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A NEW RECURSIVE FORMULATION FOR FAST COMPUTING OF UNKNOWN INPUT ESTIMATION

A tracking application

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Abstract: This paper introduces a new form for the Recursive Input Estimation problem. The proposed algorithm needs the inversion at each sample time of one $n_y \times n_y$ and one $n_u \times n_u$ matrices, whereas the classical formulation requires the inversion of two $n_y \times n_y$ matrices. Consequently, a reduction of computation time occurs in the case where $n_y > n_u$, n_y being the number of measurements and n_u the number of inputs of the system under consideration. The new formulation obtained is tested on a tracking application, and its results compared with those issued from the classical formulation.

1 INTRODUCTION

The problem of both estimating the state and input of a dynamical system from noisy measurements has been leading for a long time to intensive research. The solution to such a problem can help in applications like fault tolerant control (through actuator bias detection and estimation) (Hou and Patton, 1998a), manoeuvring targets tracking (Aley and Delinger, 1973), evaluation of reaction rates in chemical reactors (Mhamdi and Marquardt, 1999) or estimation of accelerometers and gyroscopes errors in inertial navigation (Grewal et al., 2001).

In order to solve this kind of problem, different approach can be found, such as model inversion (Silverman, 1969), Unknown Input Observers (Darouach et al., 1994) and Filters (Hou and Patton, 1998b), Interacting Multiple Model (IMM) (Mazor et al., 1998) or inclusion of the unknown inputs into the state vector (Bar-Shalom and Birmiwal, 1982) (leading to an augmented state vector). Another approach, called *Input Estimation*, has been proposed by (Chan et al., 1979), recursively developed in (Wang and Varshney, 1993) and adapted to a class of time-varying inputs in (Lee and Tark, 1999), (Jilkov and Li, 2002). Its advantage compared to the others approach is undeniable: Indeed, its applicability conditions are far less

restrictive than the model inversion or Unknown Input Observers/Filters ones; Its computational burden is lower than the IMM one (Mazor et al., 1998); its structure allows decoupling of state and input estimation, contrary to the augmented state vector approach.

However, this algorithm presents a drawback : It needs the inversion of two squares matrices of dimension $n_y \times n_y$, where n_y is the number of measurements done on the system under consideration, while a Kalman filter (Kalman, 1960) using an augmented state vector needs only one inversion. The purpose of this article is to propose a new formulation of least squares estimator, by using the information form (Anderson and Moore, 1979). The two matrices to inverse are then of dimension $n_y \times n_y$ and $n_u \times n_u$, where n_u is the number of inputs. Such a formulation reduces the computation time in the case where n_y is greater than n_u , and moreover, provides a simpler algorithm.

In this paper, the Recursive Input Estimation (RIE) is firstly presented. Then, the new formulation of RIE using information matrices is proposed in section 2. Finally, the proposed method is applied on a tracking application in section 3.

2 THE INPUT ESTIMATION APPROACH

Consider the following linear discrete time system:

$$\begin{cases} \mathbf{x}_{k+1} = \mathbf{A}_k \mathbf{x}_k + \mathbf{B}_k \mathbf{u} + \mathbf{G}_k \mathbf{w}_k \\ \mathbf{y}_k = \mathbf{C}_k \mathbf{x}_k + \mathbf{v}_k \end{cases} \quad (1)$$

where $\mathbf{x} \in \mathbb{R}^{n_x}$ is the state vector, $\mathbf{u} \in \mathbb{R}^{n_u}$ the input vector, $\mathbf{y} \in \mathbb{R}^{n_y}$ the measurement vector, $\mathbf{w} \in \mathbb{R}^{n_w}$ the process noise, $\mathbf{v} \in \mathbb{R}^{n_v}$ the measurement noise and $k \in \mathbb{N}$ the (discrete) time. Theoretically, \mathbf{u} is a constant vector, but, from a practical point of view, it can be slowly time varying. \mathbf{x}_0 , \mathbf{w}_k and \mathbf{v}_k are supposed to be gaussian, with respective covariance \mathbf{P}_0 , \mathbf{Q}_k and \mathbf{R}_k . Moreover, \mathbf{w}_k and \mathbf{v}_k are supposed to be mutually uncorrelated, and uncorrelated with \mathbf{x}_0 . \mathbf{A}_k , \mathbf{B}_k , \mathbf{C}_k and \mathbf{G}_k are known matrices, potentially time varying. The system described by (1) is assumed to be observable if \mathbf{u} is known.

