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## Contact impact elements

Hachmi Ben Dhia

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## CONTACT IMPACT ELEMENTS

**H. Ben Dhia** *Laboratoire de Mécanique des Sols Structures et Matériaux, Ecole Centrale de Paris, UMR 85 89, Grande Voie des Vignes, 92 295 Chatenay Malabry Cedex. Téléphone : 01 41 13 13 56, Télécopie : 01 41 13 14 42 Adresse(s) électronique(s) : bendhia@mssmat.ecp.fr*

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### 1 INTRODUCTION

Partant du continu pour arriver au discret, nous essayons d'éclairer les rôles joués par les différents composants participant de l'élaboration d'éléments numériques de contact en dynamique, dans le cadre des transformations mécaniques finies. Cette démarche permet, non seulement d'avoir un référent utile en cas de besoin d'adaptativité, mais également de retrouver en les consolidant, les éléments de contact de type Noeud à Noeud, Noeud à facette et facette à facette, éléments qui ont été directement (et progressivement) développés au niveau discret (éléments finis) par une large communauté en mécanique numérique du contact.

Les modèles et formulations continus classiques de ces problèmes (e.g. Duvaut et Lions, 1972, Kikuchi et Oden, 1988, Wriggers, 2002, Laursen, 2002) sont brièvement évoqués dans la deuxième section. Un accent particulier est mis sur une formulation "généralisante" que nous qualifions de *lagrangienne stabilisée*. Cette formulation, mentionnée brièvement dans Ben Dhia et Zammali (2004) fera l'objet final de la section 2.

Le problème adossé à cette formulation que nous adoptons, est mixte et comporte des Champs de Signes inconnus (Ben Dhia, 1988). Ces champs sont ceux intrinsèques à la mécanique du contact. Ils en relèvent les inconnues géométriques que sont les interfaces effectives de contact.

Après avoir discuté les difficultés liées à l'approximation numérique de ce problème, utilisé comme "paradigme" aux problèmes de contact, des outils de modélisation numérique (éléments et particules finis dans les cas des maillages incompatibles, en particulier (Ben Dhia et Zarroug, 2002)) sont proposés et analysés dans la section 3. Dans un souci de clarté, les développements sont réalisés en négligeant les frottements; l'extension au frottement étant brièvement commentée en avant dernière section.

Les différents points seront éclairés par des tests numériques (Ben Dhia et Zarroug 2002, Ben Dhia et Zammali 2004, pour l'essentiel), développés et obtenus dans Code-Aster d'EdF. Nous relèverons et illustrerons enfin l'apport de l'approche multi-échelle Arlequin (Ben Dhia, 1998 et 1999) au traitement du caractère multi-échelle de l'impact par couplage de modèles, pouvant explorer le couplage dynamique moléculaire/MMC (Ben Dhia, 2004).

### 2 A STABILIZED LAGRANGIAN FORMULATION OF CONTACT PROBLEMS

For clarity, the frictionless static contact problem is considered here. After the introduction of some basic notations, the Virtual Work Principle (VWP) is formulated for a system of two solids which may come dynamically into contact with each other in the finite transformations framework. In the VWP, and as classically done (e.g. Klarbring, 1995) we use the Action and Reaction Principle, extended to the case where the contact is not effective on the potential contact interface by defining pairing mappings. Finally, by using fields of Level-Sets type (Ben Dhia, 1988), the Signorini-Moreau dynamic contact laws are written merely in terms of equations standing for boundary conditions. This allows a straightforward derivation of, first a lagrangian formulation, and second a stabilized lagrangian one, generalizing both the lagrangian and the augmented lagrangian formulations. The main interests of this new formulation of contact problems are listed.

## 2.1 Introduction of the problem, hypotheses and notations

We consider the problem of dynamic contact between two deformable solids  $S^1$  and  $S^2$  occupying the closures of the domains  $\Omega_0^1$  and  $\Omega_0^2$  of  $R^d$ , ( $d = 2,3$ ) in their reference respective configurations and the closures of  $\Omega_t^1$  and  $\Omega_t^2$  of  $R^d$  in their current ones (see figure 1). The deformation  $\varphi^i$  of the solid  $S^i$  is an application (assumed to be sufficiently regular), defined from  $\Omega_0^i \times I$  into  $(\Omega_t^i)_{t \in I}$ , with  $I = ]0, T[$  standing for the time interval of study of the mechanical problem.

