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Xavier Margueron, Jean-Pierre Keradec, Abdelhadi Besri. Complete analytical calculation of static leakage parameters. Application to planar HF transformer optimization. IEEE Industrial Application Society 2007, Sep 2007, New Orléans, United States. hal-00289145

HAL Id: hal-00289145

<https://hal.science/hal-00289145>

Submitted on 13 Feb 2009

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Complete Analytical Calculation of Static Leakage Parameters. Application to HF Transformer Optimization

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Abstract—The pre-determination of transformer leakage inductances is essential for component designers. Except for special winding disposal, usual analytical methods of evaluation are useless and fem simulation evaluation, even more widely applicable, requires a time unsuitable with any optimization process. Work presented here is based on an analytical method we published recently about leakage inductances calculation. In the present paper, an explicit expression of leakage inductances is given. This expression does not appear as series so it requires short computer time and it opens a lot of future applications relying on optimization software. Moreover, the calculation is not restricted to leakage inductances: it has been extended to all the parameters characterizing the whole leakage behavior part of any transformer.

Keywords—leakage inductances; analytical calculation; transformer parameters; planar technology.

I. INTRODUCTION

Leakage inductances of a transformer have a strong impact on stresses sustained by switches so they are of major concern for power electronic designers. Contrary to magnetizing inductances, their values can't be easily predetermined by computation so they are often measured in a post-conception phase or evaluated before, using long fem simulations.

Because leakage inductances may cause over voltages on and extra losses in power electronic switches, designers must predict their values to be sure that converters will work properly. Some analytical methods are available [1-3] but they are restricted to particular geometries. Some others, which rely on series expansion, are more widely applicable but they require a lot of terms that slow the calculation.

Three years ago we worked on a semi-analytical method based on 2D PEEC formulae [4]. Potential vector was analytically expressed but, in order to reach energy values, a last integration was still carried out numerically. This last integration is now solved analytically so leakage inductances can be evaluated using only one explicit formula. The method can be applied to any kind of transformer winding, with quite no restriction on conductor's arrangement. Such a presentation suits very well to an optimization process [5].

This energy calculation only assumes that total current is null in transformer window, so it can be used also to determine all the parameters which characterize the behavior of any n-winding transformer (which has, at least, one winding short-circuited). These parameters are grouped in what we call "leakage transformer" [6]. This approach allows these parameters to be evaluated at frequencies where eddy current effects are negligible. The method is very useful, particularly in the case of planar transformer, where windings arrangements may be more complicated than in usual round winding transformers. An industrial planar transformer supplies a realistic example. Measurements carried out on it will be compared to our analytical results.

II. LEAKAGE ENERGY CALCULATION

A. Definition of Leakage Inductance

Inductive equivalent circuit of a two winding transformer can be represented by two inductances, three resistances and one perfect coupler (Fig. 1):

- r_1 and r_2 : Winding resistances seen from primary.
- L_m : Magnetizing inductance.
- L_f : Leakage inductance.
- R : Magnetic losses resistance.
- η : Coupler ratio.

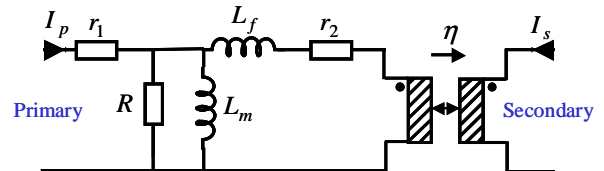


Figure 1. Two winding transformer inductive equivalent circuit

If coupling is strong, transformation ratio η is very close to the turn number ratio and leakage inductance L_f is far smaller than magnetizing one L_m . L_f influence appears on an

impedance measurement when secondary is short circuited (Fig. 2). Its measurement can be realized only in a specific frequency range (zone 4). Others zones are respectively dominated by primary resistance (zone 1), magnetizing inductance (zone 2), resistances of both windings (zone 3) and capacitances in high frequency (zone 5).

If both windings (primary and secondary) are supplied by opposites currents whose ratio is equal to coupling ratio η , current in magnetizing inductance is null. Ampere turns are then compensated and, in frequency range 4, magnetic energy is stored in leakage inductance (1).

