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# Comments on Vasyliunas' and Pontius' studies of the effects of the planetary ionosphere and of the Coriolis force on the interchange instability

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[1] Recent Galileo and Cassini plasma and magnetic field observations indicate that the centrifugal/interchange instability plays a critical role in radial plasma transport in the rapidly rotating magnetospheres of giant planets. These observations have stimulated considerable interest in understanding the development and the properties of the instability. A complete description of the interchange instability involves an electric coupling between the magnetosphere and the planetary ionosphere. Vasyliunas (1994) pointed out that when the ionosphere is ineffective at constraining interchange motions, local magnetospheric conditions govern the plasma dynamics. In that case the Coriolis force leads to a dramatic decrease in the instability growth rate according to Pontius (1997), whereas it has a neutral influence according to Ferrière et al. (2001). The purpose of the present paper is to critically reexamine Vasyliunas' and Pontius' results and investigate the origin of their discrepancies with Ferrière et al.'s results. To that end, we consider the case of a thin plasma disk in an axisymmetric, rapidly rotating magnetosphere with no field-aligned electric currents at equilibrium.

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## 1. Introduction

[2] The structure and dynamics of giant planet magnetospheres are dominated by the presence within the magnetospheric cavity of several plasma sources and by the fast planetary rotation. The various plasma populations are trapped by the planetary magnetic field and confined near the equatorial plane by the centrifugal force, thereby giving rise to a thin disk of corotating plasma. The redistribution of the locally created plasma throughout the magnetospheric system is one of the fundamental dynamical processes occurring in giant planet magnetospheres. This redistribution is achieved through plasma transport perpendicular to magnetic field lines. The exact mechanisms responsible for this transport are not yet completely understood, with the transport probably operating through different modes and on different scales. It is, however, widely believed that the outward plasma transport is triggered by the centrifugal instability, a Rayleigh-Taylor type instability in which the centrifugal force plays the role of the gravitational force and that it proceeds through the interchange of magnetic flux tubes [e.g., Melrose, 1967; Ioannidis and Brice, 1971; Hill,

1976]. Physically, under the action of the centrifugal and magnetic buoyancy forces, mass-loaded flux tubes tend to exchange positions with nearly empty flux tubes located farther out.

[3] The recent Galileo and Cassini space missions have amply documented the Jovian and Saturnian magnetospheric systems. In particular, these missions have provided us with new observations and new insights into the mechanisms responsible for the outward plasma transport. There is now considerable observational evidence that centrifugally driven flux tube interchange is at play in the corotation-dominated regions of the Jovian and Saturnian magnetospheres and that it takes part in the redistribution of plasma throughout these systems. Signatures of intermittent, short-lived, mass-loaded, and empty flux tubes in the Io torus have been detected by the Galileo spacecraft, in orbit around Jupiter from December of 1995 to September of 2003 [Kivelson et al., 1997; Thorne et al., 1997; Bolton et al., 1997]. Similar signatures have been observed in the E-ring of Saturn by the Cassini spacecraft, in orbit around Saturn since July 2004 [André et al., 2005; Burch et al., 2005; Hill et al., 2005; Leisner et al., 2005; Mauk et al., 2005] and have lent further support to the notion that the centrifugal instability lies at the root of the outward transport.

[4] The influence of planetary rotation is felt through the centrifugal force and the Coriolis force. Whereas the centrifugal force is included (through an effective gravity) in most studies of the interchange instability, the Coriolis force is generally not taken into account, and its impact on the

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interchange instability has never been completely and rigorously addressed.

[5] In addition, the adjustment between the magnetospheric plasma flow driven by the interchange instability and the ionospheric electric field is imposed by an electric coupling between the magnetosphere and the ionosphere. Therefore a complete description of the interchange instability and of the resulting plasma motions requires the inclusion of the system of electric currents closing through the ionosphere.

[6] *Vasyliunas* [1994] discussed the efficiency of the ionosphere in controlling the magnetospheric plasma flow. He pointed out that when the ionospheric Pedersen conductivity is very small, the magnetosphere decouples from the ionosphere and the magnetospheric plasma flow is governed by local magnetospheric conditions.