The space  $R^d$  is supposed to be endowed with an orthonormal basis  $(\mathbf{e}_1, \dots, \mathbf{e}_d)$ .

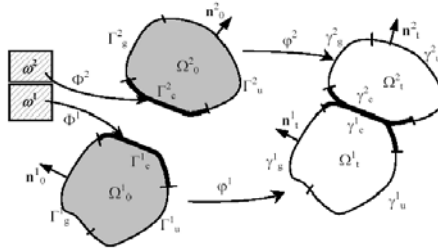


FIG. 1 – The contact problem

The boundary of  $\Omega_0^i$  is divided into  $\Gamma_u^i$ , the part on which the displacements are prescribed,  $\Gamma_g^i$  the one on which the surface loads are given and  $\Gamma_c^i$  the final part on which contact may occur we call the potential contact surface. These three parts are assumed to make a partition of the boundary in the classical sense. The deformed boundary portions, assumed to constitute a partition of the boundary of  $\Omega_t^i$  are denoted by  $\gamma_u^i$ ,  $\gamma_g^i$  and  $\gamma_c^i$ . Moreover, the potential contact surfaces are assumed to be parametrized *via* two regular mappings, denoted by  $\Phi^i$  and defined from given domains  $\omega^i$  included in  $R^{d-1}$  into  $R^d$  (see figure 1). The classical outward unit normals to the boundaries of the solids  $S^i$  are denoted by  $\mathbf{n}_0^i$  and  $\mathbf{n}_t^i$  in the reference and current configurations, respectively.

## 2.2 Virtual Work Principle

For clarity and without restriction, we assume that the applied surface loads are equal to zero on  $\gamma_g^i$  and neglect the body forces. The VWP reads then for each time  $t \in I$ :

Find  $\mathbf{u} = (\mathbf{u}^1, \mathbf{u}^2) \in \mathbf{CA}_u = \mathbf{CA}_u^1 \times \mathbf{CA}_u^2$ ;  $\forall \mathbf{w} = (\mathbf{w}^1, \mathbf{w}^2) \in \mathbf{CA}_u$

$$\sum_{i=1}^2 \int_{\Omega_0^i} \rho_0^i \ddot{\mathbf{u}}^i \cdot \mathbf{w}^i d\Omega_0^i + \sum_{i=1}^2 \int_{\Omega_0^i} \text{Tr}[\Pi^i(\mathbf{u}^i)(\nabla_p(\mathbf{w}^i))^T] d\Omega_0^i - \int_{\Gamma_c} \mathbf{R} \cdot [[\mathbf{w}]] d\Gamma_p = 0 \quad (1)$$

In system (1),  $\mathbf{CA}_u^i$ ,  $i = 1,2$ , denotes the space of kinematically admissible fields,  $\mathbf{u}^i$  and  $\ddot{\mathbf{u}}^i$  are the displacement and acceleration fields,  $\Gamma_c (= \Gamma_c^1)$  is the potential contact “slave” surface,  $\rho_0^i$  denotes the mass density in the reference state of solid  $S^i$  and  $\Pi^i$  is the first Piola-Kirchhoff stress tensor defined in  $\Omega_0^i$ . The nominal vector valued unknown density of contact forces is denoted by  $\mathbf{R} (= \mathbf{R}^1)$ . This density of forces is experienced by solid  $S^1$  from solid  $S^2$ . Moreover, the Action and Reaction Principle is used. In the present context, this principle is extended as following:

$$\mathbf{R}^1(\mathbf{p}, t) = -\mathbf{R}^2(\bar{\mathbf{p}}(t), t) \quad \text{a.e. } (\mathbf{p}, t) \in \Gamma_c \times I \quad (2)$$