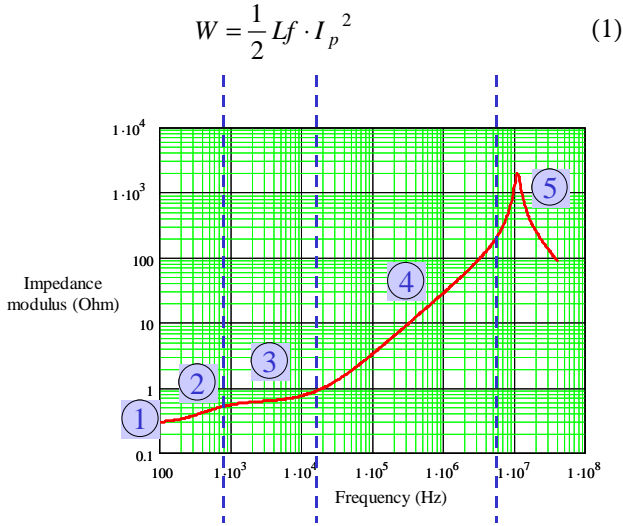


Figure 2. Impedance measurement- secondary short circuited

In this paper we focus on static leakage inductances. They correspond to the inductances seen at low frequencies (i.e. such that current inside conductors is uniform). For higher frequencies, eddy currents reduce these values.

B. Leakage Energy Calculation Method

Leakage inductance calculation methods are based on magnetic energy stored when ampere-turns are compensated [1-3]. This energy can be computed with magnetic excitation \vec{H} (2) or potential vector \vec{A} (3). Integral of (2) must be extended to infinity while integral (3) is only limited to conductors, where \vec{J} is not null.

$$W = \frac{1}{2} \iiint \mu \cdot \vec{H}^2 \cdot dV \quad (2)$$

$$W = \frac{1}{2} \iiint \vec{A} \cdot \vec{J} \cdot dV \quad (3)$$

with μ : permeability
 \vec{J} : current density

In order to simplify calculations, transformer windings are considered as 2D plan systems with a length equal to the mean length turn. Energy per unit of length is noted Wl (4) (5).

$$Wl = \frac{1}{2} \iint \mu \cdot \vec{H}^2 \cdot dS \quad (4)$$

$$Wl = \frac{1}{2} \iint \vec{A} \cdot \vec{J} \cdot dS \quad (5)$$

According to conductor shape and arrangement in planar transformer (Fig. 3), the method introduced in [4] is based on exact formulation of potential vector created by a straight rectangular cross section conductor (Fig. 4).

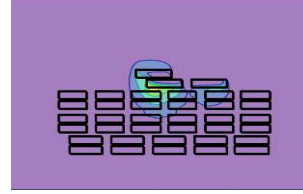


Figure 3. Planar transformer section

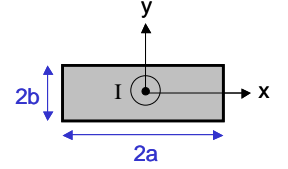


Figure 4. Rectangular conductor

All materials are considered as linear, homogenous and isotropic. Then, three more hypotheses have to be made:

- Studied device is supposed to be 2D plan. Currents are perpendicular to conductor sections and they are also uniform inside each wire (static case).
- Transformer is in leakage inductance "measurement situation": Total current in each window is null.
- Windings are made of either rectangular wires either round conductors grouped into rectangular regions.

Our elementary field source is an infinite straight wire, having a rectangular cross-section (Fig. 4), in which a homogenous current flows. Vector potential (6) is obtained by integrating that of a thin wire, from $x = -a$ to $+a$ and from $y = -b$ to $+b$. In a $Oxyz$ base, potential vector is directed along Oz .

In our previous paper [4], the double integration which appears in (5) still had to be made numerically. Recently, we find the analytical expression (7) of this integral which is that of the $F(X, Y)$ function written in (6).

C. Example for Two Rectangular Conductors

To illustrate the computing method and then, obtain only a unique formula for leakage inductance calculation, let's take the example of a transformer window occupied by only one pair of rectangular wires (Fig. 5) indexed 1 and 2.