[7] *Pontius* [1997] later extended *Vasyliunas*' [1994] study by tackling the problem of the influence of the Coriolis force on the interchange instability. He came to the conclusion that when the ionosphere is ineffective at constraining interchange motions, the Coriolis force governs the plasma dynamics and leads to a dramatic decrease in the instability growth rate. In this paper, we show that *Pontius*' predictions are not consistent with the properties of interchange motions found by *Ferrière et al.*'s [2001], who concluded that the Coriolis force has a neutral influence on the interchange instability.

[8] Our paper is divided as follows: In section 2, relying on the results of *Ferrière et al.* [2001], we write down the dispersion relation of the short-wavelength interchange mode in an axisymmetric, rapidly rotating magnetosphere with no field-aligned electric currents at equilibrium. We successively consider the general case of a plasma with nonvanishing temperature (section 2.1) and the particular case of a cold plasma (section 2.2), both under the thin-disk approximation. In section 3, we compare our dispersion relation with those obtained earlier by *Vasyliunas* [1994] and by *Pontius* [1997], and we discuss in detail the origin of the discrepancies. In section 4, we state the conclusions of our study.

## 2. Dispersion Relation of the Interchange Mode

[9] A detailed derivation of the dispersion relation of the interchange mode in an axisymmetric, rapidly rotating magnetosphere can be found in the work of *Ferrière et al.* [2001]. Here, we will be content to remind the reader of the underlying assumptions, describe the physical properties of the interchange mode, and write down its equation of motion as well as its dispersion relation.

[10] We use an orthogonal curvilinear coordinate system  $(\alpha, \beta, \zeta)$ , with unit vectors  $\hat{e}_\alpha, \hat{e}_\beta, \hat{e}_\zeta$  and scale factors  $h_\alpha, h_\beta, h_\zeta$ , defined as follows:  $\alpha$  and  $\beta$  are the Euler potentials [Northrop, 1963] representing the equilibrium magnetic field,  $\mathbf{B}_0 = \nabla\alpha \times \nabla\beta$ , and  $\zeta$  is the field-aligned coordinate, such that  $\hat{e}_\zeta \parallel \mathbf{B}_0$ . This choice of coordinates is motivated by our desire to avoid any mathematical complications resulting from the use of a nonorthogonal system, but it is clearly restrictive. First, orthogonal field-based coordinates do not exist for arbitrary magnetic field configurations (see *Salat and Tataronis* [2000] for a discussion on the conditions under which they exist). Second, the condi-

tion  $\mathbf{B}_0 \parallel \nabla\zeta$  implies  $\nabla \times \mathbf{B}_0 \perp \mathbf{B}_0$ , which means that the equilibrium state has no field-aligned electric currents. Let us emphasize that this restriction applies only to the equilibrium state, not to perturbations. Even so, it is generally not strictly satisfied in rotating magnetospheres.

[11] We consider the evolution of an ideal magnetohydrodynamic (MHD) plasma characterized by a mass density  $\rho$  and a thermal pressure  $P$ , rotating at rigid angular velocity  $\Omega$ , and subject to an effective gravity  $\mathbf{g}$  including a centrifugal component. The set of governing equations is composed of the continuity equation (with no mass loading, consistent with our assumption of rigid rotation), the momentum equation, the induction equation in the frozen-in approximation, and the adiabatic equation of state, all expressed in the rotating frame.

[12] Here, we only need to discuss the momentum equation. In the equilibrium state (denoted by a subscript 0), the plasma is assumed at rest in the rotating frame. All perturbations from the equilibrium state (denoted by a prefixed  $\delta$ ) are supposed to vary in time as  $\exp(-i\omega t)$ , with the wave frequency  $\omega$  possibly complex. The perturbed momentum equation can then be written as

