Half-order modelling of electrical networks Application to stability studies

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Abstract: The fractional order systems has been used successfully to model in frequency domain the development of induced currents in electric machines, especially in the dampers or massive parts of large synchronous machines. The objective of this work consists in extending the field of application of fractional order models for small signal stability studies of an electrical network.

1. INTRODUCTION

For several years, a very close attention has been paid to the study of the electrical networks in order to improve their sizing, their quality, their safety and their performances. Indeed, the electrical networks become more and more complex, including an increasing number of power electronics converters, which are used as interface or control devices (Barruel, 2005, Wildrick *et al.,* 1993).

To bring solutions to the problems caused by this new context of electrical networks is a difficult task. For example, an accurate modelling over a large frequency range is delicate and generally leads to the simulation of huge systems, with a great number of parameters (Kundur, 1994). As a result, reduction techniques of model size are often required. Several works on the reduced modelling of electrical machines concern the use of the fractional order derivation (Riu, 2001). They lead to the development of models which are accurate and of reduced order. The resulting models are especially dedicated to dynamic studies of power networks.

In this paper, the authors have focused on the study of small signal stability of an on-board electrical network or any electrical systems in which the synchronous machine are modelled by half-order systems. We will compare two classical stability methods commonly used in power systems studies, one based on modal approach and another based on Nyquist criterion. For both approaches, the synchronous machine is modelled using a non integer order system.

2. HALF-ORDER MODELLING OF A SYNCHRONOUS MACHINE

2.1 Non integer order systems

The non-integer order derivation is applied since several years in many scientific fields as rheology, chemistry, signal processing, control or electromagnetic computation. It is used for an accurate modelling of some physical phenomena like diffusion, fractal structures (Oldham, *et al.,* 1974) or distributed parameter systems which are represented by partial differential equations. It is also the base of a particular technique of robust control (Oustaloup, 1995).

A non-integer order system is characterized by the presence of a non-integer order operator. In frequency domain, the system is characterized by a frequency power law. For instance, half-order systems vary with the square root of frequency.

2.2 Non integer order model of synchronous machine

It has been shown that the diffusion phenomena in alternator damping bars (Fig. 1) can be modelled by a half-order resistive system.

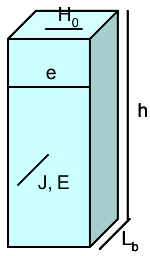


Fig. 1. Representation of a damper bar.

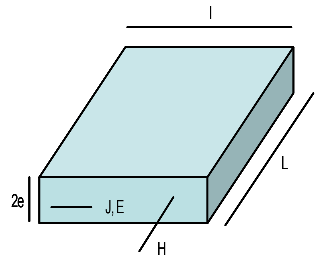


Fig. 2. Representation of a solid ferromagnetic piece.

The impedance of a rectangular bar can be expressed as a half-order system (Retière *et al.*, 1999):

$$\overline{Z}^{1/2}_{resistive} = R_o\sqrt{1+j\frac{w}{w_o}} \qquad (1)$$

where $R_o = \dfrac{L_b}{s \cdot h \cdot e}$ and $w_o = \dfrac{1}{s \cdot m \cdot h^2}$

Fig. 3 shows the relevance of this model compared with the analytical expression of the impedance (Alger, 1970):

$$\overline{Z}_{ana} = R_o \cdot \frac{a}{th(a)} \text{ with } a^2 = jwsmh^2 \qquad (2)$$

In the same way, it has been shown that the impedance characterising a solid ferromagnetic piece (Fig. 2) can be expressed as a half-order inductive system:

$$\overline{Z}^{1/2}_{inductive} = \frac{jwL_o}{\sqrt{1+j\frac{w}{w_o}}} \qquad (3)$$

where $L_o = m\frac{2\cdot e\cdot l}{L}n^2$ and $w_o = \frac{1}{s\cdot m\cdot e^2}$ (n is the number of coil turns surrounding the sheet).

The parameters R_o, L_o and ω_o depend on the geometrical and physical data (Riu, 2001).