where  $\bar{\mathbf{p}}(t)$  is, for each  $t > 0$ , a point belonging to the “master” reference surface  $\Gamma_c^2$ , paired with the point  $\mathbf{p}$  of  $\Gamma_c (= \Gamma_c^1)$  by coupling-like applications of proximity type (e.g. Curnier and Alart, 1988), or, more generally, by using any admissible direction along which the nearest point of  $\Gamma_c^2$  to  $\mathbf{p}$  is found (Ben Dhia and Durville, 1995, Ben Dhia et al., 2000). Notice here that the pairing applications are defined from the moving surface  $\gamma_c^1$  into the moving surface  $\gamma_c^2$  then transported back, at each time, to the reference contact interfaces with the help of the deformation.

With these pairing applications, a (time dependent) jump-like field is defined on  $\Gamma_c$  as follows.

For each  $\mathbf{w} = (\mathbf{w}^1, \mathbf{w}^2) \in \mathbf{CA}_u$ ,

$$[[\mathbf{w}]](\mathbf{p}) = \mathbf{w}^1(\mathbf{p}) - \mathbf{w}^2(\bar{\mathbf{p}}(t)) \quad \text{a.e. } (\mathbf{p}, t) \in \Gamma_c \times \mathbf{I} \quad (3)$$

This field is used in (1).

System (1) has now to be completed by material behaviour laws, initial conditions and contact laws. For this purpose, an hyperelastic behaviour is assumed (for clarity) for the constitutive materials of the considered solids. That is,

$$\Pi^i = \rho_0^i \frac{\partial W^i}{\partial \mathbf{F}^i} \quad (4)$$

where  $W^i$  is a local internal elastic energy per unit mass and  $\mathbf{F}^i$  is the deformation gradient tensor. The initial conditions are the following:

$$\mathbf{u}^i(\mathbf{p}, 0) = \mathbf{u}^{i0}, \quad \mathbf{v}^i(\mathbf{p}, 0) = \mathbf{v}^{i0} \quad \text{a.e. } \mathbf{p} \in \Omega_0^i \quad (5)$$

where  $\mathbf{u}^{i0}$  and  $\mathbf{v}^{i0}$  are given.

Still the mechanical problem is not complete since the contact actions are not yet defined. This is the object of the following subsection.

### 2.3 Contact laws

We denote by  $\mathbf{n}$  the unit inward normal to the “master” current surface transported back to the reference one. More precisely,

$$\mathbf{n}(\mathbf{p}, t) = -\mathbf{n}_t^2(\varphi^2(\bar{\mathbf{p}}(t), t)) \quad \text{a.e. } (\mathbf{p}, t) \in \Gamma_c \times \mathbf{I} \quad (6)$$

Now, using a classical decomposition of the nominal contact density of loads  $\mathbf{R}$ , we set:

$$\mathbf{R}(\mathbf{p}, t) = \lambda(\mathbf{p}, t)\mathbf{n}(\mathbf{p}, t) + \mathbf{R}_\tau(\mathbf{p}, t) \quad \text{a.e. } (\mathbf{p}, t) \in \Gamma_c \times \mathbf{I} \quad (7)$$

where  $\mathbf{R}_\tau$  refers to the density of tangential contact loads and  $\lambda$  is the normal contact pression. These fields are defined by means of interface laws. Signorini and Coulomb’s models are used in the sequel and one of our key points is the equivalent setting of the latters in terms of equations in which *Level-Sets* are introduced as intrinsic contact unknown fields. Notice that for clarity reasons, we first neglect friction phenomena.

#### 2.3.1 The classical Signorini model - The displacement-based Signorini contact conditions read:

$$d_n(\mathbf{p}, t) \leq 0 \quad \text{a.e. } (\mathbf{p}, t) \in \Gamma_c \times \mathbf{I} \quad (8)$$

$$\lambda(\mathbf{p}, t) \leq 0 \quad \text{a.e. } (\mathbf{p}, t) \in \Gamma_c \times \mathbf{I} \quad (9)$$

$$d_n(\mathbf{p}, t)\lambda(\mathbf{p}, t) = 0 \quad \text{a.e. } (\mathbf{p}, t) \in \Gamma_c \times \mathbf{I} \quad (10)$$

where  $d_n$  denotes the signed distance defined by:

$$d_n(\mathbf{p}, t) = (\varphi^1(\mathbf{p}, t) - \varphi^2(\bar{\mathbf{p}}(t), t)) \cdot \mathbf{n}(\mathbf{p}, t) \quad \text{a.e. } (\mathbf{p}, t) \in \Gamma_c \times \mathbf{I} \quad (11)$$

