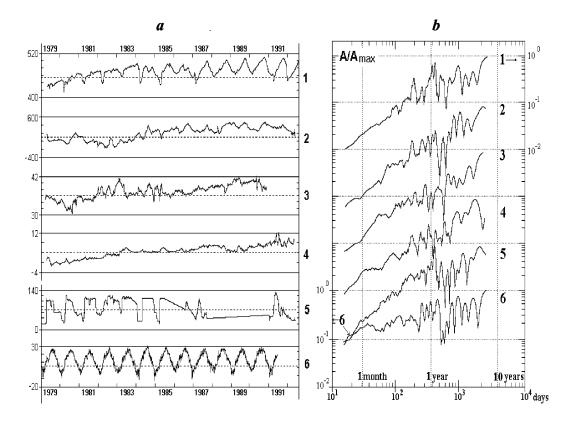


Fig. 1. The initial time series of some geophysical parameters the monitoring of which was realized at the Garm test-site in Tadjikistan (a) and normalized to the maximum amplitude spectra of the realizations filtered from a seasonal component bf (b). 1 – the potential (mV) of an electrode pair copper-lead (ECP) at the Garm geophysical observatory; 2 – the potential (mV) of the electrotelluric field (ETF) at the Khazor-Chashma observatory; 3 – apparent electrical resistivity (ρ_a , Ohm·m), measured by a method of vertical electrical sounding (VES) at AMNB array of AB = 50 m and MN = 2 m; 4 – apparent electrical resistivity (Ohm·m), measured by the VES method at AMNB array of AB = 3000 m and MN = 500 m; 5 – a series of values of the ground water level (cm) near the Khazor-Chashma observatory; 6 – a series of air temperature ($^{\circ}$ C) at the Garm observatory.

$$<(...) > \overrightarrow{T} \to \infty \frac{1}{T} \int_{-T/2}^{T/2} (...) dt,$$

$$\Phi^{(p)}(\tau) = < \left[V(t) - V(t+\tau) \right]^p > . \tag{2}$$

In this case, $\Phi^{(p)}(\tau)$ is formed only by jumps of the dynamic variable for different space-time hierarchical levels of the system, and S(f) by bursts and jumps.

(3) The "certification data", which are extracted from S(f) and $\Phi^{(p)}(\tau)$ represent correlation times and parameters characterizing a loss of "memory" (correlation); these data refer to irregularities like "bursts" and "jumps". For irregularities like "derivative discontinuities", the parameters are extracted from power spectra and difference moments based on time series of the form $\Delta_n^m V(t_k)/\Delta_n t^m (m \geq 1)$, where, $\Delta_n^m V(t_k) = \Delta_n^{m-1} V(t_k) - \Delta_n^{m-1} V(t_{k-n})$ and $\Delta_n t = (t_k - t_{k-n})$ is the sampling interval for the dynamic variable measured at points in time t_k . A degree of reproduction of values of the main "certification data" determined from the corresponding power spectra and difference moments when varying the intervals $\Delta_n t = (t_k - t_{k-n})$ serve as the adequacy criterion of these procedures of formation of different time

series.

When analyzing chaotic time series obtained in the course of experimental measurements for the different discretization frequencies, the problem of smoothing the initial series often arises. There are many ways of digitized signals filtration to separate the "low frequency" component: by the use of smoothing multinomials, wavelets, etc. We use a method of splitting the signal into the "low frequency" $V_R(t)$ and "high frequency" $V_F(t)$ components which was proposed in (Timashev and Vstovsky, 2003). In accordance with this paper, the extraction of "high frequency" components is based on a "relaxation procedure" by analogy with a solution of diffusion (heat conductivity) equation presented in the form of a finite difference equation, corresponding to a simplest explicit difference scheme of numeric solution of diffusion (heat conductivity) equation. In fact, the smoothing procedure corresponds to a sequential decrease in the gradients of local values with their mutual closing to each other in each of taken triples. In this way we obtain the "low frequency" component V_R . The use of a diffusion equation permits us to speak about "evolution of dynamic variable values" of the chaotic series under the chosen smoothing proce-

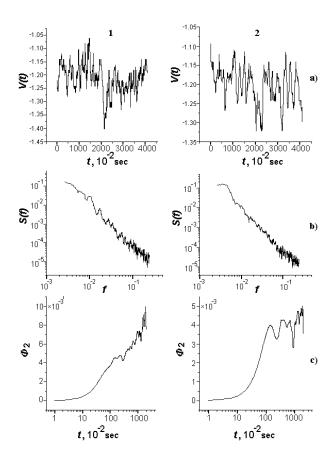


Fig. 2. An example of identification of "certification data" of a dynamic process – electric potential fluctuations in the electromembrane system – from a standard power spectrum S(f) and a difference moment of the second order Q_2 . In spite of closeness of the variation structure of two initial realizations at different time intervals (1a and 2a) and their power spectra (1b and 2b) a significant difference of the values of different moments (1b and 2b) testifying in favor of existence of differences in the process dynamics at various time intervals is observed.

