



**HAL**  
open science

## Automatic Cuts for Magnetic Scalar Potential Formulations

Ahn Tuan Phung, Olivier Chadebec, Patrice Labie, Yann Le Floch, Gerard Meunier

► **To cite this version:**

Ahn Tuan Phung, Olivier Chadebec, Patrice Labie, Yann Le Floch, Gerard Meunier. Automatic Cuts for Magnetic Scalar Potential Formulations. IEEE Transactions on Magnetics, 2005, 41 (5), pp.1668-1671. 10.1109/TMAG.2005.846105 . hal-00290009

**HAL Id: hal-00290009**

**<https://hal.science/hal-00290009v1>**

Submitted on 24 Jun 2008

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Automatic Cuts for Magnetic Scalar Potential Formulations

Anh Tuan Phung<sup>1</sup>, Olivier Chadebec<sup>1</sup>, Patrice Labie<sup>1</sup>, Yann Le Floch<sup>2</sup>, and Gérard Meunier<sup>1</sup>

<sup>1</sup>LEG/ENSIEG, 38402 Grenoble Cedex, France

<sup>2</sup>Laboratoire d, ENSIEG, Saint Martin d, France

<sup>2</sup>CEDRAT, 38246 Meylan Cedex, France

The approach of magnetic scalar potential  $T_0\phi$ - $\phi$  is widely popular for solving magnetic problems coupled with electric circuits. However, cuts are required to preserve the Ampere's law for the case of ferromagnetic circuits with holes surrounded by coils. The manual definition of cuts might not be evident even for experimented finite-element (FE) users. This paper presents a new automatic cuts generation algorithm. First, we present a tool that allows determining if a ferromagnetic region has holes or not. If holes are found, the region is then inflated to get automatically needed cuts. The presented algorithm is also applicable to generate electric cuts in massive conductors.

*Index Terms*—Automatic cut, finite-element (FE) method, multiply connected regions, total magnetic scalar potential.

## I. INTRODUCTION

THE formulation in term of total magnetic scalar potential  $T_0\phi$ - $\phi$  is known for its robustness and efficiency to solve magnetic problems coupled with electric circuits [1], [2]. The main advantage of this formulation is to lead to one unknown per node in air and ferromagnetic regions and then to save significant computation time.

Nevertheless, in case of ferromagnetic regions with holes surrounded by coils, the well-known connexity problem occurs and cuts are then required to verify Ampere's law. The manual definition of cuts could not be obvious even for experimented FE users. This paper proposes an algorithm to detect ferromagnetic region with holes and subsequently to find appropriate cut surfaces.

In the next section, we will describe the  $T_0\phi$ - $\phi$  formulation and clarify the reason that makes us to generate automatically the cuts.

## II. $T_0\phi$ - $\phi$ FORMULATION

The  $T_0\phi$ - $\phi$  formulation is able to treat problems with magnetic devices coupling with electric circuits. By using this formulation, we have in the air region  $\Omega_0$

$$\mathbf{H} = I \cdot \mathbf{t}_0 - \mathbf{grad}\phi \quad (1)$$

$$\mathbf{B} = \mu_0 \mathbf{H} \quad (2)$$

where  $\mathbf{t}_0$  is computed in  $\Omega_0$  region with a unit current in the inductor and satisfying  $\mathbf{t}_0 \times \mathbf{n} = 0$  on its boundary  $\partial\Omega_0$ .

In magnetic circuit region  $\Omega_m$ , in order to circumvent the so-called cancellation error caused by the subtraction of two great numbers, it is convenient to use the total scalar magnetic potential  $\phi$

$$\mathbf{H} = -\mathbf{grad}\phi \quad (3)$$

$$\mathbf{B} = \mu \mathbf{H}. \quad (4)$$

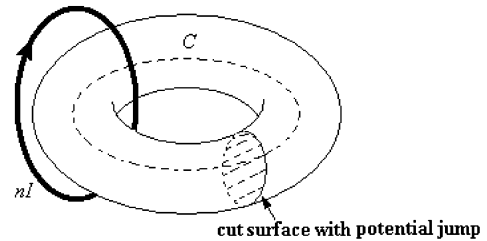


Fig. 1. Closed magnetic circuit surrounding by coils.