2.3.2 A level-Set based Signorini model - Denoting by  $\Gamma_c^{eff}(t)$  the subset of points of  $\Gamma_c$  on which the contact is effective at a given time  $t$ , the classical Signorini conditions can equivalently be written as:

$$d_n = 0 \text{ and } \lambda \leq 0 \quad \text{on } \Gamma_c^{eff}(t) \quad \text{a.e. } t \in \mathbf{I} \quad (12)$$

$$d_n < 0 \text{ and } \lambda = 0 \quad \text{on } \Gamma_c \setminus \Gamma_c^{eff}(t) \quad \text{a.e. } t \in \mathbf{I} \quad (13)$$

By introducing a *Sign*-like field, as done by Ben Dhia (1988) for a penalized unilateral contact model, the conditions above can equivalently be written as alternative of interface transmission conditions. Indeed, if:

$$S = 1_{R^-}(\lambda - k_1 d_n) \quad \text{on } \Gamma_c \times \mathbf{I} \quad (14)$$

where, for a set  $K$ ,  $1_K$  is the indicator function of  $K$  ( $1_K(x) = 1$  if  $x \in K$  and  $0$ , otherwise),  $k_1$  is any strictly positive real parameter (homogeneous to a spring rigidity), then the local system (12, 13) can be written as:

$$S d_n = 0 \quad \text{on } \Gamma_c \times \mathbf{I} \quad (15)$$

$$(1 - S)\lambda = 0 \quad \text{on } \Gamma_c \times \mathbf{I} \quad (16)$$

Equation (15), both with the definition (14) of  $S$ , is an interface normal kinematic continuity condition on  $\Gamma_c^{eff}$ , whilst equation (16) is a free static normal boundary condition that imposes  $\lambda = 0$  on  $\Gamma_c \setminus \Gamma_c^{eff}$ . The unknown *Sign*-like field  $S$ , defined on the whole potential contact zone  $\Gamma_c$  can also be seen as a *Level-Set* field; the unknown effective contact zone corresponding to the iso-1 *Level-Set*. The fact that in our setting of the contact laws only equalities are involved is to be underlined. Another aspect intimately linked to the definition (14) of the *Level-Set* field  $S$  is that the (alternative)-interface conditions (15) and (16) can be gathered in only one interface equation. Indeed by introducing another homogenization parameter  $k_2 \in R^*$ , one can write the Signorini contact laws as follows.

$$S - 1_{R^-}(\lambda - k_1 d_n) = 0 \quad \text{on } \Gamma_c \times \mathbf{I} \quad (17)$$

$$S d_n + \frac{(1 - S)}{k_2} \lambda = 0 \quad \text{on } \Gamma_c \times \mathbf{I} \quad (18)$$

A straightforward weighted residual system can be derived from (17) and a first global mixed continuous frictionless dynamic formulation of the contact problem could be obtained. However, it is well known that, due to shocks, this formulation would be a formal one and in practice, the numerical fields which can be calculated by classical numerical schemes show spurious oscillations (see numerical section). More or less *a priori* or *a posteriori* physically-based treatments have been designed by several authors to attenuate or kill these numerical oscillations (eg. Hughes et al. (1976), Armero and Petocz, 1998, Vola et al. (1998)). Here, by following the ideas of Moreau (1988), the Signorini conditions are written in terms of placement and relative velocities of the contact surfaces.