dure as a realization of minimum high frequency" information in the $V_R(t)$ component. This means that a defining part of the high frequency information is contained in the function $V_F(t) \equiv V(t) - V_R(t)$. The described splitting of the initial signal V(t) into the two components $V_R(t)$ and $V_F(t)$ enables us to calculate the dependencies S(f) and $\Phi^{(p)}(\tau)$, introduced above, for each of the functions $V_J(t)(J=R, F)$ or G, where subindex G is used in the cases when the initial signal V(t) is also used for calculations. The cumulative experience of FNS analysis of chaotic signals of different phenomena clearly shows justification of its use for the search of otherwise hidden information (see examples below).

4 Results of application of FNS

An example of extraction of the "certification data" of a dynamic process (electrical potential fluctuations in electromembrane system) is given in Fig. 2. Chaotic series of two time realizations $V_i(t)$ of this process in a field of the "beyond the limits" current fixed at a distance from the surface of cation changeable membrane by two adjacent electrodes were analyzed.

The initial time series $V_i(t)$ obtained directly from the observations of the membrane potentials are shown in the upper part of the figure. The frequency of discretization of the analyzed series is 100 Hz, and the number of observations is 4096 for each series. The relations S(f) and $\Phi^{(p)}(\tau)$ for p=2, are shown in Fig. 2b and 2c. It is clear that the difference moment Φ_2 demonstrates discernibility of the measured signals to a much greater extent in comparison with the usual power spectra.

Not all the "certification data" are shown in the figure – only an illustration of selective possibilities of one of the proposed criterion Φ_2 is given compared to the standard spectral statistics S(f). Additional criterion relations make it possible not only to classify the observed signals depending on the structure of their nonstationarities, but also to evaluate dynamics of the nonstationarities in an explicit form.

While studying nonstationary processes, dynamics of S(f) and $\Phi^{(p)}(\tau)$ have been analyzed varying time with averaging interval $[k\Delta T, t_k]$ and T extension, where k = 0, 1,2,... and $t_k = T + k\Delta T$, with the constraint $T_{tot}(T + \Delta T <$ T_{tot}), T_{tot} being the total time period. The time intervals Tand ΔT should be selected on the basis of a physical sense of the problem revealing the typical time of a process which determines the most important internal structural reconstruction of the studied evolution. Therefore, if some "secondary" processes with typical times τ_i slightly influencing the main nonstationary process of the structure reconstruction occur, we have to select T so that $\tau_i \ll T$. In general, if a complex system is involved in a nonstationary evolution, it is characterized by a set of typical times T_{sr} (called times of "structural reconstruction"), for a corresponding set of scales of the system's spatial organization, and a problem of prognosis becomes multi-parametric. Therefore, not single but a number of "precursors" of a catastrophic event has to be examined. Each of these "precursors" can be revealed by analyzing the dynamic variables describing the system, selecting the averaging interval $T < T_{sr}$.

It is obvious to associate a phenomenon of a "precursor" with the sharp variations of S(f) and $\Phi^{(p)}(\tau)$ when approaching t_k to the time t_c of a catastrophic event when a reconstruction takes place at all the possible spatial scales in the system. It should to be expected, that the time t_k of the precursor should be from the moment t_c not less than the interval ΔT , i.e. $\Delta T_{cn} = t_c - t_K \geq \Delta T$, at the realization of the inequality $\Delta T_{cn} \ll T_{tot}$. In this sense we can speak about a "precursor". When revealing a "precursor", it is important to distinguish when sharp variations in S(f)and $\Phi^{(p)}(\tau)$ are caused by significant signal variations on the "front" or "back" boundary of the interval T, by approaching the "front" boundary t_k to a moment t_c of the expected event. The given problem can be solved by the analysis of the time behavior of the corresponding criteria by varying T: it is obvious that when T increases by the value ΔT_1 the