Subsequently, we will denote by $\hat{\mathbf{x}}_{k|n}$ and $\hat{\mathbf{u}}_{k|n}$ the estimates at time k of \mathbf{x}_k and \mathbf{u} using $\{\mathbf{y}_1, \dots, \mathbf{y}_n\}$, and their respective covariance by $\mathbf{P}_{k|n}$ and $\mathbf{\Gamma}_{k|n}$. We also adopt the shorthand notation $\mathbf{V}_i = \mathbf{V}_{i|i}$ for any variable \mathbf{V} .

The RIE algorithm as described in (Wang and Varshney, 1993) and (Keche et al., 1997) consists in a Kalman filter estimating the state of the system without taking into account \mathbf{u} , and a minimum variance least squares algorithm estimating \mathbf{u} by using the Kalman filter innovation. These two estimates are then summed in order to give the best estimate $\hat{\mathbf{x}}_k$ of \mathbf{x}_k possible. The RIE algorithm is described in figure 1, and synthesized afterwards. The values computed by the Kalman filter (which do not take into account the input \mathbf{u}) are denoted with the superscript 0. T symbolizes the transposition operation, and \mathbf{I}_\times , $\mathbf{0}_\times$ stand for the identity and nil matrices of appropriate dimension.

1. Initialisation with $\hat{\mathbf{x}}_0^0 = \hat{\mathbf{x}}_0$, $\mathbf{P}_0^0 = \mathbf{P}_0$, $\hat{\mathbf{u}}_0$, $\mathbf{\Gamma}_0$ and $\mathbf{F}_0 = \mathbf{0}_{n_x \times n_u}$

2. \mathbf{H}_k update: $\mathbf{H}_k = \mathbf{A}_{k-1} \mathbf{F}_{k-1} + \mathbf{B}_{k-1}$

3. State prediction:

$$\begin{aligned} \bullet \hat{\mathbf{x}}_{k|k-1}^0 &= \mathbf{A}_{k-1} \hat{\mathbf{x}}_{k-1}^0 \\ \bullet \mathbf{P}_{k|k-1}^0 &= \mathbf{A}_{k-1} \mathbf{P}_{k-1}^0 \mathbf{A}_{k-1}^T \\ &\quad + \mathbf{G}_{k-1} \mathbf{Q}_{k-1} \mathbf{G}_{k-1}^T \end{aligned}$$

4. State update:

$$\begin{aligned} \bullet \hat{\mathbf{y}}_{k|k-1}^0 &= \mathbf{y}_k - \mathbf{C}_k \hat{\mathbf{x}}_{k|k-1}^0 \\ \bullet \mathbf{\Sigma}_{k|k-1}^0 &= \mathbf{C}_k \mathbf{P}_{k|k-1}^0 \mathbf{C}_k^T + \mathbf{R}_k \\ \bullet \mathbf{K}_k^0 &= \mathbf{P}_{k|k-1}^0 \mathbf{C}_k^T (\mathbf{\Sigma}_{k|k-1}^0)^{-1} \\ \bullet \hat{\mathbf{x}}_k^0 &= \hat{\mathbf{x}}_{k|k-1}^0 + \mathbf{K}_k^0 \hat{\mathbf{y}}_{k|k-1}^0 \\ \bullet \mathbf{P}_k^0 &= (\mathbf{I}_{n_x} - \mathbf{K}_k^0 \mathbf{C}_k) \mathbf{P}_{k|k-1}^0 \end{aligned}$$

5. \mathbf{F}_k update: $\mathbf{F}_k = (\mathbf{I}_{n_x} - \mathbf{K}_k^0 \mathbf{C}_k) \mathbf{H}_k$

6. Estimated state and covariance correction:

$$\begin{aligned} \bullet \hat{\mathbf{x}}_k &= \hat{\mathbf{x}}_k^0 + \mathbf{F}_k \hat{\mathbf{u}}_{k-1} \\ \bullet \mathbf{P}_k &= \mathbf{P}_k^0 + \mathbf{F}_k \mathbf{\Gamma}_{k-1} \mathbf{F}_k^T \end{aligned}$$