On using nodal variable, we compute  $\mathbf{t}_0$  in the region  $\Omega_0$  such as

$$\mathbf{t}_0 = \mathbf{h}_0 - \mathbf{grad}(\delta\phi) \quad (5)$$

where

- $\mathbf{h}_0$  is the field in  $\Omega_0$  region created by a unit current in the inductor, calculated with Biot-Savart law.
- $\delta\phi$  is the reduced-total increment [3], [4] calculated with a unit current in the inductor with the respect of the condition of changing between different domains (in this case, between  $\Omega_0$  and  $\Omega_m$ ). Therefore, we compute  $\delta\phi$  such as  $[\mathbf{h}_0 - \mathbf{grad}(\delta\phi)] \times \mathbf{n} = 0$  on  $\partial\Omega_0$ .

This formulation is very attractive because only one unknown per node in  $\Omega_0$  and  $\Omega_m$  is needed. However, in case of a multiply connected region  $\Omega_m$  surrounded by coils, Ampere's law is not verified. This problem is known as connexity problem. Let us consider Fig. 1.

On the closed-loop C, the  $\mathbf{H}$  circulation must be equal to  $nI$  such as (6)

$$\oint_C \mathbf{H} \cdot d\mathbf{l} = nI. \quad (6)$$

Whereas it is equal to zero in this configuration

$$\oint_C \mathbf{H} \cdot d\mathbf{l} = \oint_C -\mathbf{grad}(\phi) \cdot d\mathbf{l} = 0. \quad (7)$$

The solution which was found to solve this problem is to allow a discontinuity of the potential in the magnetic circuit by creating

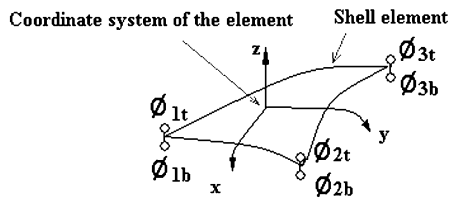


Fig. 2. Shell element.

a cut. The value of the discontinuity of potential will be imposed by the total current (nI) which surrounds the magnetic circuit [3]. The cut is then modeled by shell elements [5] with double nodes (see Fig. 2). Each couple of double nodes is at the same coordinates and each node is at different sides of the cut (top and bottom in the example). We can now consider that  $\Omega_m$  have no more holes and  $\delta\phi$  is now computed on  $\partial\Omega_0$  and on the cut itself. In such a way, Ampere's law is naturally restored.

Then, we can see that it could be difficult for an FE user to understand the problem and even for an experimented one to avoid mistakes. The best way seems to automate the total procedure. In the next section, we will investigate in a new method to generate the cut. We will then clarify its pertinence compared to others approaches.

### III. INFLATING TO GENERATE CUTS

#### A. Classical Approach

Lots of works have already been realized on the subject [3], [6], [7], and [8]. Most part of them search cutting surfaces by deflating scalar magnetic region around each coil. They obtain a surface which blocks up the hole of the coil. The cut is then the intersection between this surface and the ferromagnetic region. These approach leads to efficient results but has some drawbacks. First, this deflation algorithm works on a high number of elements (air and ferromagnetic region), and then the method might be time consuming. Moreover, for a magnetic circuit with two surrounding coils and one hole only; two cutting surfaces are found and only one is needed.

We propose a new approach similar to previous method which enables to suppress theses problems. Instead of deflating the scalar potential regions, the basic idea is to inflate the ferromagnetic circuit region.

In this section, we study a magnetic circuit which needs to be cut. First, we are going to describe a tool that permit to compute the number of holes for a given magnetic circuit. Second, we will explain the procedure to generate the required cuts.

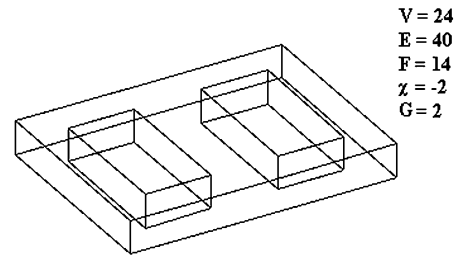
#### B. With Holes or Not?