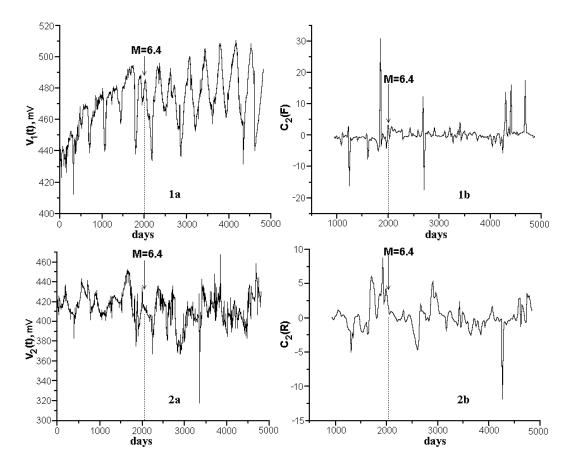


Fig. 3. An example of separation of a signal-precursor prior to a devastating Dzhirgatal earthquake with M=6.4 within the Garm test-site from the time realizations of electro-chemical potential applying the FNS analysis. 1a – the potential (mV) of an electrode pair copper-lead (ECP) at the Garm observatory (the curve 1 in Fig. 1); 2a – the same at the Khazor-Chashma observatory; 1b – a prognostic criterion C_2 filtered from a low frequency component of a high frequency residual of the initial series $V_1(t)$; 2b – the same for a low frequency component of the initial series $V_2(t)$.

nonstationarity effects associated with the signal behavior at the "back" boundary should be displayed with the same time delay ΔT_1 , when the factor display caused by sharp signal variations in the area of the front boundary does not depend so strongly on the averaged interval value.

We consider below the "precursors" based on the difference moments $\Phi^{(p)}(\tau)$, that are turned out to be more informative. The relations $\Phi^{(p)}(\tau)$ are reliably calculated in the interval $[0, \Delta T]$ with $\alpha < 0.5$. We introduce infinite relations:

$$C_{J}(t_{k+1}) = \frac{\alpha T + (k+1)\Delta T}{\int\limits_{(k+1)\Delta T} Q_{J}^{(p)}(\tau)d\tau - \int\limits_{k\Delta T} Q_{J}^{(p)}(\tau)d\tau} Q_{J}^{(p)}(\tau)d\tau} \frac{\Delta T}{T}, \quad (3)$$

where we may consider $\Phi^{(p)}(\tau)$ or their derivatives by the "delay" parameter τ calculated using the functions $V_J(t)(J=R, F \text{ or } G \text{ as } Q_J^{(p)}(\tau)$. Here we relate the last index to the initial relation V(t). The introduced relations characterize "a nonstationarity measure" of the process at the

averaging interval T by the time axis on a value ΔT , in particular, when approaching the upper boundary of the averaging time interval t_k to a moment t_c of a catastrophic event. If the processes is stationary, $C_J(t_{k+1}) = 0$.

In Fig. 3 an example is given of the identification of precursors prior to the devastating Dzhirgatal earthquake with M = 6.4 (which occurred on 26 October 1984 within the Garm test-site in Tadjikistan) using the method outlined above. The time realizations of the electrical/chemical potential (ECP) at two observation points with a sampling frequency of 1 reading per day were used as the initial time series. Here $V_1(t)$ represents the time series of the daily values of the potential (mV) of an electrode pair - copper-lead at the Garm valley observatory (Fig. 1a), and $V_2(t)$ is the potential at the Khazor-Chashma highland observatory (Fig. 2a) measured during the period 1979–1991. The time evolution of the C_2 criterion, obtained respectively for "high frequency" and "low frequency" components of the initial series $V_i(t)$, is calculated according to Eq. (3) (Fig. 1b and 2b). C_2 criterion has been calculated in a sliding time window of 846 days with a 20 days shift. In both cases we can note the presence of a signal-precursor prior to the Dzhirgatal earthquake

– a more intensive positive burst of the criterion C_2 approximately 100–150 days before the earthquake is indicated by a vertical arrow in one of the figures. The intensive negative spikes are caused by back bound effects discussed above.

It is hard to see by inspection any change in the variation of the initial series before the earthquake. So it is unlikely that any significant prognostic information could be obtained in a given case using standard methods of the initial series processing. These results confirm the good possibilities of the proposed FSN approach to reveal prognostic information from chaotic series of geophysical observations.

5 Conclusions

The method of flicker-noise spectroscopy to derive information on the dynamics of an earthquake preparation process is proposed. Integral indices of analyzed time realizations, in particular power spectra and various moments ("structural functions") of different character, are used as information relations. The method of the analysis using extended understanding of information contained in chaotic signals has shown some new approaches in the search for large earthquake "precursors". The phenomenon of a "precursor" is associated with the reconstruction of the geological medium structure at the analyzed scale, indicating that the medium is "ready" to sharp changes of its state.