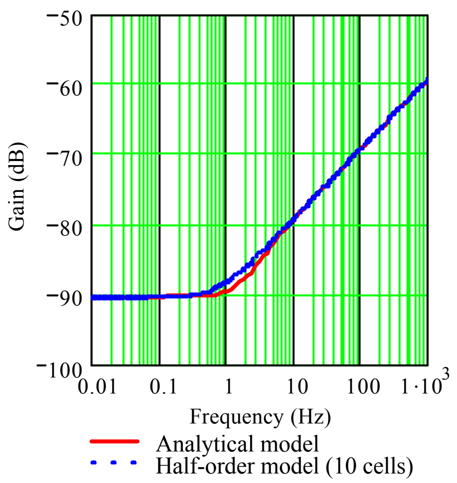


Fig. 3. Comparison between the analytical and half order approximation of the damper bar impedance.

In order to take into account the frequential effects of the induced currents in the synchronous machine, a new equivalent circuit in the Park reference frame is established from the following physical considerations:

- no induced currents expand in the armature windings for frequencies lower than 1 kHz: it is modelled by a resistance (r_s) and an inductance (l_s);.
- the inductance l_{ad}, corresponding to the stored energy in the air gap, is supposed to be constant versus frequency;
- due to the development of induced currents in the massive parts of the rotor, an “inductive” half-order impedance is added in parallel to l_{ad}: $\overline{Z}^{1/2}_{1d/q}(w) = \frac{jwL_{1d/q}}{\sqrt{1+j\frac{w}{w_{1d/q}}}}$
- eddy currents are neglected in the excitation winding, which is therefore modelled by a resistance r_f and an inductance l_f, both constant versus the frequency;
- a constant inductance l_{12d} is associated to the mutual field between the exc itation and the damper windings;
- the damper windings are modelled by a “resistive” half-order impedance: $\overline{Z}^{1/2}_{2d/q}(w) = R_{2d/q}\sqrt{1+j\frac{w}{w_{2d/q}}}$
- .the effect of speed variation on stator voltage are given by $\omega\cdot\lambda_d$ in d axis and $\omega\cdot\lambda_q$ in q axis.

The resulting “half-order” equivalent circuit of a synchronous machine in the d- and q-axes is shown in (Fig. 4) and (Fig. 5) for steady-state conditions.

It should be noted that -contrary to integer models of such alternators- each parameter has a physical signification.

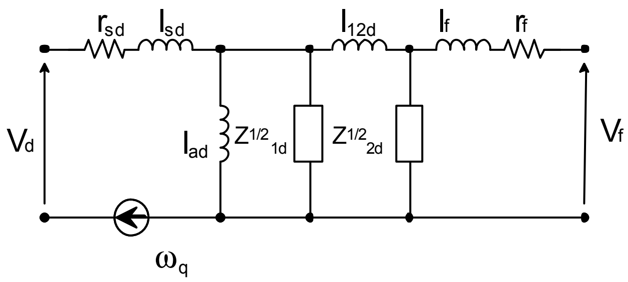


Fig. 4. Synchronous machine, direct axis equivalent half-order circuit

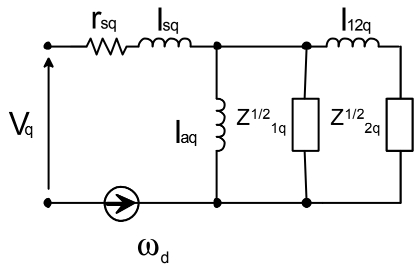


Fig. 5. Synchronous machine, quadrature axis equivalent half-order circuit

These equivalent circuits have been identified and compared to traditional integer models (Riu, 2001); their principal properties relate to their compactness (a model of a non integer order includes twice less parameters than a traditional integer model), their precision over a large frequency range for this kind of system (until approximately 1000 Hz) and their physical signification as each parameter can be linked directly to geometrical dimensions and physical characteristics of the machine.