The tool that we use to detect a region with holes is the *Euler–Poincaré* characteristic  $\chi$  and the genus  $G$  of the region [9]

$$\chi = V - E + F \quad (8)$$

$$G = (2 - \chi)/2 \quad (9)$$

where  $V$ ,  $E$ , and  $F$  are, respectively, the number of vertices, edges, and faces representing one region. The genus  $G$  is the number of holes of the region. An example is presented in Fig. 3.

Fig. 3. Characteristic *Euler–Poincaré* for two-hole magnetic circuit.

In this example, we can see that the use of *Euler–Poincaré* characteristic is able to recognize holes of one region and then to point out if this region needs cuts or not.

#### C. Inflate the Magnetic Circuit to Make Cuts

The goal of algorithm is from a mesh of a multiply connected magnetic circuit, to find out a set of facets which make the circuit simply connected.

The basic idea is to simulate the growth of a balloon injected inside the region until it occupies the whole region. Then the meeting part of balloon's boundary will form the appropriated cuts. We can easily figure out that cuts generated will make the region simply connected. However, we will see that this process is going to generate a set of surfaces. Only some of them will cut really the magnetic circuit, it will be then necessary to keep the good ones and to eliminate the others.

Let us now explain our algorithm precisely.

##### 1) Initialization:

- Grouping volume of magnetic circuits into connected regions.
- From the finite-element (FE) mesh, return all elements in each connected region.
- Set all these elements as unvisited.
- For each group, choose an arbitrary element which will be the initial set  $I$  of the balloon and mark it visited at range  $k = 0$  (see Fig. 4).

2) Iteration: We repeat the following procedure until there are no more elements unvisited.

- Return all the neighbors of the initial set  $I$ .
- For each neighbor:
  - If it was marked at rang  $r$  superior or equal to  $k$ , stock the facet between this neighbor and the involved element into the set  $K$  of acceptable facets for cut.
  - If the neighbor is unvisited, add this element to the temporary set  $T$  and mark it at rang  $k + 1$ .
  - Else, continue with the next neighbor element.
- Replace the initial set  $I$  by the temporary set  $T$  and so on.

We can see that the previous procedure ensures that the region defined by all the visited elements remains simply connected. The volume of the multiply connected magnetic circuit will be occupied step by step by the balloon's volume. The procedure works in parallel if there is more than one group of multiply connected magnetic circuits. This allows the algorithm to go faster.

3) Filter: After having visited all elements, we obtain a set  $K$  of acceptable facets cut. These facets are grouping in different

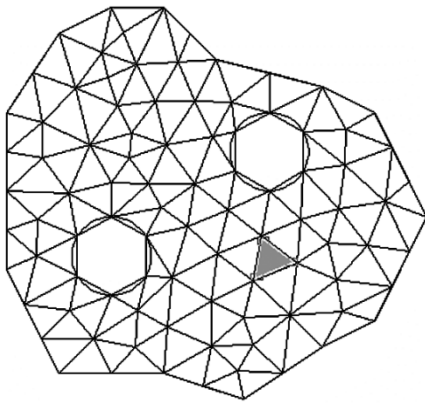


Fig. 4. Mesh and an arbitrary initial element.

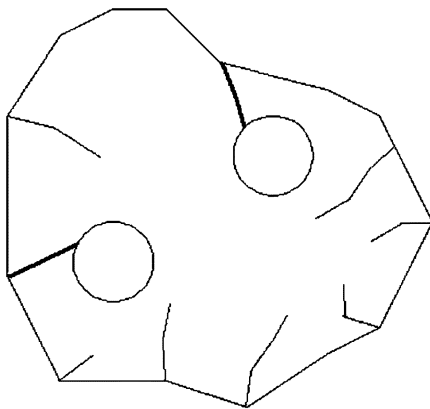


Fig. 5. Acceptable facets for cut AF.

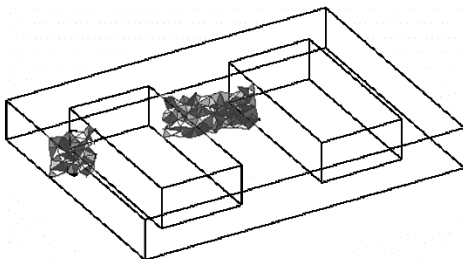


Fig. 6. Final cut obtained on 3-D multiply connected magnetic circuit.

connected surfaces. Only some of them are the real cut. For example, in Fig. 5, the real cut must be the both bold lines.