$$\begin{aligned} -\rho_0\omega^2\delta\mathbf{r} = & -\nabla_\perp\delta P_T - \nabla_\parallel\delta P + \delta T_M\mathbf{c}_0 \\ & + \frac{B_0}{\mu_0} \left[ \frac{1}{h_\alpha} \frac{\partial}{\partial s} (h_\alpha\delta B_\alpha) \hat{e}_\alpha + \frac{1}{h_\beta} \frac{\partial}{\partial s} (h_\beta\delta B_\beta) \hat{e}_\beta \right] \\ & - (-\nabla_\perp P_{M0} + T_{M0}\mathbf{c}_0) \cdot \frac{\delta\mathbf{B}_\perp}{B_0} \hat{e}_\zeta \\ & + \delta\rho\mathbf{g} + 2i\omega\rho_0\Omega \times \delta\mathbf{r}, \end{aligned} \quad (1)$$

where  $\delta\mathbf{r}$  is the displacement vector,  $ds = h_\zeta d\zeta$  is the field-aligned increment,  $P_M = B^2/(2\mu_0)$  is the magnetic pressure,  $P_T = P + P_M$  is the total (thermal + magnetic) pressure,  $T_M = 2P_M$  is the magnetic tension, and  $\mathbf{c}_0$  is the magnetic curvature vector. As usual, the symbols  $\perp$  and  $\parallel$  indicate the directions perpendicular and parallel, respectively, to the magnetic field.

[13] The different terms on the right-hand side of equation (1) successively represent the perpendicular gradient in total pressure, the parallel gradient in thermal pressure, the magnetic buoyancy force, the magnetic tension force resulting from field line bending, the projection of the perturbed magnetic force onto the direction of the background magnetic field, the gravitational buoyancy force, and the Coriolis force. Equation (1) shows that the Lagrangian displacement vector  $\delta\mathbf{r}$  depends on the perturbations in mass density  $\delta\rho$ , in magnetic field  $\delta\mathbf{B}$ , and in thermal pressure  $\delta P$ . These perturbations, in turn, are given as functions of  $\delta\mathbf{r}$  by the perturbed continuity equation, induction equation, and equation of state, respectively.

[14] We now restrict our attention to the case of a rapidly rotating magnetosphere whose equilibrium state is axisymmetric about the spin axis and symmetric with respect to the equatorial plane (in the sense that all scalar quantities as well as the latitudinal component of the magnetic field are symmetric, whereas the radial and azimuthal field components are antisymmetric, with respect to the equatorial plane). In this special case, the curvilinear coordinate  $\beta$  may be identified with azimuthal angle  $\phi$  and the associated scale factor  $h_\beta$  with distance to the spin axis. The coordinate  $\alpha$  may then be taken independent of  $\beta$ , such that the

orthogonality of our chosen coordinate system is satisfied. In addition, all the equilibrium parameters are independent of  $\beta$ , and the rotation vector, effective gravity, and magnetic curvature vector are orthogonal to  $\hat{e}_\beta$ :  $\Omega, \mathbf{g}, \mathbf{c}_0 \perp \hat{e}_\beta$ .

[15] An analytical dispersion relation can be obtained for the interchange mode in the framework of the WKB approximation, which supposes that perturbations have a wavelength short compared to the typical scale height of the equilibrium,  $H$ . In fact, the requirement of short wavelength applies only to the perpendicular direction. At this point, no restriction needs to be imposed in the parallel direction, insofar as field-aligned variations in the perturbations will be integrated out in the final dispersion relation. Accordingly, perturbations are taken to vary in space as  $\exp[i(\mathbf{k}_\perp \cdot \mathbf{r})]$  times an unspecified function of the field-aligned coordinate  $s$ , with  $k_\perp \gg (1/H_\perp)$ .