7. Input estimation:

$$\begin{aligned} \bullet \mathbf{S}_k &= \mathbf{C}_k \mathbf{H}_k \mathbf{\Gamma}_{k-1} \mathbf{H}_k^T \mathbf{C}_k^T + \mathbf{\Sigma}_{k|k-1}^0 \\ \bullet \mathbf{L}_k &= \mathbf{\Gamma}_{k-1} \mathbf{H}_k^T \mathbf{C}_k^T \mathbf{S}_k^{-1} \\ \bullet \hat{\mathbf{u}}_k &= \hat{\mathbf{u}}_{k-1} + \mathbf{L}_k (\hat{\mathbf{y}}_{k|k-1}^0 - \mathbf{C}_k \mathbf{H}_k \hat{\mathbf{u}}_{k-1}) \\ \bullet \mathbf{\Gamma}_k &= \mathbf{\Gamma}_{k-1} - \mathbf{\Gamma}_{k-1} \mathbf{H}_k^T \mathbf{C}_k^T \mathbf{S}_k^{-1} \mathbf{C}_k \mathbf{H}_k \mathbf{\Gamma}_{k-1} \\ &= (\mathbf{I}_{n_u} - \mathbf{L}_k \mathbf{C}_k \mathbf{H}_k) \mathbf{\Gamma}_{k-1} \end{aligned}$$

Classical RIE algorithm

As one can see, this algorithm needs at each sample time to compute the inverses of $\mathbf{\Sigma}_{k|k-1}^0$ and \mathbf{S}_k . These two matrices are of dimension $n_y \times n_y$. The purpose of the next section is to find an algorithm that needs to inverse $\mathbf{\Sigma}_{k|k-1}^0$ and instead of \mathbf{S}_k , a matrix of lower dimension. Afterwards, in order to simplify some expressions, we write:

$$\mathbf{D}_k = \mathbf{C}_k \mathbf{H}_k \quad (2)$$

In addition, 7. will stand for the seventh step of the classical RIE information, given above.

3 THE INPUT ESTIMATION APPROACH

For the sake of convenience, let us remember the matrix inversion lemma, α and γ being square and regular matrices and α , β , γ and δ having compatible dimensions:

$$\begin{aligned} (\alpha + \beta \gamma \delta)^{-1} &= \alpha^{-1} \\ &\quad - \alpha^{-1} \beta (\gamma^{-1} + \delta \alpha^{-1} \beta)^{-1} \delta \alpha^{-1} \end{aligned} \quad (3)$$

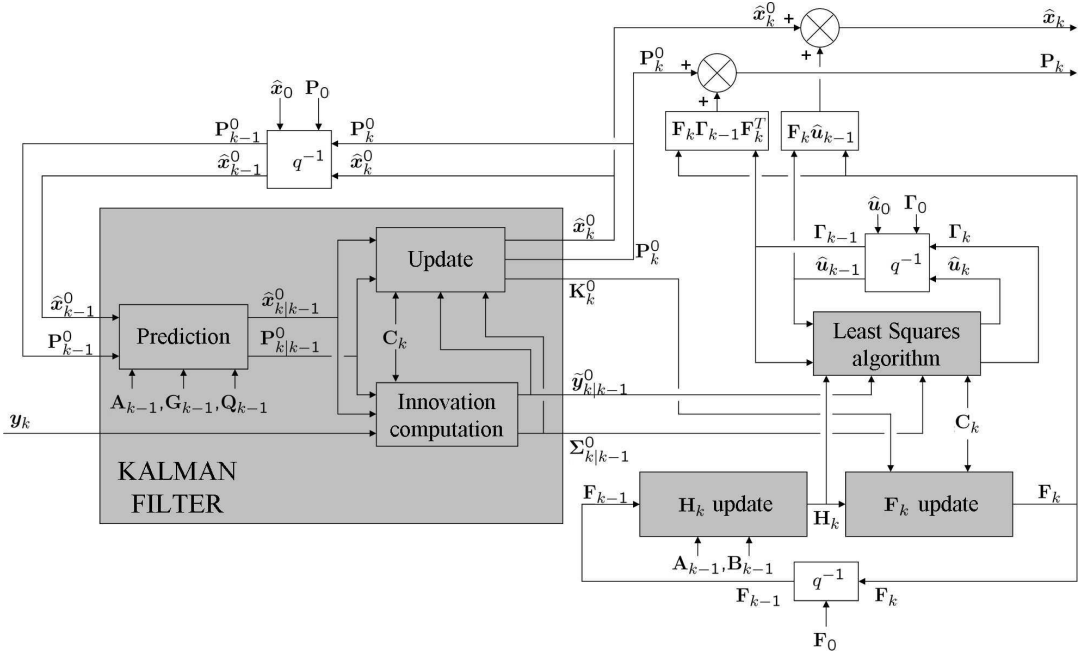


Figure 1: Recursive Input Estimation Principle.