3. SMALL SIGNAL STABILITY ANALYSIS OF A NON-INTEGER ORDER SYSTEM

For an alternator, it is possible to study his voltage stability, speed (or frequency) stability and rotor angle stability (Kundur, 1994):

- voltage stability refers to the ability of a power system to maintain voltage at all buses in the system after a perturbation from a given initial condition;
- frequency stability refers to the ability of a power system to maintain frequency following a severe system disturbance resulting in a significant imbalance between generation and load;
- rotor angle stability refers to the ability of synchronous machines to remain in synchronism after being subjected to a disturbance.

Disturbances may be small or large, leading to the analysis of small-signal or large-signal stability. In this article, the authors are just interested in the study of small-signal voltage stability of an electrical system. The small-signal conditions enable us to work directly on a linearized model of the system around the operating point.

As mentioned above, we have been interested in the stability study of an electrical network. Sources of instability are related to interactions between devices connected to the network (electrical motors, generators, power converters, filters). For stability aspects, power converters are crucial devices. Indeed, their control mode is so that they behave as constant power loads. From a small-signal point of view, it means that a power converter can be represented by a negative resistance, which is typical of an unstable load.

In this paper, a basic power system as described in figure 6 is analyzed. It includes a synchronous generator and a constant power load. The alternator is supposed driven by a constant speed turbine. For simplicity purpose, the voltage alternator is not controlled. This simplification does not change the relevancy of the methodology given here after.

3.1 Frequency approach for stability analysis

Stability analysis of an electrical system can be based on an impedance method (Sudhoff *et al.*, 2000). The most straightforward application of the impedance method to power systems is called "Middlebrook criterion" (Middlebrook, 1979). According to this criterion, a system is stable if the Nyquist contour of the ratio Z_{source}/Z_{load} stays within the unit circle, where Z_{source} is the alternator impedance and Z_{load}, the constant power load impedance (Fig. 6). The transmission line is included in the source impedance. All these impedances are analytically calculated using Thevenin theorem. The synchronous machine being modelled by two equivalent half-order circuits, one in d-axis and the other in q-axis, we need to apply the Middlebrook criterion in both axes.

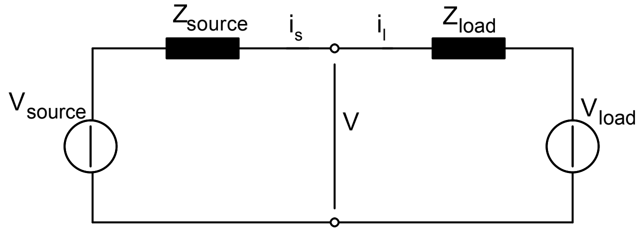


Fig. 6. Thevenin equivalent circuit of the studying system.

3.2 Modal approach in stability analysis

From the electrical equation given by the equivalent half-order circuits (Fig. 4 and Fig. 5), we can build a generalised state system including the non integer order derivation of state variables. This system equation can be written as follows (Oustaloup, 1995):

$$[\underline{x}]^{(n)} = [A]\cdot[\underline{x}] + [B]\cdot[\underline{u}] \tag{4}$$

where n is a real number, [A] the state matrix, [x] the state vector, [B] the input matrix and [u] the input vector.

If n is equal to unity, we find the well-known expression of a classical state system. From this relation, it is possible to calculate the eigenvalues and their associated vectors (right and left vectors). For a non-integer order system with a fractional order between 0 and 1, the small signal stability condition is given by the following relation (Matignon, 1996):

$$\arg(l_i) \geq \frac{n\cdot p}{2} \tag{5}$$

This result is well-known for n=1. Its geometric translation for n=1/2 is given in Fig. 7.

3.3 Application to electric power systems

Frequency approach. Considering a power system with a constant power load and a synchronous machine modelled using half-order systems in d- and q-axis, the Middlebrook criterion is applied. According to this criterion the transfer function T of the system is given by:

$$T(s) = 1 + \frac{Z_{source}}{Z_{load}} \tag{6}$$

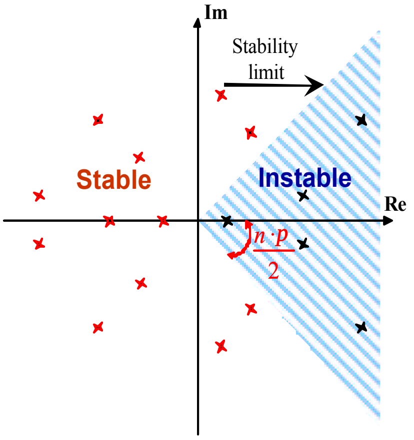


Fig. 7. Stability/instability area for a half-order system.