## 2.4 The Signorini-Moreau model

Let us assume that at a given time  $t = t_0 \in \mathbf{I}$ , the Signorini displacement-based contact conditions are satisfied. That is,

$$d_n(\mathbf{p}, t_0) \leq 0 \quad \text{a.e. } \mathbf{p} \in \Gamma_c \quad (19)$$

$$\lambda(\mathbf{p}, t_0) \leq 0 \quad \text{a.e. } \mathbf{p} \in \Gamma_c \quad (20)$$

$$d_n(\mathbf{p}, t_0)\lambda(\mathbf{p}, t_0) = 0 \quad \text{a.e. } \mathbf{p} \in \Gamma_c \quad (21)$$

The "viability lemma" of J.J. Moreau (1988, 2000) asserts that with this hypothesis, the Signorini contact conditions (8-10) are satisfied at all futur times as far as the following conditions are fulfilled: for  $(\mathbf{p}, t) \in \Gamma_c \times \mathbf{I}$ ,

$$\begin{aligned} \text{if } d_n(\mathbf{p}, t) < 0 \quad \text{then } \lambda(\mathbf{p}, t) = 0 \\ \text{otherwise} \end{aligned} \quad (22)$$

$$[[v_n(\mathbf{p},t)]] \leq 0 \quad (23)$$

$$\lambda(\mathbf{p},t) \leq 0 \quad (24)$$

$$[[v_n(\mathbf{p},t)]]\lambda(\mathbf{p},t) = 0 \quad (25)$$

where  $[[v_n]]$  stands for the normal velocity jump field in the sense of the definition (3).

For this contact model, which is called here the *Signorini-Moreau* model, one can notice that both the relative placements and relative velocities of the contact surfaces are controlled.

## 2.5 Level-Sets based Signorini-Moreau model

By following the developments carried out in subsection 3.2, we introduce now two *Sign*-like fields and write the *Signorini-Moreau* conditions as equations:

$$\lambda = S_u S_v (\lambda - k_2 [[v_n]]) \quad \text{on } \Gamma_c \times \mathbf{I} \quad (26)$$

$$S_u = 1_{R^-}(-d_n) \quad \text{on } \Gamma_c \times \mathbf{I} \quad (27)$$

$$S_v = 1_{R^-}(\lambda - k_1 [[v_n]]) \quad \text{on } \Gamma_c \times \mathbf{I} \quad (28)$$

The dynamic contact problem will be formulated with this new setting of the *Signorini-Moreau* contact conditions in the following section.

*Remark: A classical way of modelling dynamic contact loads consists in using regularized possibly damped or compliance models: (eg. Martins and Oden, 1985 and the experimental references therein)*

$$\lambda = -k_n ((d_n)^+)^{m_1} - c_n ((d_n)^+)^{m_2} [[v_n]] \quad (29)$$

where  $k_n$ ,  $c_n$ ,  $m_1$  and  $m_2$  are either numerical or material parameters characterizing the contact interface. This equation can be treated in a weak or a strong way. In all cases, one can notice that an unknown *Sign*-like field can be defined by:

$$S = 1_{R^-}(-d_n) \quad \text{on } \Gamma_c \times \mathbf{I} \quad (30)$$

and introduced as additionnal unknown field as done by Ben Dhia (1988).

## 2.6 Lagrangian and stabilized Lagrangian formulations

By using the VWP (1) and operating a weak formulation of equation (26) whilst keeping equations (27) and (28) as local strong ones, the following weak-strong mixed formulation of the dynamic frictionless contact problem is obtained:

assuming that the displacement and velocity fields  $\mathbf{u}^i$  and  $\mathbf{v}^i$  are known at a given instant  $t_0 \in \mathbf{I}$ , then for all  $t > t_0, t \in \mathbf{I}$ , the problem to be solved is the following: (where the reference to time and Lebesgue measure symbols  $d\Omega_0^i$  and  $d\Gamma_c$  are omitted)