We need then to eliminate the non part of cut. These parts are distinguished from the real cut by their floating edges in the magnetic circuit region.






Finally, we get the resulting cuts. We present here a two-hole magnetic circuit and the cuts obtained with our approach (see Fig. 6).

#### D. Computation Time

The process of inflation is very quick. We can explain this point because the method is based only on the relationship between different neighbor elements. With a standard FEM database, the process of inflation is nearly instantaneous.

The most time consuming part of the algorithm is to eliminate the non part of cuts (step 3). The total needed time to generate

TABLE I  
COMPARISON BETWEEN INFLATING METHOD AND DEFLATING METHOD

Case	Configuration	Number of required cuts	Number of cuts generated	
			By inflating	By deflating
1		1	1	1
2		1	1	2
3		1	2	1
4		2	2	2
5		1	2	1

the complete cuts for the problem presented in the following is about 7 s on a Pentium IV 1.8 GHz 512 Mb of RAM. The problem was modeled with more than 64 000 elements and most of them are in the magnetic circuit. This example gives an idea of speed of the method.

#### IV. DISCUSSION

Four points can be discussed about this method. First, the time consuming of this technique is reduced in comparison to air deflating techniques [5]. It works with elements of the magnetic circuit, region which has fewer elements than the whole problem. Therefore, the number of elements that this method has to treat is small, so do the time.

Second, most part of time, the number of generated cuts is equal to the number of holes. We present in Table I a summary of some cases in which we compare the air deflating approach and our method.

For case 1 and 4, both methods are equivalent leading to the good number of cuts. For case 2, our method returns one cut and only one is needed. For the third case, the deflating approach is superior to ours which generates too much cuts. Finally, in case 5, our method leads to a too high number of cutting surfaces. It must be pointed out that to generate a too high number of cuts is not really a problem. In fact, if a coil does not surround a closed magnetic circuit, the reduced-total increment  $\delta\phi$  calculated on a nonneeded cut will be naturally imposed to zeros by the formulation. The problem of nonneeded cut generation will only increase the computation time of the method by useless integration.

Third, the generated cut does not depend on any auxiliary object like inductors but directly on the mesh of the magnetic circuit. This is an advantage in comparison of [5] for problems with nonmesh inductors, the deflating method being based on the mesh of coils. Moreover, we can notice that for some configurations, we will get fewer cuts than holes if there are some intersections between cuts. But the important point is that the multiply connected magnetic circuit will be always cut into simply connected one.

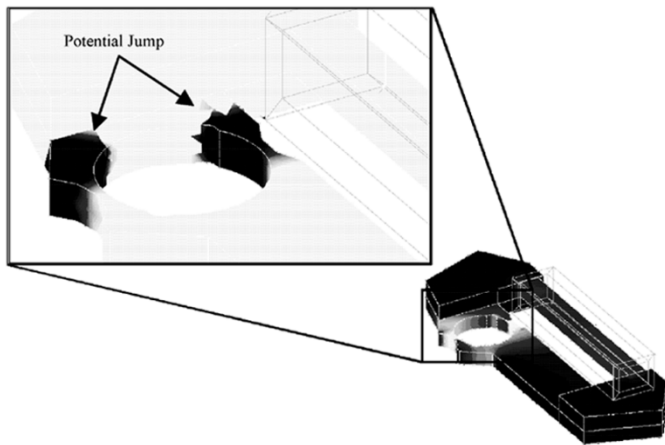


Fig. 7. Magnetic field of the watch motor obtained with automatic cuts. Zoom area represents the potential jump.

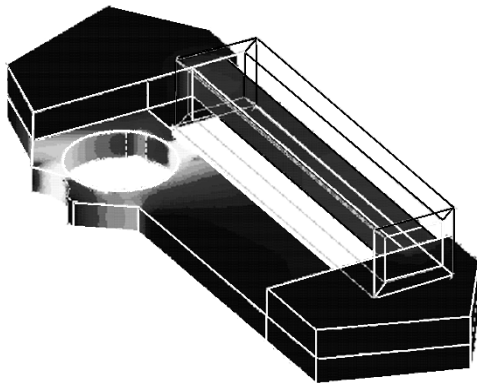


Fig. 8. Magnetic field of the watch motor with manual cuts.