[16] The physical properties of the interchange mode can be inferred from the perturbed momentum equation (1). Without going through the rigorous mathematical derivation (given in the work of *Ferrière et al.* [2001] and summarized in appendix A), we can easily recover these properties with a simple physical argument. By definition, the interchange mode is a mode driven by buoyancy (of both gravitational/centrifugal and magnetic origins), and its frequency is on the order of the magnetic Rayleigh-Taylor frequency:  $\omega^2 \sim \omega_{\text{RT}}^2 \sim (C_s^2 + V_A^2)/H_\perp^2$ , where  $C_s$  is the adiabatic sound speed and  $V_A$  the Alfvén speed. This implies that thermal and magnetic forces, which are the drivers of the standard MHD modes in a uniform medium, must remain comparatively weak. More specifically, the magnetic tension force, which drives the Alfvén wave ( $\omega^2 = V_A^2 k_\parallel^2$ ), the parallel gradient in thermal pressure, which drives the sound wave ( $\omega^2 = C_s^2 k_\parallel^2$ ), and the perpendicular gradient in total pressure, which drives the fast magnetosonic wave ( $\omega^2 = (C_s^2 + V_A^2) k_\perp^2$ ), must all be comparatively weak, i.e., much weaker than for the standard MHD modes. This, in turn, implies that field-line bending, plasma compression in the parallel direction, and total (plasma + field-line) compression in the perpendicular direction must all be nearly “turned off.” In other words, the interchange mode must have weak gradients along field lines ( $\partial/\partial s \ll k_\perp$ ), be quasi-incompressible, both parallel and perpendicular to the magnetic field ( $\partial\delta s/\partial s \simeq 0$  and  $\mathbf{k}_\perp \cdot \delta\mathbf{r}_\perp \simeq 0$ ), and remain very close to total pressure balance ( $\delta P_T \simeq 0$ ). For the following, we note that to leading order in the small parameter  $\varepsilon \equiv 1/(k_\perp H_\perp)$ , the last two approximate identities may be considered exact, i.e.,  $\simeq 0$  may be replaced by  $= 0$ .

[17] We can now use the perpendicular quasi-incompressibility condition,  $k_\alpha \delta r_\alpha + k_\beta \delta r_\beta = 0$ , to eliminate  $\delta r_\beta$  from the perturbed momentum equation (1) and write its component along  $\mathbf{g}_\perp$  in the form

$$\begin{aligned} \omega^2 \rho_0 k_\perp^2 \delta r_\alpha = & -k_\beta^2 (g_\alpha \delta \rho + c_\alpha \delta \mathcal{T}_M) \\ & - i B_0^2 k_\beta \frac{\partial}{\partial s} \left( \frac{\delta j_\zeta}{B_0} \right) \\ & - 2i \omega \rho_0 \Omega_\alpha k_\alpha k_\beta \delta s, \end{aligned} \quad (2)$$

where  $\delta j_\zeta = \frac{i}{H_0} (k_\alpha \delta B_\beta - k_\beta \delta B_\alpha)$  is the perturbation in parallel electric current density, given by the parallel component of Ampère’s equation.

## 2.1. Plasma With Nonvanishing Temperature

[18] To obtain the dispersion relation of the interchange mode, we integrate equation (2) over a displaced flux tube, between its ionospheric footpoints assumed to be located at  $s = \pm L_i$  (ionospheric quantities are denoted by a subscript  $i$ ). Since in a rapidly rotating magnetosphere, most of the plasma is confined near the equator, we may approximate the slowly varying quantities appearing in mass-weighted integrals of the type  $\int \dots \rho_0 \frac{ds}{B_0}$  by their equatorial value (denoted by a subscript  $e$ ) and take them out of the integral sign (thin-disk approximation). For the Coriolis force (last term on the right-hand side of equation (2)),  $\Omega_{\alpha,e} = 0$  by symmetry, so that we may not simply replace  $\Omega_\alpha$  by  $\Omega_{\alpha,e}$ . However, it can be shown that the Coriolis term is negligible with respect to the inertial term (see Appendix B). In the end, the dispersion relation of the interchange mode in an axisymmetric, rapidly rotating magnetosphere can be cast in the simple form

$$\omega^2 = \frac{k_{\beta,e}^2}{k_{\perp,e}^2} (\omega_g^2 + \omega_m^2) - i \left( \frac{1}{\xi} \frac{k_{\alpha,e}^2}{k_{\perp,e}^2} + \xi \frac{k_{\beta,e}^2}{k_{\perp,e}^2} \right) \omega_\Sigma \omega, \quad (3)$$

where  $\xi$  is a geometrical factor defined by

$$\xi = - \frac{r_e}{\sin \theta_i} \frac{d\theta_i}{dr_e} \sin I, \quad (4)$$

with  $r_e$  the radial distance of the equatorial crossing point,  $\theta_i$  the colatitude of the ionospheric footpoint, and  $I$  the inclination of the ionospheric field lines to the local horizontal. In the particular case of a dipolar magnetic field, one has  $\xi = (1 + 3 \cos^2 \theta_i)^{-1/2}$ , which reduces to  $\xi = 1/2$  in the high-latitude limit ( $\theta_i \rightarrow 0$ ).