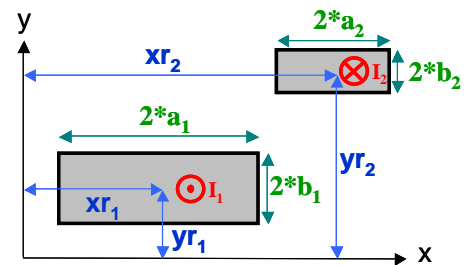


Figure 5. Rectangular conductors

$$A(x, y) = \frac{-\mu_0 \cdot J}{4\pi} \left[F(X, Y) \right]_{X=x-a}^{X=x+a} \Big|_{Y=y+b}^{Y=y-b}$$

$$\text{with } F(X, Y) = XY \ln(X^2 + Y^2) + X^2 \arctan\left(\frac{Y}{X}\right) + Y^2 \arctan\left(\frac{X}{Y}\right) \text{ and } J = \frac{I}{2a \cdot 2b} \quad (6)$$

$$G(X, Y) = \iint F(X, Y) \cdot dXdY$$

$$= -\frac{1}{24} (X^4 - 6X^2Y^2 + Y^4) \cdot \ln(X^2 + Y^2) + \frac{1}{3} XY \left(X^2 \arctan\left(\frac{Y}{X}\right) + Y^2 \arctan\left(\frac{X}{Y}\right) \right) - \frac{7}{24} X^2Y^2 \quad (7)$$

Each wire is supplied with a total current equal respectively to I_1 and I_2 , that, at the end, are supposed to be opposite.

Vector potential due to these two wires (8) is the sum of two contributions similar to (6). Combining (5) and (8) leads to the expression of total energy per unit of length Wl (9).

$$A(x, y) = \sum_{k=1}^2 Az_k(x - xr_k, y - yr_k) \quad (8)$$

$$Wl = \frac{1}{2} \iint_{Wcs \cup Wcs2} J \cdot [A_1(x - xr_1, y - yr_1) + A_2(x - xr_2, y - yr_2)] \cdot dxdy$$

$$= \frac{1}{2} J_1 \iint_{Wcs1} (A_1 + A_2) \cdot dxdy + \frac{1}{2} J_2 \iint_{Wcs2} (A_1 + A_2) \cdot dxdy \quad (9)$$

with $Wcs1$ and $Wcs2$: Wire cross section areas

Value of (9) is obtained by varying the $G(X, Y)$ function (7) over wire cross section Wcs (10).

Vector potential created by the conductor centred in (0,0) is deduced from the double variation (6). This variation must be included in the $G(X, Y)$ expression. Other wires are located by (xr_k, yr_k) and they all must be taken into account.

Formula (10) can then be rewritten as (11) to obtain the exact formulation of magnetic energy per length unit for two

$$Wl = \frac{1}{2} J_1 \iint_{Wcs1} (A_1 + A_2) \cdot dxdy + \frac{1}{2} J_2 \iint_{Wcs2} (A_1 + A_2) \cdot dxdy$$

$$= \frac{1}{2} J_1 \left[[G_1 + G_2]_{X=x-a_1}^{X=x+a_1} \Big|_{Y=y-b_1}^{Y=y+b_1} \right] + \frac{1}{2} J_2 \left[[G_1 + G_2]_{X=x-a_2}^{X=x+a_2} \Big|_{Y=y-b_2}^{Y=y+b_2} \right] \quad (10)$$

$$Wl = \frac{1}{2} J_1 \left[\left[-\frac{\mu_0}{4\pi} J_1 [G_1(X - xr_1, Y - yr_1)]_{X=x-a_1}^{X=x+a_1} \Big|_{Y=y+b_1}^{Y=y-b_1} - \frac{\mu_0}{4\pi} J_2 [G_2(X - xr_2, Y - yr_2)]_{X=x+a_2}^{X=x-a_2} \Big|_{Y=y+b_2}^{Y=y-b_2} \right]_{x=xr_1-a_1}^{x=xr_1+a_1} \Big|_{y=yr_1-b_1}^{y=yr_1+b_1} \right]$$