Find  $(\mathbf{v}, \lambda; \mathbf{u}, S_u, S_v) \in \mathbf{CA}_v \times H \times \mathbf{CA}_u \times (\mathbf{S}(\Gamma_c; R))^2; \forall (\mathbf{w}, \lambda^*) \in \mathbf{CA}_v \times H$

$$\sum_{i=1}^2 \int_{\Omega_0^i} \rho_0^i \dot{\mathbf{v}}^i \cdot \mathbf{w}^i + \sum_{i=1}^2 \int_{\Omega_0^i} \text{Tr}[\Pi^i (\nabla_p(\mathbf{w}^i))^T] - \int_{\Gamma_c} S_u S_v \lambda [[w_n]] = 0 \quad (31)$$

$$-\frac{1}{k_2} \int_{\Gamma_c} [\lambda - S_u S_v (\lambda - k_2 [[v_n]])] \lambda^* = 0 \quad (32)$$

$$\mathbf{u}^i(t) = \mathbf{u}^i(t_0) + \int_{t_0}^t \mathbf{v}^i(s) ds \quad \text{in } \Omega_0^i \quad (33)$$

$$S_u - 1_{R^-}(-d_n) = 0 \quad \text{on } \Gamma_c \times \mathbf{I} \quad (34)$$

$$S_v - 1_{R^-}(\lambda - k_1[[v_n]]) = 0 \quad \text{on } \Gamma_c \times I \quad (35)$$

where  $\mathbf{CA}_v$  is the space of kinematically admissible velocity fields,  $H$  is the space of contact Lagrange multiplier,  $S(\Gamma_c; R)$  is the space of (possibly multivalued-) functions defined from  $\Gamma_c$  into  $R$ ,  $\Pi^i$  is given by the behaviour law (4) and where the initial conditions are given by (5).

As a matter of fact equations (31-35) can be splitted into two parts expressing in a weak sense the fact that either the gap is closed ( $S_u S_v = 1$ ) or a free normal natural condition ( $S_u S_v = 0$ ). In the former alternative, it is clear that the kinematical constraint is taken into account by means of the lagrangian multiplier  $\lambda$  and the formulation is a lagrangian one. In the litterature and by following pioneering works of Hestenes and Powell (1969), Alart and Curnier (1991) derived, in the discrete finite element framework, an augmented lagrangian formulation for contact problems (see also Simo and Laursen (1992), for a rather similar continuous augmented formulation). Incidentally and very surprisingly, one can not derive the weak lagrangian formulation from these augmented ones! In the contrary, since in (32) the virtual multipliers impose that the jump of the normal velocity field is weakly null when the contact is effective, one can obviously add the following term to the weak equilibrium equations (31), without changing the continuous problem:

$$\int_{\Gamma_c} S_u S_v k_3 \Pi_H[[v_n]] \Pi_H[[w_n]] \quad (36)$$

with  $k_3 \geq 0$  standing for a parameter and  $\Pi_H$  for an orthogonal projection on the space  $H$ . Notice here that while important, this projection seems not to have been considered by the contact community.

One can now check that the obtained formulation we call a stabilized lagrangian contact formulation is a continuous generalization of the discrete augmented one of Alart and Curnier (1991) (and also of the continuous ones of Pietrzak and Curnier, 1999, Ben Dhia et al., 2000), in the following sense:

- the latter is recovered, in the discret range, by taking  $k_3 = k_1 = k_2 > 0$ ,
- the lagrangian formulation is obtained by taking  $k_3 = 0, k_1 > 0, k_2 \in R^*$

This unification is believed to be important from both a numerical (conditionning issue) and a practical (coding issue) points of view.

*Remark: The derived formulation is called stabilized since the main feature of the additional term (36) is a numerical stabilization effect whenever the standard kinematical boundary conditions do not cancel rigid body motions of one of the two solids, in the static or quasi-static regimes.*

### 3 SOLUTION STRATEGY

As a matter of fact, the problem above depends both on time and space. To be solved numerically, appropriate time and space schemes have to be used. This is briefly done in the following subsections.

#### 3.1 Time discretization

The fact that the problem (31-35) is both displacement and velocity-based, we only need first order time discretization schemes. The first order derivative of the velocity field with respect to time (in the inertial virtual work term in (31)) is approximated by a first order Finite Difference  $\theta$ -scheme. The same kind of scheme is used to compute the displacement field in (33)). This leads to a static-like semi-discretized continuous contact problem, at each time step. We notice here that either implicit, explicit or hybrid schemes (Carpenter et al., 1991, Ben Dhia et al., 2003) can be recovered with this procedure, the latters being of course conditionnally stable (eg. Belytschko et Neal, 1991). The influence of  $\theta$  on temporal integration can be found for example in the works of Vola et al. (1998), Jean (1999).