Modal approach. For the same power system, a generalised state system including the non integer order derivation of state variables is calculated from the differential equations describing the system :

$$\begin{cases} V_f = -r_f \cdot i_f - l_f \cdot s \cdot i_f - l_{ad} \cdot s \cdot \left(i_d + i_f + i_{1d} + i_{2d}\right) \\ \quad - l_{12d} \cdot s \cdot \left(i_f + i_{2d}\right) \\ 0 = -R_{2d} \cdot i_{2d} - \dfrac{R_{2d}}{\sqrt{w_{2d}}} s^{1/2} \cdot i_{2d} - l_{ad} \cdot s \cdot \\ \quad \cdot \left(i_d + i_f + i_{1d} + i_{2d}\right) - l_{12d} \cdot s \cdot \left(i_f + i_{2d}\right) \\ 0 = -L_{1d} \cdot s \cdot i_{1d} - l_{ad} \cdot s \cdot \left(i_d + i_f + i_{1d} + i_{2d}\right) - \\ \quad - \dfrac{l_{ad}}{\sqrt{w_{1d}}} \cdot s^{3/2} \cdot \left(i_d + i_f + i_{1d} + i_{2d}\right) \\ V_d = -r \cdot i_d - l_d \cdot s \cdot i_d - l_{ad} \cdot s \cdot \left(i_d + i_f + i_{1d} + i_{2d}\right) \\ V_q = -r \cdot i_q - l_q \cdot s \cdot i_q - l_{aq} \cdot s \cdot \left(i_q + i_{1q} + i_{2q}\right) \\ 0 = -r_{2q} \cdot i_{2q} - l_{2q} \cdot s \cdot i_{2q} - l_{aq} \cdot s \cdot \left(i_q + i_{1q} + i_{2q}\right) \\ 0 = -L_{1q} \cdot s \cdot i_{1q} - l_{aq} \cdot s \cdot \left(i_q + i_{1q} + i_{2q}\right) - \\ \quad - \dfrac{l_{aq}}{\sqrt{w_{1q}}} \cdot s^{3/2} \cdot \left(i_q + i_{1q} + i_{2q}\right) \end{cases} \tag{7}$$

Introducing $\overline{Z}^{1/2}_{1d/q}$ and $\overline{Z}^{1/2}_{2d/q}$ into (7), the generalised state system can then be written as :

$$\begin{cases} x_1^{(1/2)} = x_2; \\ x_2^{(1/2)} = a_{2,1} \cdot x_1 + a_{2,5} \cdot x_5 + a_{2,6} \cdot x_6 + a_{2,7} \cdot x_7 + a \cdot V_d; \\ x_3^{(1/2)} = x_4; \\ x_4^{(1/2)} = a_{4,3} \cdot x_3 + a_{4,5} \cdot x_5 + a_{4,6} \cdot x_6 + b \cdot V_f; \\ x_5^{(1/2)} = x_6; \\ x_6^{(1/2)} = a_{6,1} \cdot x_1 + a_{6,3} \cdot x_3 + a_{6,5} \cdot x_5 + a_{6,6} \cdot x_6 + a_{6,7} \cdot x_7 + c \cdot V_d + d \cdot V_f; \\ x_7^{(1/2)} = a_{7,1} \cdot x_1 + a_{7,2} \cdot x_2 + a_{7,3} \cdot x_3 + a_{7,5} \cdot x_5 + a_{7,6} \cdot x_6 + a_{7,7} \cdot x_7 \\ \quad + e \cdot V_d + f \cdot V_f + j \cdot V_d^{(1/2)}; \\ x_8^{(1/2)} = x_9; \\ x_9^{(1/2)} = a_{9,8} \cdot x_8 + a_{9,10} \cdot x_{10} + a_{9,12} \cdot x_{12} + g \cdot V_q; \\ x_{10}^{(1/2)} = x_{11}; \\ x_{11}^{(1/2)} = a_{11,8} \cdot x_8 + a_{11,10} \cdot x_{10} + a_{11,12} \cdot x_{12} + h \cdot V_q; \\ x_{12}^{(1/2)} = a_{12,8} \cdot x_8 + a_{12,9} \cdot x_9 + a_{12,10} \cdot x_{10} + a_{12,11} \cdot x_{11} + a_{12,12} \cdot x_{12} + l \cdot V_q; \end{cases} \tag{8}$$