$$+ \frac{1}{2} J_2 \left[\left[-\frac{\mu_0}{4\pi} J_1 [G_1(X - xr_1, Y - yr_1)]_{X=x-a_1}^{X=x+a_1} \Big|_{Y=y+b_1}^{Y=y-b_1} - \frac{\mu_0}{4\pi} J_2 [G_2(X - xr_2, Y - yr_2)]_{X=x+a_2}^{X=x-a_2} \Big|_{Y=y+b_2}^{Y=y-b_2} \right]_{x=xr_2-a_2}^{x=xr_2+a_2} \Big|_{y=yr_2-b_2}^{y=yr_2+b_2} \right] \quad (11)$$

rectangular conductors. With $J_2 = -J_1$, This energy is proportional to J_1^2 . This leads to the inductance per length unit. Neglecting the influence of the magnetic core, the leakage inductance is obtained by multiplying the previous value by the mean turn length.

D. Generalization for Transformer Winding

Formula (11) is easily extended to N conductors (12). Always assuming that total current in the window is null, this expression gives the exact value of energy per length unit. This leakage energy depends on $N-1$ independent currents and it is characterized by $N(N-1)/2$ independent coefficients.

The formula is completely analytical and explicit so it is quickly computed and well suited to be used in an optimization software.

III. ACCOUNTING FOR MAGNETIC MATERIAL

Although magnetic core has a small impact [4] on values regarding short-circuit conditions (often less than 5% on leakage inductances), it must be taken into account, even roughly, to obtain corresponding parameters with a suitable accuracy.

Magnetic image technique [7] supplies a simple way to approximately account for magnetic material influence.

$$Wl = -\frac{1}{2} \frac{\mu_0}{4\pi} \sum_{n=1}^N J_n \left[\sum_{k=1}^N \left(J_k \left[G(X - xr_k, Y - yr_k) \right]_{\substack{X=x-a_k \\ X=x+a_k}}^{\substack{Y=y-b_k \\ Y=y+b_k}} \right) \right]_{\substack{x=xr_n-a_n \\ x=xr_n+a_n}}^{\substack{y=yr_n-b_n \\ y=yr_n+b_n}} \quad (12)$$

A. Magnetic Images for Infinite Material

The goal of this technique is to replace the magnetic material by currents located in its volume in order to obtain the same magnetic field in the air.

To begin, let's recall a well known problem. A thin straight wire, parallel to the plan which limits two semi-infinite materials of respective relative permeability 1 and μ_r (Fig. 6) carries a current I . One can show that the field in the air is the sum of the one (H_i) created by the wire alone and another one (H_r) created by another wire, called "image wire". This wire is symmetrically located on the other side of this interface, and a current I' flows through it (13).

$$I' = I \frac{\mu_r - 1}{\mu_r + 1} \quad (13)$$

More generally, if a current density parallel to the initial wire exists in the air, the magnetic material can be replaced by a symmetrical current density after multiplication by the same ratio.

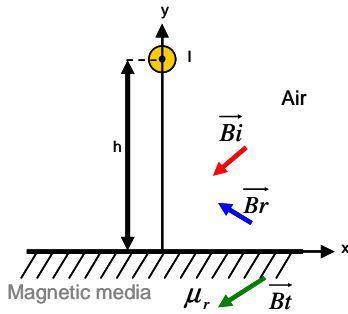


Figure 6. Wire and infinite magnetic material

B. Influence of Ferrite Thickness on Images

Before applying the image technique to the magnetic core, it is indispensable to answer this question: is the same technique still applicable if magnetic material has a finite thickness? The answer cannot be always positive, especially when the thickness tends to zero or the relative permeability reaches 1.

With this in mind, the influence of magnetic core thickness has been investigated. Semi-infinite material of Fig. 6 has been replaced by a magnetic layer of thickness ep (Fig. 7). In this case, reflected field H_r , is expressed as (14).

This expression (14) is divided in two terms:

- The first one is identical to the field reflected by an infinite media. This field is created by an image wire located in $(0, -h)$ and supplied by a current given by the expression given above (13).