By putting the first equation of 7. into the fourth, and using (2), we have:

$$\Gamma_k = \Gamma_{k-1} - \Gamma_{k-1} \mathbf{D}_k^T \left(\Sigma_{k|k-1}^0 + \mathbf{D}_k \Gamma_{k-1} \mathbf{D}_k^T \right)^{-1} \mathbf{D}_k \Gamma_{k-1} \quad (4)$$

One can easily see, by identification between (4) and the matrix inversion lemma (3), that (4) can be rewritten as follow:

$$\Gamma_k = \left(\Gamma_{k-1}^{-1} + \mathbf{D}_k^T \left(\Sigma_{k|k-1}^0 \right)^{-1} + \mathbf{D}_k \right)^{-1} \quad (5)$$

or equivalently:

$$\mathcal{J}_k = \mathcal{J}_{k-1} + \mathbf{D}_k^T \left(\Sigma_{k|k-1}^0 \right)^{-1} \mathbf{D}_k \quad (6)$$

where \mathcal{J}_k is the information matrix (i.e. the inverse of the covariance matrix) of the least squares algorithm:

$$\mathcal{J}_k = \Gamma_k^{-1} \quad (7)$$

Now, let us apply the matrix inversion lemma to \mathbf{S}_k , given in the first equation of 7.. Thanks to definition (2), we have:

$$\begin{aligned} \mathbf{S}_k^{-1} &= \left(\mathbf{D}_k \Gamma_{k-1} \mathbf{D}_k^T + \Sigma_{k|k-1}^0 \right)^{-1} \\ &= \left(\Sigma_{k|k-1}^0 \right)^{-1} - \left(\Sigma_{k|k-1}^0 \right)^{-1} \mathbf{D}_k \\ &\quad \times \left(\Gamma_{k-1}^{-1} + \mathbf{D}_k^T \left(\Sigma_{k|k-1}^0 \right)^{-1} \mathbf{D}_k \right)^{-1} \quad (8) \\ &\quad \times \mathbf{D}_k^T \left(\Sigma_{k|k-1}^0 \right)^{-1} \end{aligned}$$

Or, thanks to (6) and (7), we can see that :

$$\left(\Gamma_{k-1}^{-1} + \mathbf{D}_k^T \left(\Sigma_{k|k-1}^0 \right)^{-1} \mathbf{D}_k \right)^{-1} = \Gamma_k \quad (9)$$

Inserting (9) into (8) gives:

$$\mathbf{S}_k^{-1} = \left(\Sigma_{k|k-1}^0 \right)^{-1} - \left(\Sigma_{k|k-1}^0 \right)^{-1} \mathbf{D}_k \Gamma_k \mathbf{D}_k^T \left(\Sigma_{k|k-1}^0 \right)^{-1} \quad (10)$$

Inserting (10) into the expression of \mathbf{L}_k (second equation of 7.) and using (2) leads to:

$$\begin{aligned} \mathbf{L}_k &= \Gamma_{k-1} \mathbf{D}_k^T \left[\left(\Sigma_{k|k-1}^0 \right)^{-1} \right. \\ &\quad \left. - \left(\Sigma_{k|k-1}^0 \right)^{-1} \mathbf{D}_k \Gamma_k \mathbf{D}_k^T \left(\Sigma_{k|k-1}^0 \right)^{-1} \right] \quad (11) \\ &= \Gamma_{k-1} \left[\mathbf{I}_{n_u} - \mathbf{D}_k^T \left(\Sigma_{k|k-1}^0 \right)^{-1} \mathbf{D}_k \Gamma_k \right] \\ &\quad \times \mathbf{D}_k^T \left(\Sigma_{k|k-1}^0 \right)^{-1} \end{aligned}$$

By using (6), we can substitute $\mathbf{D}_k^T \left(\Sigma_{k|k-1}^0 \right)^{-1} \mathbf{D}_k$ in (11) by $\mathcal{J}_k - \mathcal{J}_{k-1}$. It leads to:

$$\begin{aligned} \mathbf{L}_k &= \Gamma_{k-1} [\mathbf{I}_{n_u} - (\mathcal{J}_k - \mathcal{J}_{k-1}) \Gamma_k] \\ &\quad \times \mathbf{D}_k^T \left(\Sigma_{k|k-1}^0 \right)^{-1} \quad (12) \\ &= \Gamma_k \mathbf{D}_k^T \left(\Sigma_{k|k-1}^0 \right)^{-1} \end{aligned}$$