[19] The first term on the right-hand side of equation (3) represents the total (gravitational/centrifugal plus magnetic) buoyancy force. The natural frequencies of gravitational/centrifugal and magnetic buoyancy waves are defined by

$$\omega_g^2 = - \frac{1}{\delta r_{\alpha,e}} g_{\alpha,e} \left\langle \frac{\delta \rho}{\rho_0} \right\rangle \quad (5)$$

and

$$\omega_m^2 = - \frac{1}{\delta r_{\alpha,e}} \left\langle \frac{h_\alpha}{h_{\alpha,e}} c_\alpha \frac{\delta \mathcal{T}_M}{\rho_0} \right\rangle, \quad (6)$$

where the angle brackets denote a mass-weighted flux tube average,

$$\langle \dots \rangle = \frac{1}{\eta_0} \int \dots \rho_0 \frac{ds}{B_0}, \quad (7)$$

and

$$\eta = \int \rho \frac{ds}{B} \quad (8)$$

is the mass per unit magnetic flux (flux tube content). The physical interpretation of equations (5)–(6) is rather straightforward if one remembers that the gravitational/



centrifugal buoyancy force results from a density perturbation in an effective gravitational field, while the magnetic buoyancy force results from a magnetic tension perturbation in a curved magnetic field. These buoyancy forces act in the  $\alpha$  direction, but because the motion is required to occur perpendicular to  $\mathbf{k}_\perp$  (by virtue of the perpendicular quasi-incompressibility condition,  $\mathbf{k}_\perp \cdot \delta \mathbf{r}_\perp = 0$ ), their efficiency is reduced by a factor ( $k_{\beta,e}^2/k_{\perp,e}^2$ ) (prefactor of ( $\omega_g^2 + \omega_m^2$ ) in equation (3)).

[20] The second term on the right-hand side of equation (3) arises from the electric coupling between the magnetosphere and the ionosphere and represents the friction force exerted by the ionospheric Pedersen conductivity on the ionospheric footpoints of magnetospheric flux tubes. The associated frequency depends on the ratio of ionospheric to equatorial displacements and is given by

$$\omega_\Sigma = \frac{1}{\tau_a} \frac{\delta \alpha_i}{\delta \alpha_e}, \quad (9)$$

where

$$\tau_a = \frac{\eta_0}{2\Sigma_P B_{0,e}} \quad (10)$$

is the ionospheric acceleration time (as defined by *Vasyliunas* [1994]) and  $2\Sigma_P$  is the ionospheric Pedersen conductivity integrated across the thickness of both ionospheres. The second term on the right-hand side of equation (3) contains two contributions: the first contribution is inversely proportional to  $\xi \tau_a$ , which corresponds to the ionospheric acceleration time for flow in the  $\beta$  (azimuthal) direction, while the second contribution is inversely proportional to  $\tau_a/\xi$ , which corresponds to the ionospheric acceleration time for flow in the  $\alpha$  (radial) direction. The ionospheric acceleration time for radial flow always exceeds the acceleration time for azimuthal flow by a factor  $\xi^{-2}$ , with  $\xi^{-2} = 4$  in the high-latitude limit of a dipolar magnetic field. Again, because of the perpendicular quasi-incompressibility condition, the efficiency of the friction force is reduced, by a factor ( $k_{\alpha,e}^2/k_{\perp,e}^2$ ) in the  $\beta$  direction and by a factor ( $k_{\beta,e}^2/k_{\perp,e}^2$ ) in the  $\alpha$  direction.