The proposed general view on the development of disastrous geodynamic events corresponds to the conceptual ideas by S. Moiseev with colleagues (see Branover et al., 1999; Edelman et al., 2000). This view suggests stability loss of an open nonlinear system takes place after an increase in small-scale dynamic fluctuations determining the following growth of large-scale perturbations, i.e. an "inverse" transfer of perturbations along the scale axis takes place – from the smaller to the larger ones. In this case, the transfer of excitations from large-scale fluctuations to small-scale ones according to Richardson's hypothesis accepted in the Kolmogorov's theory of fully developed isotropic turbulence is being understood as "a direct" transfer.

It should be noted that we can also arrive at a conclusion on information-significant increase in small-scale ("high frequency") dynamic fluctuations in an open system prior to large-scale ("low frequency") structure transformation from the computer analysis of a dynamics of structure rearrangements in complex systems just before phase transitions. It was demonstrated, in particular, in (Klochikhin et al., 2000).

It is obvious that more detailed information on the geological medium conditions before a large earthquake may be obtained if multi-point simultaneous measurements of dynamic variables $V_i(t)$ (i=1,2,.....N) are realized in the area of an impending earthquake. The analysis of multi-point correlations permits one to make conclusions on spatial/temporal dynamics of realized correlations in energetically stimulated geophysical medium – on the direction of stimulation transfer between the regions in which the signals $V_i(t)$ are registered.

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References

- Branover, H., Eidelman, A., Golbraikh, E., and Moiseev, S.: Turbulence and structures, chaos, fluctuations and helical self-organizing in nature and the laboratory, Academic Press, San Diego, USA, 1999.
- Deshcherevsky, A. V., Lukk, A. A., and Sidorin, A. Ya.: Flickernoise structure in the time realizations of geophysical fields, Izvestia, Phys. Solid Earth, 33, 7, 515–529, 1997.
- Deshcherevsky, A. V., Lukk, A. A., and Sidorin, A. Ya.: Evidences of self-organization in geophysical fields' temporal variations, Phys. Chem. Earth (A), 25, 12, 775–779, 2000.
- Edelman, A., Branover, H., Moiseev, S. S., and Gordienko, S.: Large-scale turbulence universality and the study of extreme weather events, Phys. Chem. Earth (B), 25, 1, 35–38, 2000.
- Frisch, U.: Turbulence, The legacy of A. N. Kolmogorov, Cambridge University Press, 1995.
- Hayakawa, M.: Electromagnetic phenomena associated with earthquakes, Bull. Univ. Electro-Communications, 13-1, 1-6, 2000.
- Klochikhin, V. L., Lakeev, S. G., Timashev, S. F., and Zaripov, R. V.: Fluctuation dynamics of the Ising two-dimensional spin model, Russian Journal of Physical Chemistry, 74, 1, S13–S19, 2000.
- Lukk, A. A., Deshcherevsky, A. V., Sidorin, A. Ya., and Sidorin, I. A.: Variation of geophysical fields as a manifistation of determinate chaos in fractal medium, Moscow, UIPE Rus. Acad. Sci., (in Russian), 210, 1996.
- Sadovsky, M. A. and Pissarenko, V. F.: Randomness and instability in geophysical processes, Izvestiya, Earth Physics, 25, 2, 79–85, 1989.
- Scholz, C. H.: Earthquakes and faulting: Self-organized critical phenomena with a characteristic dimension, in: Spontaneous Formation of Space-Time Structures and Criticality, (Eds) Riste, T. and Sherrington, D., Kluwer Dordrecht, 41–56, 1991.
- Telesca, L., Cuomo, V., and Lapenna, V.: A new approach to investigate the correlation between geoelectrical time fluctuations and earthquakes in a seismic area of southern Italy, Geophys. Research Lett., 28, 23, 4375–4378, 2001.
- Timashev, S. F.: Flicker-Noise spectroscopy in analysis of chaotic fluxes in distributed dynamical dissipative systems, Russian Journal of Physical Chemistry, 75, 10, 1742–1749, 2001a.
- Timashev, S. F.: Flicker-noise spectroscopy as a tool for analysis of fluctuations in physical systems, in: Noise in Physical Systems and 1/f Fluctuations ICNF 2001, (Ed) Bosman, G., World Scientific, New Jersey-London, 775–778, 2001b.
- Timashev, S. F. and Vstovsky, G. V.: Flicker-Noise Spectroscopy in analysis of chaotic time series of dynamical variables and a problem of signal – noise relation, Russian J. of Electrochemistry, 39, 1, 2003.
- Turcotte, D. L.: Crustal deformation and fractals, a review, in: Fractals and dynamic systems an geoscience, (Ed) Kruhl, J. H., 7–23, 1994.