This last result is the adaptation to recursive minimum variance least squares of a well known equation

of Kalman filtering (Anderson and Moore, 1979). Let us use it into the update equation of $\hat{\mathbf{u}}_k$ (third equation of 7.):

$$\begin{aligned} \hat{\mathbf{u}}_k = & \left[\mathbf{I}_{n_u} - \mathbf{\Gamma}_k \mathbf{D}_k^T \left(\mathbf{\Sigma}_{k|k-1}^0 \right)^{-1} \mathbf{D}_k \right] \hat{\mathbf{u}}_{k-1} \\ & + \mathbf{\Gamma}_k \mathbf{D}_k^T \left(\mathbf{\Sigma}_{k|k-1}^0 \right)^{-1} \tilde{\mathbf{y}}_{k|k-1}^0 \end{aligned} \quad (13)$$

Now, let us introduce the following variable change:

$$\hat{\mathbf{z}}_k = \mathcal{J}_k \hat{\mathbf{u}}_k \quad (14)$$

Consequently, (13) becomes:

$$\begin{aligned} \hat{\mathbf{z}}_k = & \mathcal{J}_k \left[\mathbf{I}_{n_u} - \mathbf{\Gamma}_k \mathbf{D}_k^T \left(\mathbf{\Sigma}_{k|k-1}^0 \right)^{-1} \mathbf{D}_k \right] \\ & \times \mathbf{\Gamma}_{k-1} \hat{\mathbf{z}}_{k-1} + \mathbf{D}_k^T \left(\mathbf{\Sigma}_{k|k-1}^0 \right)^{-1} \tilde{\mathbf{y}}_{k|k-1}^0 \end{aligned} \quad (15)$$

By using (6), (15) can be rewritten as follow:

$$\begin{aligned} \hat{\mathbf{z}}_k = & \mathcal{J}_k [\mathbf{I}_{n_u} - \mathbf{\Gamma}_k (\mathcal{J}_k - \mathcal{J}_{k-1})] \mathbf{\Gamma}_{k-1} \hat{\mathbf{z}}_{k-1} \\ & + \mathbf{D}_k^T \left(\mathbf{\Sigma}_{k|k-1}^0 \right)^{-1} \tilde{\mathbf{y}}_{k|k-1}^0 \end{aligned} \quad (16)$$

Thanks to definition (7), one can easily see that $\mathcal{J}_k [\mathbf{I}_{n_u} - \mathbf{\Gamma}_k (\mathcal{J}_k - \mathcal{J}_{k-1})] \mathbf{\Gamma}_{k-1} = \mathbf{I}_{n_u}$. Consequently:

$$\hat{\mathbf{z}}_k = \hat{\mathbf{z}}_{k-1} + \mathbf{D}_k^T \left(\mathbf{\Sigma}_{k|k-1}^0 \right)^{-1} \tilde{\mathbf{y}}_{k|k-1}^0 \quad (17)$$

Equations (2), (6), (7), (14), and (17) describe the information form of the least squares algorithm. The information form of the RIE algorithm is described below, and illustrated in figure 2:

1) to 5) Identical to classical RIE algorithm.

6) Estimated state and covariance correction:

- o $\hat{\mathbf{x}}_k = \hat{\mathbf{x}}_k^0 + \mathbf{F}_k (\mathcal{J}_{k-1})^{-1} \hat{\mathbf{z}}_{k-1}$
- o $\mathbf{P}_k = \mathbf{P}_k^0 + \mathbf{F}_k (\mathcal{J}_{k-1})^{-1} \mathbf{F}_k^T$

7) Input estimation through information least squares algorithm:

- o $\hat{\mathbf{z}}_k = \hat{\mathbf{z}}_{k-1} + \mathbf{H}_k^T \mathbf{C}_k^T \left(\mathbf{\Sigma}_{k|k-1}^0 \right)^{-1} \tilde{\mathbf{y}}_{k|k-1}^0$
- o $\mathcal{J}_k = \mathcal{J}_{k-1} + \mathbf{H}_k^T \mathbf{C}_k^T \left(\mathbf{\Sigma}_{k|k-1}^0 \right)^{-1} \mathbf{C}_k \mathbf{H}_k$

Information form RIE algorithm

As one can see, the matrices to be inverted in this algorithm are $\mathbf{\Sigma}_{k|k-1}^0$ (which is used in the Kalman filter and least squares algorithm) and \mathcal{J}_{k-1} , with respective dimension $n_y \times n_y$ and $n_u \times n_u$. It leads to

computation time saving and a simpler algorithm in the case where $n_u < n_y$.

We also could have develop an information form for the classical Kalman filter (see (Anderson and Moore, 1979), p.139 for the equations), but that would lead to several difficulties and drawbacks: Firstly, it needs the inversion of \mathbf{A}_k , which is not always regular (for example, in some models of inertial navigation errors). Secondly, an information form for a Kalman filter needs to inverse another matrix, which is not $\mathbf{\Sigma}_{k|k-1}^0$; As this inverse is required for the least square algorithm, that would be one more matrix to inverse.