3.4 Comparison between modal and frequency approaches

The studied system is composed by a (1101 MVA, 22 kV, 60 Hz) synchronous machine and a constant power load of 650 MW.

In Fig. 8.a and 8.b, we have represented the frequency evolution of the d-axis operational inductance using SSFR tests given in (Canay, 1993), as well as inductances resulting from the classical and fractional modelling. It should be also specified that implicit half order systems has been approximated by explicit half-order systems, thanks to the induced simplifications to construct the generalized state-space system.

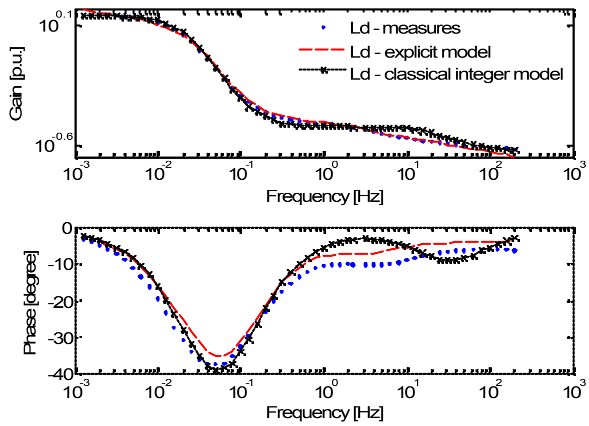


Fig. 8.a. D-axis operational inductance identification.

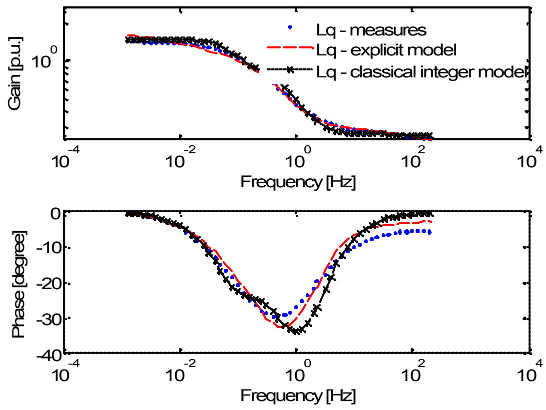


Fig. 8.b. Q-axis operational inductance identification.

Fig. 9 represents the poles of the system calculated from the state-space system and the transfer function T. They are of course strictly superposed.

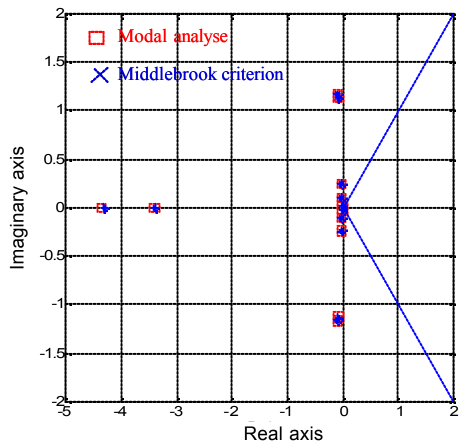
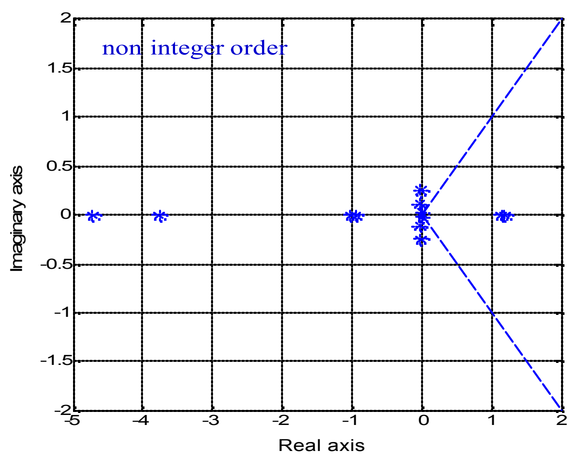