#### 3.2 Space discretization

An overview of the spatial discretization of the semi-discretized problems is described here. The velocity, displacement and lagrangian multiplier fields are approximated by means of the classical Finite Element method. By contrast, the dramatically irregular *Level-Sets* fields  $S_u$  and  $S_v$  can not reasonably be approximated by the same method. These are approximated by means of a collocation

method which, when combined with the finite element method, consists in evaluating the *Level-Sets* fields in a finite collection of particles of  $\Gamma_c$ . For consistency, the collocation set of points has then to cope with the numerical integration points used to approximate the integrals involving contact in the the whole system. As a matter of fact, classical contact elements (node-to-node, etc.) can be recovered with both appropriate choices of these particles and appropriate pairing applications.

3.2.1 FE Approximation - The finite element discretization of the semi-discretized mixed problems has been discussed in Ben Dhia and Zarroug (2002) in the quasi-static framework. The same is done here in the dynamic context regime and the reader is referred to (Ben Dhia and Zammali, 2004), for details. Let us here just stress the fact that the finite element spaces used for the discretization of the speeds and the contact loads respectively, have to be compatible for stability reasons (e.g. Bathes and Brezzi, 2001). Let us also underline the fact that for curved contact surfaces, the definition of compatible choices of finite element spaces seems to be still an open question; though it has been shown numerically in Ben Dhia and Zarroug (2002) that a simple regularization of the contact surfaces may numerically stabilize the finite element scheme. A nonlocal procedure that might be also well appropriate consists in averaging the non interpenetration condition on surfaces whose sizes could be related to the space discretization parameters.

3.2.2 Collocation method - Now, once the classical mechanical fields (speeds, displacements and loads) have been (correctly) approximated, the associated problem has still a continuous dimension intimately related to the continuous character of the non classical, but still intrinsic contact fields, namely the *Level-Sets* ( $S_u$  and  $S_v$ ). To close the discretization, a collocation method (finite particles approach) is used to approximate these irregular fields. It has been shown (Ben Dhia and Zarroug, 2002) that the contact loads may be dramatically oscillating for the the simple classical Taylor patch-test if the choice of these points is not sufficiently appropriate. This important aspect will be discussed during the conference.

The nonlinear finite systems of equations obtained this way are then solved by a Newton-like method.

## 4 DYNAMIC FRICTIONAL CONTACT PROBLEMS AND SOLUTION METHOD

When taking into account friction phenomena through the Coulomb's laws, for instance, and as done by Ben Dhia (1990), the same methodology as the one developed above still applies and one can introduce an additional unknown Level-Set like field separating sticking from sliding regions in the effective contact zones. Moreover, the additional friction terms are discretized by following basically the lines developed in the frictionless case. The numerical algorithm could no more be of Newton type. A fixed point strategy on the normal contact threshold coupled with a  $\theta$ -tangent algorithm are used (Ben Dhia and Zarroug, 2002). Incidentally, this  $\theta$ -tangent algorithm is to our best knowledge the own available explicit generalized-Newton algorithm, using non extreme (binary) values of the subgradient (in the sense of Clarke (1983)) of the irregular projection application.

## 5 NUMERICAL RESULTS AND MULTISCALE ASPECTS

The points mentioned above will be exemplified by numerical results, developed in Code-Aster of EdF.

As perspective, we show the ability of the multiscale Arlequin framework (Ben Dhia, 1998, 1999) to superpose to a Macroscopic model a local refined (Ben Dhia et Zammali, 2004) or even better a discrete molecular dynamics model (Ben Dhia, 2004) in the vicinity of impact zones. As a matter of fact, local molecular dynamics models in the volume, both with physical interface potentials seem clearly more relevant (no discontinuities) to the impact problem than macroscopic continuum mechanics models with Signorini-like contact laws.

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