Finally, it should be noted that the condition $n_y > n_u$ does not necessarily cause a controllability problem. Indeed, \mathbf{y} represents some measurements, which are *a priori* different from the output to control. As example, we can mention the problem of tracking a moving vehicle: The driver adjusts the acceleration (the input of the system) in order to control the position (the output to control). But an external observer uses measurements which are the position and/or velocity of the vehicle in order to estimate its state (position, velocity) and its unknown input (acceleration).

4 APPLICATION

In this section, we apply the information form of RIE on a two-dimension tracking application, and compare with the results obtained by the classical RIE. In such an application, the RIE is one of the most widely used tools (Wang and Varshney, 1993), (Lee and Tark, 1999). However, others approach can be found, such as the inclusion of the RIE into a multiple hypothesis setting (Bogler, 1987), the variable dimension filter (Bar-Shalom and Birmiwal, 1982) or the IMM (Blom and Bar-Shalom, 1988). A complete state of the art can be found in (Bar-Shalom, 2001).

4.1 Model Considered

Let $\vec{r} = \overrightarrow{OM}$ be the position vector of a vehicle moving on the earth surface, considered as a plane. O is fixed with regard to the earth, and taken for the sake of convenience into the studied plane. M is the vehicle gravity centre. If we consider the earth as inertial (neither entrainment nor Coriolis acceleration), then we have:

$$\vec{V} = \left. \frac{d\vec{r}}{dt} \right|_{[T]} \quad \vec{\gamma} = \left. \frac{d\vec{V}}{dt} \right|_{[T]} \quad (18)$$

where $[T]$ is an earth-fixed frame, \vec{V} the vehicle velocity (relatively to the earth), and $\vec{\gamma}$ the vehicle ac-

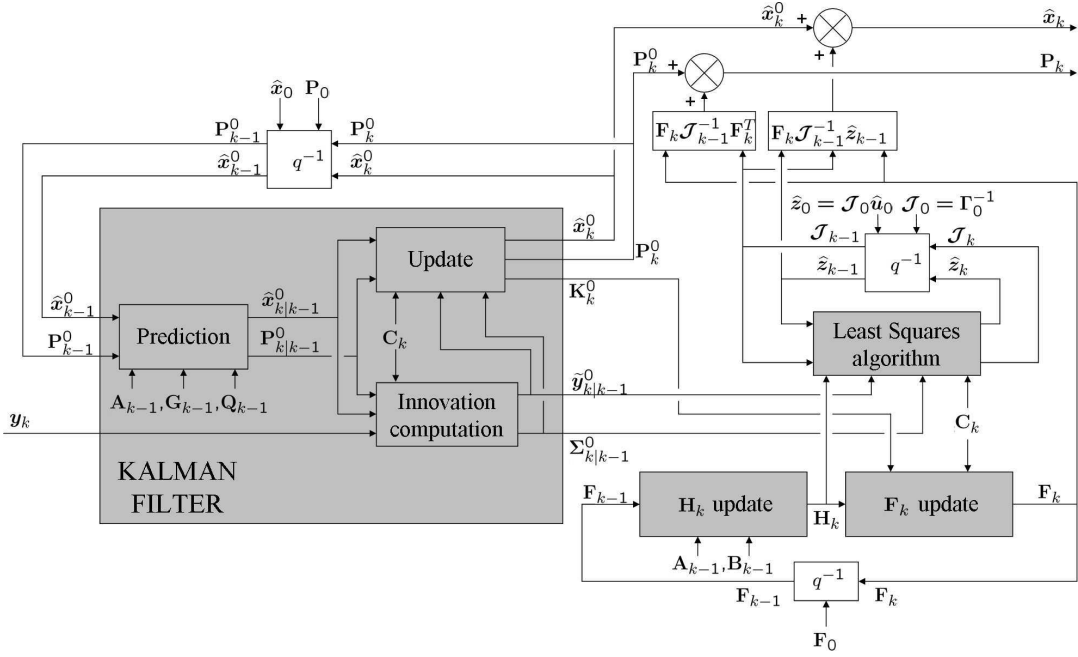


Figure 2: Information form for recursive input estimation.