[21] *Vasyliunas* [1994] pointed out the role played by the ionospheric acceleration time in the radial transport of plasma and in the inertial limit to corotation [*Hill*, 1979]. The outward transport of plasma in giant planet magnetospheres tends to decrease its azimuthal velocity, and the outward moving plasma needs to be continually accelerated back to corotation by ionospheric drag. When the ionospheric acceleration time for azimuthal flow ( $\xi \tau_a$ ) is longer than the time for outward radial transport, corotation can no longer be maintained. Therefore as noted by *Vasyliunas* [1994], the ionospheric acceleration time measures the ability of the planetary ionosphere to enforce corotation.

[22] In principle, solving the dispersion relation (3) requires relating ( $\delta\rho/\rho_0$ ) ( $s$ ) and  $\delta T_M$  ( $s$ ) to  $\delta r_{\alpha,e}$  and  $\delta \alpha_i$  to  $\delta \alpha_e$ . In general, this cannot be done analytically, because of the  $s$ -dependence of the equilibrium parameters. It is only in the particular case when the plasma is cold and thermal pressure forces can be neglected that the dispersion relation

of the interchange mode can be solved analytically. This particular case is the subject of the next subsection.

## 2.2. Particular Case of a Cold Plasma

[23] In the absence of thermal pressure ( $P = 0$ ), the requirement of total pressure balance ( $\delta P_T = 0$ ) to leading order in  $\varepsilon$  implies that magnetic pressure must remain unperturbed ( $\delta P_M = 0$ ) to leading order in  $\varepsilon$ , which in turn implies  $\delta B = 0$  and  $\delta T_M = 0$ . It then follows from equation (6) that

$$\omega_m^2 = 0. \quad (11)$$

For  $\omega_g^2$ , equation (5) successively leads to

$$\begin{aligned} \omega_g^2 &= -\frac{1}{\delta r_{\alpha,e}} g_{\alpha,e} \frac{1}{\eta_0} \int \delta \rho \frac{ds}{B_0} \\ &= -\frac{1}{\delta r_{\alpha,e}} g_{\alpha,e} \frac{\delta \eta}{\eta_0} \\ &= \mathbf{g}_e \cdot \frac{\nabla_e \eta_0}{\eta_0}, \end{aligned} \quad (12)$$

when use is made of equation (7), the perturbed form of equation (8) with  $\delta B = 0$ , and the conservation of flux tube content ( $\delta \eta + \nabla_e \eta_0 \cdot \delta \mathbf{r}_{\perp,e} = 0$ ).

[24] For  $\omega_\Sigma$ , we must relate the ionospheric displacement,  $\delta \alpha_i$ , to the conjugate equatorial displacement,  $\delta \alpha_e$ , in equation (9). Once again, we will skip the details of the derivation (which can be found in the work of *Ferrière et al.* [2001]) and only explain how the final result can be obtained.

[25] We first note that for a cold plasma, the magnetospheric disk becomes extremely thin and the transverse displacement  $\delta \alpha$  remains virtually constant across its thickness. Outside the magnetospheric disk,  $\delta \alpha$  necessarily varies along field lines because field lines are deformed by the perturbed parallel electric currents flowing between the magnetospheric disk and the ionosphere. Mathematically, the parallel component of Ampère's equation combined with the perpendicular components of the induction equation and with the perpendicular quasi-incompressibility condition implies that the perturbation in parallel electric current per unit magnetic flux is related to the parallel gradient in  $\delta \alpha$  through

$$\frac{\delta j_\zeta}{B_0} = -\frac{i}{\mu_0} \frac{k_\perp^2}{k_\beta} h_\alpha \frac{\partial \delta \alpha}{\partial s}. \quad (13)$$

[26] The next point to realize is that the perturbation in parallel electric current per unit magnetic flux is constant along any field line between the magnetospheric disk and the ionosphere because this region is devoid of plasma ( $\rho_0, \delta \rho = 0$ ) and experiences no magnetic buoyancy ( $\delta T_M = 0$ ), so that the magnetic tension force must vanish (second term on the right-hand side of equation (2)). As a result,  $\delta j_\zeta/B_0$  all along a field line equals its value at the ionospheric footpoints. The latter is given by the divergence in ionospheric perpendicular currents, which for their part are proportional to the ionospheric Pedersen conductivity,  $\Sigma_P$ , times the ionospheric perpendicular electric field (Ohm's law), itself proportional to the velocity of the ionospheric footpoints,