Fig. 9. System poles given by modal analysis and Thevenin-based impedance approach

In Fig. 10, poles are also given for an equivalent integer-order model of the system (Riu, 2001). Both cases show that the system is unstable because poles are located in the unstable area. This is due to the constant power load.



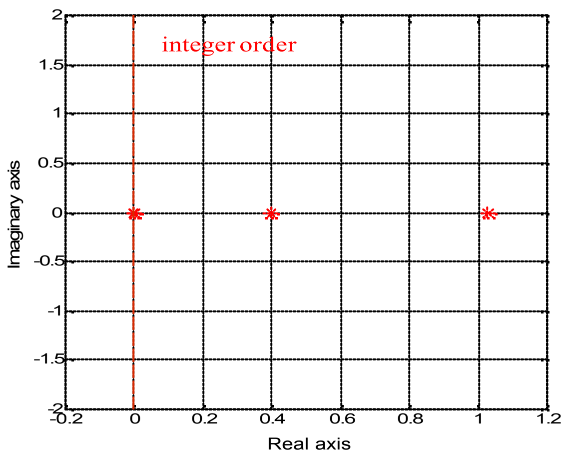


Fig. 10. Poles migration in case of instability.

Using the Middlebrook criterion, e.g. by examining the Nyquist contour of the transfer function Z_{source}/Z_{load}, we can also show that the system is unstable if the load is a constant power load. If the load is a constant impedance one, the system is stable. Nyquist diagrams are presented in (Fig. 11) for d-axis and in (Fig. 12) for q-axis.

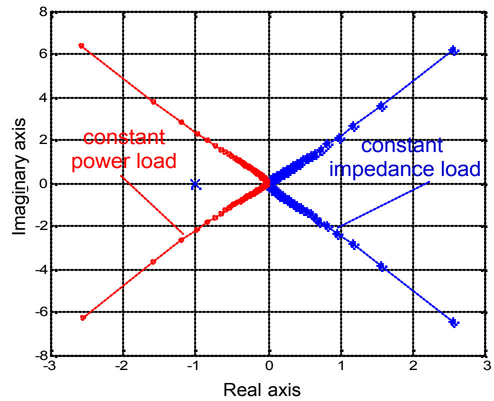


Fig. 11. d-axis Nyquist diagram :

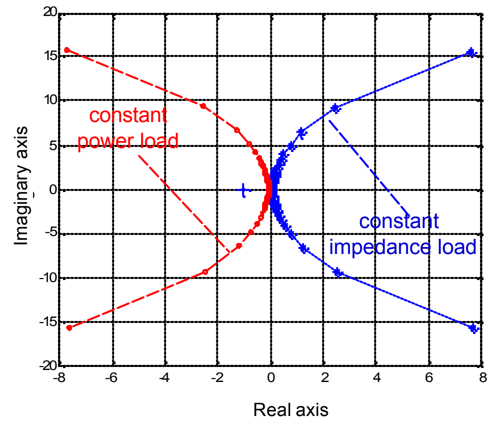


Fig. 12. q-axis Nyquist diagram

4. CONCLUSIONS

In this paper, we have modelled a synchronous machine using non-integer order systems. These systems allowed to carry out equivalent circuits in d- and q-axis which are accurate and compact over a large frequency range. This modelling has been used for the stability study of a system connecting a synchronous machine and a constant power load.

It has been showed that by generalizing classical integer order methods for stability study (i.e. frequency approach by Nyquist criterion and modal approach), it was possible to predict the stability.

Prospects of this work concern the computation of participation factors. These factors should give us an idea of the links between dynamic performances of the system and the constructive parameters of the systems.

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