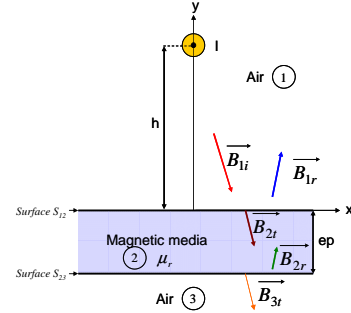


Figure 7. Influence of finite magnetic material (thickness ep)

- The second one corresponds to the field of an infinite set of thin wires. The n^{ieth} is placed at $(0, -h - 2n \times ep)$ and it carries a current $I[(\mu_r - 1)/(\mu_r + 1)]^{2n}$. This sum comes from multiple reflections on magnetic material faces. It is also multiplied by a factor $-4\mu_r/(\mu_r^2 - 1)$.

The field H_r , added to the incident field H_i , gives the total field in the air, on the wire side.

C. Comparison Between Finite and Infinite Thickness

Both analytical expressions have been presented in previous paragraph. In order to know what result (infinite or finite) must be used, results of both techniques must be compared. First term of (14) is identical to the field of infinite media. The second one decreases as n increases because $(\mu_r - 1)/(\mu_r + 1) < 1$. Thus, as n increases, image wire is less and less influent. For some situations, infinite model may be sufficient.

To find an acceptable approximation, we consider a wire placed at $h = 50mm$ and $x = 0mm$ of a ferrite media (Fig. 7). Tangent magnetic field on the surface at $x = 50mm$ is used to check both formulations (finite and the infinite) for two different values of permeability $\mu_r = 40$ and 2000 (Fig. 8).

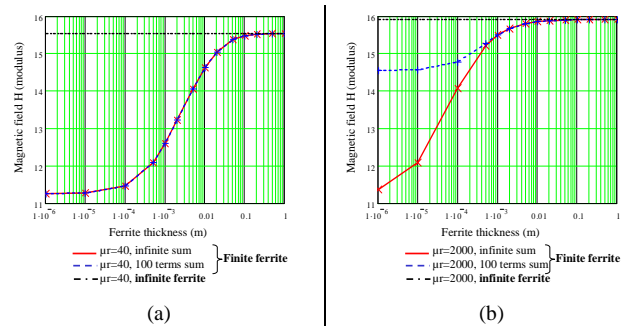


Figure 8. Comparison of finite and infinite formulation
(a) : $\mu_r=40$ | (b) : $\mu_r=2000$

$$H_{1r} = \frac{1}{2\pi} \left[H_{li} \left(x, y+h, I \frac{\mu_r - 1}{\mu_r + 1} \right) - \frac{4\mu_r}{\mu_r^2 - 1} \sum_{n=1}^{\infty} H_{li} \left(x, y+h+2nep, I \left(\frac{\mu_r - 1}{\mu_r + 1} \right)^{2n} \right) \right] \quad (14)$$

For both permeabilities, field obtained with finite thickness reaches the one obtained with an infinite magnetic material when thickness reaches a sufficient value. As thickness decreases test field reaches a limit value independent of the permeability: that of the incident field. However, this value is reached for a lower thickness when permeability is high. To sum up, as thickness grows, test field increases from the limit related to no magnetic material to another limit given by an infinite media. This growth reaches 90% of its maximum at 1mm for $\mu_r = 2000$ and 20mm with $\mu_r = 40$. For thickness higher than these values, reflected field is very close to that of a semi-infinite material.

In practical cases, thickness is not an important parameter for material whose permeability is upper than 2000. For permeability around 40, computation must be done using multiple reflections (14). The right resulting field is obtained by adding at least 20 to 25 images.

D. Application to Leakage Energy Calculation

In order to compute leakage energy, magnetic material can be taken into account with the use of magnetic images. Formulation depends on permeability and ferrite thickness. Application of such method is not very different with infinite or finite material. Only current magnitude in images, wire positions and necessary numbers of images are different.

In Fig. 9, 8 magnetic images are added to initial transformer windows. In most cases, with high permeability ferrite, these images are sufficient. Indeed, because total current in the windows is null, radiated field decreases so fast that images located further has negligible influence on that existing inside the window. Formula (12) is then modified into (15) in order to account for the effect of magnetic material.