$-i \omega h_{\alpha,i} \delta\alpha_i$  (frozen-in approximation). The exact expression works out to be

$$\left(\frac{\delta j_\zeta}{B_0}\right)_{\pm L_i} = \pm \Sigma_P \frac{k_{\perp,i}^2}{k_{\beta,i}} h_{\alpha,i} \omega \delta\alpha_i. \quad (14)$$

[27] By substituting equation (14) for the left-hand side of equation (13) and integrating equation (13) over  $s$  from the magnetospheric disk (identified with the equatorial plane) to the northern ionosphere, we obtain

$$\delta\alpha_i - \delta\alpha_e = i \mu_0 \Sigma_P \mathcal{L}_i \omega \delta\alpha_i, \quad (15)$$

where

$$\mathcal{L}_i = \int_0^{L_i} \frac{k_\beta}{k_{\beta,i}} \frac{k_{\perp,i}^2}{k_\perp^2} \frac{h_{\alpha,i}}{h_\alpha} ds. \quad (16)$$

Finally, with the ratio  $\delta\alpha_i/\delta\alpha_e$  taken from equation (15), equation (9) leads to

$$\omega_\Sigma = \frac{1}{\tau_a} \frac{1}{1 - i \mu_0 \Sigma_P \mathcal{L}_i \omega}, \quad (17)$$

where  $\tau_a$  is the ionospheric acceleration time defined by equation (10).

[28] Altogether, in the particular case of a cold plasma, the dispersion relation of the interchange mode (equation (3)) reduces to

$$\omega^2 = \frac{k_{\beta,e}^2}{k_{\perp,e}^2} \left( \mathbf{g}_e \cdot \frac{\nabla_e \eta_0}{\eta_0} \right) - i \left( \frac{1}{\xi} \frac{k_{\alpha,e}^2}{k_{\perp,e}^2} + \xi \frac{k_{\beta,e}^2}{k_{\perp,e}^2} \right) \frac{1}{1 - i \mu_0 \Sigma_P \mathcal{L}_i \omega} \frac{\omega}{\tau_a}. \quad (18)$$

[29] We now rewrite equation (18) in the form of a third-degree equation for  $\omega$  and use Routh-Hurwitz' theorem in matrix algebra to derive the stability criterion of the interchange mode. We find that the interchange mode is stable for all values of  $\mathbf{k}_\perp$  if and only if

$$\mathbf{g}_e \cdot \nabla_e \eta_0 \geq 0. \quad (19)$$

Thus we recover *Hill's* [1976] stability criterion, according to which a cold plasma is stable against interchange motions if and only if its mass per unit magnetic flux,  $\eta_0$ , increases in the direction of the effective gravity. If  $\mathbf{g}_e \cdot \nabla_e \eta_0 < 0$ , one of the three roots of equation (18) is purely imaginary with  $\Im(\omega) > 0$ , and the interchange mode is unstable. In the axisymmetric magnetospheric configuration considered here, neither the ionospheric drag nor the Coriolis force affects the stability of the interchange mode.

### 3. Comparison With Previous Work

[30] In order to compare our results with the previous cold plasma approaches to the problem by *Vasyliunas* [1994] and *Pontius* [1997], we neglect the contribution from pure gravity in our effective gravity, which then

reduces to the centrifugal acceleration (directed outward), and we assume that the background flux tube content,  $\eta_0$ , varies locally with radial distance from the planet as  $R^{-\nu}$ . In that case, the stability condition (19) is not fulfilled and the interchange mode is unstable. If we further replace  $-i\omega$  by the growth rate  $\gamma$ , our dispersion relation (18) becomes

$$\gamma^2 - \frac{k_{\beta,e}^2}{k_{\perp,e}^2} \nu \Omega^2 + \left( \frac{1}{\xi^2} \frac{k_{\alpha,e}^2}{k_{\perp,e}^2} + \frac{k_{\beta,e}^2}{k_{\perp,e}^2} \right) \frac{1}{1 + \mu_0 \Sigma_P \mathcal{L}_i \gamma} \xi \frac{\gamma}{\tau_a} = 0, \quad (20)$$

where the second and third terms represent the centrifugal buoyancy and the ionospheric drag, respectively.