celeration (relatively to the earth). In order to simplify, the two axes of $[T]$ (we do not need three axis as the vehicle motion take place in a plane) are chosen included in the plane. In addition, we consider that:

$$\vec{\gamma} = \vec{a} + \vec{\eta} \quad (19)$$

where \vec{a} is fixed with regard to the earth, and $\vec{\eta}$ a white noise. Projecting these equations into $[T]$ leads to the following model:

$$\begin{bmatrix} \dot{r} \\ \dot{V} \end{bmatrix} = \begin{bmatrix} \mathbf{0}_{2 \times 2} & \mathbf{I}_2 \\ \mathbf{0}_{2 \times 2} & \mathbf{0}_{2 \times 2} \end{bmatrix} \begin{bmatrix} r \\ V \end{bmatrix} + \begin{bmatrix} \mathbf{0}_{2 \times 2} \\ \mathbf{I}_2 \end{bmatrix} (a + \eta) \quad (20)$$

where, for any vector $\vec{\lambda}$, λ is the projection of $\vec{\lambda}$ onto the axis of $[T]$. \vec{a} being fixed with regard to $[T]$, then $\hat{a} = \mathbf{0}_{2 \times 1}$. This model is discretized thanks to an Euler scheme, with a sample time T :

$$\begin{bmatrix} r_{k+1} \\ V_{k+1} \end{bmatrix} = \begin{bmatrix} \mathbf{I}_2 & T\mathbf{I}_2 \\ \mathbf{0}_{2 \times 2} & \mathbf{I}_2 \end{bmatrix} \begin{bmatrix} r_k \\ V_k \end{bmatrix} + \begin{bmatrix} \mathbf{0}_{2 \times 2} \\ T\mathbf{I}_2 \end{bmatrix} a + \begin{bmatrix} \mathbf{0}_{2 \times 2} \\ \mathbf{I}_2 \end{bmatrix} \rho_k \quad (21)$$

where $\rho_k = \int_{kT}^{(k+1)T} \eta(t)dt$ is a random walk, with covariance $\sqrt{T}\mathbb{E}(\eta\eta^T)$, \mathbb{E} being de mathematical expectation (Grewal et al., 2001). From (21), one can

easily see that:

$$\begin{aligned} A_k &= \begin{bmatrix} \mathbf{I}_2 & T\mathbf{I}_2 \\ \mathbf{0}_{2 \times 2} & \mathbf{I}_2 \end{bmatrix} & x_k &= \begin{bmatrix} r_k \\ V_k \end{bmatrix} & u &= a \\ B_k &= \begin{bmatrix} \mathbf{0}_{2 \times 2} \\ T\mathbf{I}_2 \end{bmatrix} & G_k &= \begin{bmatrix} \mathbf{0}_{2 \times 2} \\ \mathbf{I}_2 \end{bmatrix} & w_k &= \rho_k \end{aligned} \quad (22)$$

The measurements are done at each sample time thanks to a GPS which measure the vehicle position and velocity. Through an appropriate coordinate change, we have:

$$y_k = x_k + v_k \quad (23)$$

where v_k is the GPS measurement error, modelled by a white noise.

4.2 Experiments

(21) and (23) allow the use of a RIE. We decide to make our RIE works in SI (International System of Units), and with sample time $T = 1s$. The vehicle acceleration is assumed to be $u = [2 \ 3]^T$, and its initials conditions $x_0 = [70 \ 20 \ 0 \ 0]^T$. The RIE covariances are $Q_k = \mathbf{I}_2$, $R_k = \text{diag}(10^2, 10^2, 1^2, 1^2)$, and initials conditions $P_0 = \text{diag}(50^2, 50^2, 0.1^2, 0.1^2)$, $\hat{x}_0 \sim \mathcal{N}(x_0, P_0)$, $\Gamma_0 = \text{diag}(10^2, 10^2)$, $\hat{u}_0 = \mathbf{0}_{2 \times 1}$. Here, " $\mathbf{M} = \text{diag}(\dots)$ " is a shorthand notation indicating that \mathbf{M} is a diagonal matrix fulfilled in order, and $x \sim \mathcal{N}(m, c)$ means that x is normally distributed, with mean m and covariance c .

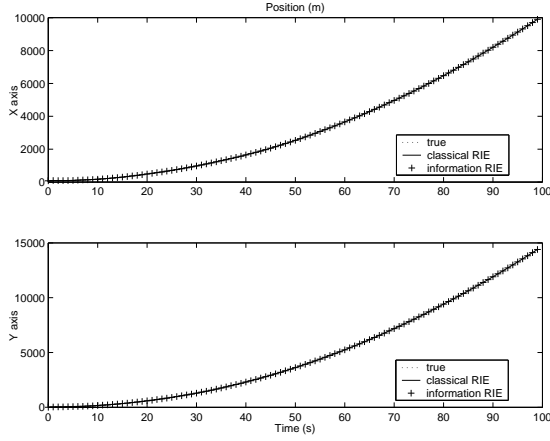


Figure 3: Real and estimated positions.