Finally, (15) is the complete expression of leakage energy per unit of length inside transformer windows. This formula is totally analytical and can be used with optimization software.

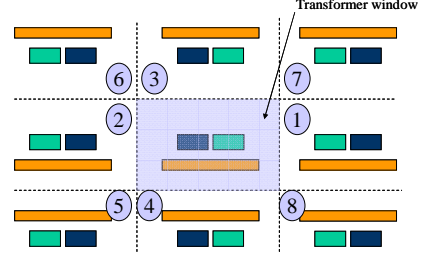


Figure 9. 8 magnetic images

IV. FROM LEAKAGE ENERGY TO LEAKAGE TRANSFORMER

Not only leakage inductances can be computed with this formula (15) but, in case of a transformer owning more than two windings, if leakage inductances are coupled, parameters depicting these properties, can be too.

The leakage transformer (which is defined for any transformer) describes the magnetic behavior that remains when one winding is short-circuited [6]. It is composed by leakage inductances and couplers that describe coupling between these inductances. For a 3 winding transformer (Fig. 10), leakage energy is computed when current in magnetizing inductance is null (16) (i.e. assuming primary is short-circuited). In this case, Ampere-turns are compensated.

In order to determine all the parameters of the leakage transformer, computation is made as in section II and III. Stored energy appears as a quadratic function of two independent currents so this function is fully described by three coefficients. Firstly, to evaluate inductance Lf_a (17), secondary current must be null ($I_s = 0$). Secondly, for inductance Lf_s defined by (18), auxiliary current must be null ($I_a = 0$).

Finally, a third combination of winding currents, the sum of which is always null ($I_p = -\eta_{ap}I_a - \eta_{sp}I_s$), allows the determination of the coupling ratio η_{sa} between secondary and auxiliary (19).

$$Wl = -\frac{1}{2} \frac{\mu_0}{4\pi} \sum_{n=1}^N J_n \left[\sum_{k=1}^N \left(J_k \left[G(X - xr_k, Y - yr_k) \right]_{X=x-a_k}^{X=x+a_k} \right)_{Y=y-b_k}^{Y=y+b_k} \right] + \sum_i MI_i \left[\right]_{x=xr_n-a_n}^{x=xr_n+a_n} \left[\right]_{y=yr_n-b_n}^{y=yr_n+b_n} \quad (15)$$

$$\text{With } MI_i = \sum_{k=1}^N \left(J_k \frac{\mu_r - 1}{\mu_r + 1} \left[G(X - xr_k - x_i, Y - yr_k - y_i) \right]_{X=x-a_k}^{X=x+a_k} \right)_{Y=y-b_k}^{Y=y+b_k} \quad (\text{infinite case})$$

and i : image number

(x_i, y_i) : image coordinates center

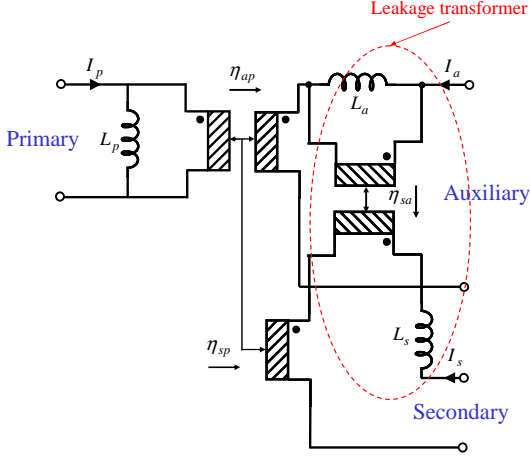


Figure 10. Leakage transformer from 3-winding transformer

$$I_p + \eta_{ap}I_a + \eta_{sp}I_s = 0 \quad (16)$$

$$Wm_1 = \frac{1}{2}L_a I_a^2 \Rightarrow Lf_a = L_a \quad (17)$$

$$Wm_2 = \frac{1}{2}(L_s + \eta_{sa}^2 L_a) I_s^2 \Rightarrow Lf_s = L_s + \eta_{sa}^2 L_a \quad (18)$$

$$Wm_3 = \frac{1}{2}Lf_s I_s^2 + \frac{1}{2}Lf_a I_a^2 + Lf_a \eta_{sa} I_s I_a \quad (19)$$

$$\Rightarrow \eta_{sa} = \frac{Wm_3 - \frac{1}{2}Lf_s I_s^2 + \frac{1}{2}Lf_a I_a^2}{Lf_a I_s I_a}$$