Fourth, this method works with all kind of multiply connected region. Therefore, we can use it for the case of multiply connected massive conductor. In this configuration, we also need cuts, but electric ones [10].

## V. RESULTS ON AN INDUSTRIAL EXAMPLE

We have implemented the algorithm in FLUX—software package which enable to take into account magnetic problems coupled with electric circuits. The example presented in Fig. 7 is a watch motor which is composed by a permanent magnet, a multiply connected magnetic circuit highly saturated. It is coupled with an external electric circuit. The total problem was modeled with 46 379 elements.

The computation of characteristic *Euler–Poincaré* returns two holes. The execution of the algorithm of inflation returns

two sets of cuts. The total time needed to execute all the inflating process was 2 s on a Pentium IV 1.8 GHz 512 Mb of RAM.

We present here the magnetic field in the magnetic circuit for a current of 0.02 mA in the inductor.

The Fig. 8 shows results obtained in the same condition but with a manual cuts. Both results present a very good adequation.

## VI. CONCLUSION

We have presented a new algorithm to generate the automatic cut in case of magnetic circuits with holes. The scheme proposes a powerful tool to generate cuts on any multiply connected configuration without taking into account any auxiliary object. This contribution strongly improves the computation time of cuts generation by working within in the low number of elements. Moreover, the approach is general and can be applied for the generation of electric cuts.

## REFERENCES

- [1] S. Bouissou, F. Piriou, C. Kieny, and G. Tanneau, "A numerical simulation of a power transformer using 3-D finite element method coupled to circuit equation," *IEEE Trans. Magn.*, vol. 30, no. 5, pp. 3224–3227, Sep. 1994.
- [2] Y. LeFloch, G. Meunier, C. Guérin, P. Labie, X. Brunotte, and D. Boudaud, "Coupled problem computation of 3-D multiply connected magnetic circuits and electric circuits," *IEEE Trans. Magn.*, vol. 39, no. 3, pp. 1725–1728, May 2003.
- [3] H. T. Luong, "Amélioration de la formulation en potentiel scalaire magnétique et génération au couplage entre équations de champ et de circuit électrique," Ph.D. dissertation, Institut National Polytechnique de Grenoble, Grenoble, France, Sep. 1994.
- [4] I. D. Mayergoyz, M. V. K. Chari, and J. D'Angelo, "A new scalar potential formulation for the three-dimensional magnetostatic problems," *IEEE Trans. Magn.*, vol. MAG-23, no. 6, pp. 3889–3894, Nov. 1987.
- [5] C. Guérin, G. Tanneau, G. Meunier, X. Brunotte, and J. B. Albertini, "Three dimensional magnetostatic finite elements for gaps and iron shells using magnetic scalar potential," *IEEE Trans. Magn.*, vol. 30, no. 5, pp. 2885–2888, Sep. 1994.
- [6] P. J. Leonard, H. C. Lai, R. J. Hill-Cottingham, and D. Rodger, "Automatic implementation of cuts in multiply connected magnetic scalar region for 3-D eddy current models," *IEEE Trans. Magn.*, vol. 29, no. 2, pp. 1368–1371, Mar. 1993.
- [7] C. S. Harold and J. Simkin, "Cutting multiply connected domains," *IEEE Trans. Magn.*, vol. MAG-21, no. 6, pp. 2495–2498, Nov. 1985.
- [8] P. R. Kotiuga, "An algorithm to make cuts for magnetic scalar potentials in tetrahedral meshes based on the finite element method," *IEEE Trans. Magn.*, vol. 25, no. 5, pp. 4129–4131, Sep. 1989.
- [9] W. Fulton, "Algebraic topology: A first course," in *Graduate Texts in Mathematics*. New York: Springer-Verlag, 1995, vol. 153.
- [10] G. Meunier, Y. Le Floch, and C. Guérin, "A nonlinear circuit coupled  $t$ - $0$ - $\Phi$  formulation for solid conductors," *IEEE Trans. Magn.*, vol. 39, no. 3, pp. 1729–1732, May 2003.