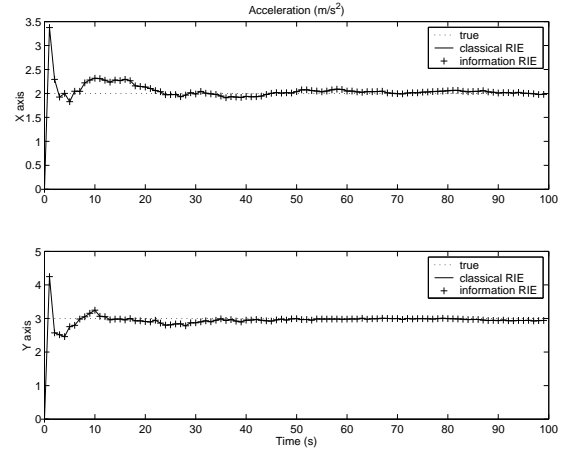


Figure 5: Real and estimated accelerations.

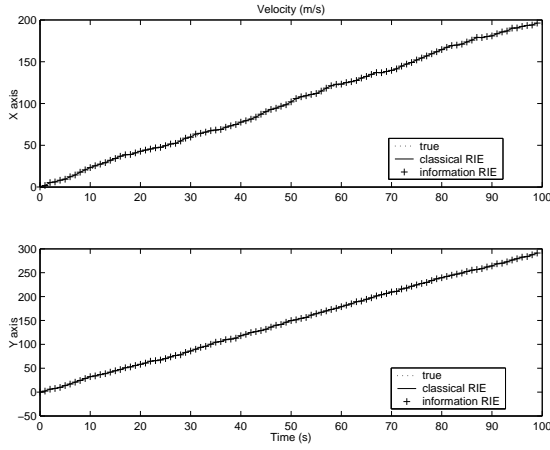


Figure 4: Real and estimated velocities.

The results obtained by the information and classical RIE, issued from a 100 iterations simulation, are given in figures 3, 4, 5.

We can clearly see that the performances of classical and information RIE are exactly the same, which is not a surprise. Indeed, from a mathematical point of view, these algorithms are identical: Only the practical implementation change. We can also see that the accelerations are pretty well estimated.

Now, let us compare the computational performances of these algorithms. By using the tic-toc MATLAB's function, which enables one to measure the time needed for executing a routine, we can appreciate the computation time of the classical and information RIE. The results, obtained for different simulation lengths, are given in table 1, where N is the number of iterations, T_c the classical RIE computation time in seconds and T_i the information RIE com-

putation time in seconds. Then, these results are illustrated in figure 6.

Table 1: Classical and information RIE computation time.

N	100	1000	5000	10000	20000
T_c	0.063	0.312	1.406	2.703	5.547
T_i	0.047	0.250	1.235	2.438	5.047

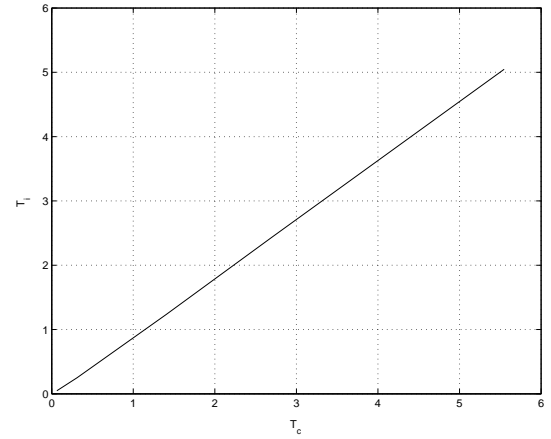


Figure 6: Information RIE computation time with respect to classical RIE computation time.

Figure 6 clearly shows that $T_i = aT_c$, with $a \in]0;1[$. By the use of least squares, we find that $a = 0.907$. This means that on the considered example, the computational gain when using the RIE information form instead of the classical one is: $1 - 0.907 = 0.093 = 9.3\%$.

5 CONCLUSION

In this article, an alternative formulation of the Recursive Input Estimation was presented. The obtained algorithm provides the same results as the baseline Recursive Input Estimation; Furthermore, it reduces the computation time in the case where the number of outputs is greater than the number of inputs. A tracking application, presenting four measurements and two constant inputs to estimate, highlighted this results, with a computation time decrease of 9.3%. An outlook of this work would be to compare these results with those obtained by others approaches, like Unknown Input Filters or model inversion.

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