It is useful to evaluate the coupling coefficient of leakage inductances (20). This coefficient is indispensable to decide if coupling of leakage inductances can be neglected or not and, as a consequence, if the related coupler can be removed.

$$k_{sa} = \frac{M}{\sqrt{L_{11}L_{22}}} = \frac{\eta_{sa}L_a}{\sqrt{L_a(L_s + \eta_{sa}^2 L_a)}} = \eta_{sa} \sqrt{\frac{Lf_a}{Lf_s}} \quad (20)$$

Finally, all the magnetic static parameters depicting leakage transformer can be evaluated analytically. This process is general and can be applied to an n -winding transformer (having an $(n-1)$ winding leakage transformer) [6]. Only the number of current combinations (having a null sum) needed to obtain all the parameters varies.

V. APPLICATION TO PLANAR TRANSFORMER

An automatic tool for computing leakage inductances (and transformers) has been developed under Mathcad® software [8]. This tool has been use to identify static leakage transformer of an industrial planar transformer (Fig. 11). This flat device contains two identical 3-winding

transformers located side by side. We now focus on one of them to compare experimental results to those given by our static leakage computing tool.

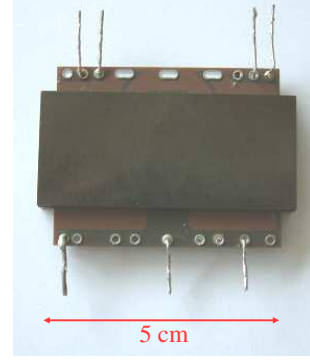


Figure 11. 3-winding planar transformer

A. Experimental Characterization

Measurements has been carried out using an Agilent 4294A impedance analyzer [9], following the method developed in [10]. Lossless equivalent circuit of such a transformer is presented in Fig. 10.

Leakage inductances are plotted in Fig. 12 and Fig. 13 versus frequency. As it has been told in section II.A, in low and high frequency, inductances are difficult to evaluate because they are masked by other components. Moreover, leakage inductances are often modified by eddy currents so, to model them in a wide frequency range, we place some Lp-Rp cells [10] in series with a pure inductance. In this case, static leakage inductance corresponds to the sum of all these serial inductances. Searched values appear on the low frequency side of model related curves shown in Fig. 12 and Fig. 13:

- $Lf_a = 72.9nH$
- $Lf_s = 69.8nH$

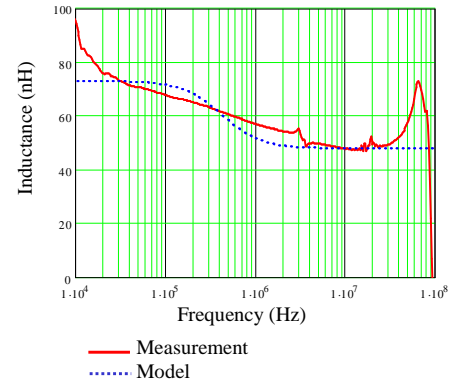


Figure 12. Auxiliary leakage inductance variation vs frequency

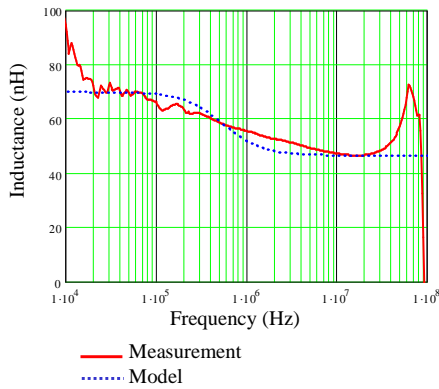


Figure 13. Secondary leakage inductance variation vs frequency

Coupling ratio η_{sa} between these two inductances has been evaluated to $\eta_{sa} = 0.64$. Then, coupling coefficient k_{sa} has been deduced: $k_{sa} = 0.65$.

B. Analytical Calculation

Conductor placement inside transformer windows has been described into our tool. Transformer windings look as shown in Fig. 14. Both windows, symmetrical relatively to central leg core, are similar. Primary is made of 17 turns and other windings are made of 3 turns.

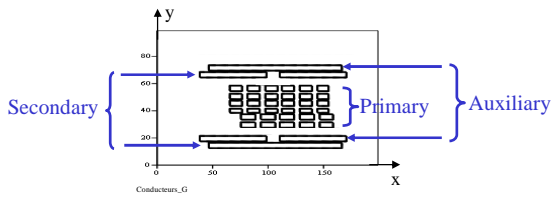


Figure 14. Transformer window description

Energy density in a window is presented in Fig. 15 for illustration.

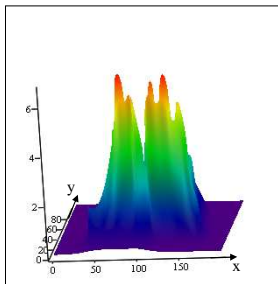


Figure 15. Energy density inside a transformer window

Leakage energy has been evaluated to $13.3 \mu J/m$ for Primary/Secondary and $13.7 \mu J/m$ for Primary/Auxiliary. These values lead respectively to $59.4 nH$ and $61.4 nH$ with a mean turn length estimated to $26 mm$. This value depends mainly on transformer geometry but energy repartition intervene too [4]. With (19) coupling ratio is evaluated to $\eta_{sa} = 0.77$. Coupling coefficient is also

calculated with (20): $k_{sa} = 0.78$. All these values are summed up in Table I.

TABLE I. LEAKAGE TRANSFORMER PARAMETERS

Parameters		Measurement	Computation	Error
P/S leakage inductance	Lf_s	$69.8 nH$	$59.4 nH$	15%
P/A leakage inductance	Lf_a	$72.9 nH$	$61.4 nH$	16%
A/S coupling ratio	η_{sa}	0.64	0.77	17%
A/S coefficient coupling	k_{sa}	0.65	0.78	17%

Finally, agreement of our computations and our experimental results is quite good: discrepancies are always less than 20%. These discrepancies can be attributed to both measurements and analytical calculation. In [10] we have shown that, even with meticulous care, we are unable to measure inductances with incertitude lower than about $8 nH$. On the other hand, our calculation tool assumes the transformer to be a 2D system, that is not exact, and it neglects extra wiring which links turns to external terminals [11]. For transformers having higher values of leakage inductances, agreement is generally far better.

Accuracy is not impressive for presented example. However, for wire arrangement shown in Fig. 14, usual methods of calculation are not applicable or they require a lot of term to converge toward a result accurate enough. Method presented here is also able to account for partial surrounding of windings by magnetic core [4]. Evaluated leakage inductances are static ones. Their values generally decrease when frequency grows because of eddy currents effects. We now work on the extension of the presented technique to high frequency.

VI. CONCLUSION

A complete analytical formula for computing static leakage parameters of a transformer has been introduced. This paper is completing a previous one by adding a fully analytical formulation. Proposed method applies as soon as wires, or winding cross sections, are rectangular.

Leakage energy computation assumes device geometry can be considered as a 2D plane. It is based on expression of vector potential created by a rectangular conductor carrying a uniform current. Magnetic material is taken into account using magnetic images, even if it doesn't completely surround the wires. Influence of magnetic material thickness has been investigated too.

A tool has been developed using Mathcad[®] software and it has been applied to industrial planar transformers. This tool proved to be effective for transformer designers. They are now able to quickly and accurately predict all static parameters, including inductances, regarding leakage.

Two extensions can now be accessed. The first one consists in coupling our developing tool with optimization software to improve device behaviour. The second one aims to account for eddy currents inside rectangular wires. This will permit these effects to be taken into account in transformer equivalent circuit.

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