[31] For comparison, the dispersion relation obtained by *Vasyliunas* [1994] (his equation (24)) reads

$$\gamma^2 - \nu \Omega^2 + \xi \frac{\gamma}{\tau_a} = 0, \quad (21)$$

while that derived by *Pontius* [1997] (his equation (12)) reads

$$\gamma^2 - \nu \Omega^2 + \xi \frac{\gamma}{\tau_a} + 4\Omega^2 \frac{\gamma}{\gamma + 1/(\tau_a \xi)} = 0. \quad (22)$$

We can see that our dispersion relation differs from both *Vasyliunas'* and *Pontius'* on three counts.

[32] First, the terms representing the centrifugal buoyancy and ionospheric drag forces in our dispersion relation (20) contain additional geometrical factors which depend on the angle between  $\mathbf{k}_{\perp,e}$  and  $\hat{\mathbf{e}}_\beta$ . As we mentioned in section 2.1, these geometrical factors arise from the perpendicular quasi-incompressibility condition ( $\mathbf{k}_\perp \cdot \delta \mathbf{r}_\perp = 0$ ), and they reduce to unity when  $\mathbf{k}_{\perp,e} \parallel \hat{\mathbf{e}}_\beta$ , i.e., when the motion takes place in the direction  $\hat{\mathbf{e}}_\alpha$  of the driving buoyancy force. These factors do not appear in equation (21) because *Vasyliunas* [1994] considered only motion in the radial direction, nor do they appear in equation (22) because *Pontius* [1997] ignored the perpendicular quasi-incompressibility condition.

[33] Second, the ionospheric drag term in our dispersion relation (20) includes the corrective factor  $1/(1 + \mu_0 \Sigma_P \mathcal{L}_i \gamma)$ . This factor comes from equation (17) and hence from the ratio  $\delta\alpha_i/\delta\alpha_e$  in equation (9). It is absent from equations (21)–(22) since *Vasyliunas* [1994] and *Pontius* [1997] both implicitly assumed that the transverse displacement  $\delta\alpha$  is constant along field lines, with the implication that  $\delta\alpha_i/\delta\alpha_e = 1$ . However, as we explained earlier in connection with equation (13), the presence of perturbed parallel electric currents between the magnetospheric disk and the ionosphere entails a deformation of field lines, and accordingly a variation of  $\delta\alpha$  along field lines. Taking this variation correctly into account automatically leads to our expression for the ionospheric drag term.

[34] Of course, one should bear in mind that the precise expression we found for  $\delta j_\zeta$  is contingent upon our choice of an orthogonal coordinate system. Nevertheless, even if this choice is invalidated by the presence of field-aligned currents at equilibrium, it remains true that perturbed parallel currents arise in the course of an interchange and that these currents deform field lines in the  $\alpha$  direction. In simple physical terms, the reason why field lines must be

deformed is because the driving buoyancy force acts at the equator, while a friction force acts at the ionospheric foot-points and causes them to lag behind.

[35] Third, Pontius' dispersion relation contains an additional term associated with the Coriolis force (last term in equation (22)). Vasyliunas, who did not include the Coriolis force in his analysis, evidently did not obtain this term. In contrast, we did include the Coriolis force, but we found that the associated term in the dispersion relation vanishes in the case of an axisymmetric, rapidly rotating magnetosphere. So why did Pontius reach another conclusion?

[36] Pontius took up Vasyliunas' approximation regarding the electric currents, namely, based on the argument that a perpendicular divergence in magnetospheric currents gives rise (via parallel currents flowing between the magnetospheric disk and the ionosphere) to an equal and opposite perpendicular divergence in ionospheric currents on the same field line, he assumed that magnetospheric currents are equal and opposite to the ionospheric currents on the same field line. By doing so, Pontius accounted only for magnetospheric currents that have nonzero perpendicular divergence.

[37] Since the equilibrium state has no parallel currents, a divergence in perpendicular currents arises only in the perturbed state, and its expression is given by

$$\nabla_{\perp} \cdot \mathbf{j}_{\perp} = \frac{B_0}{h_{\zeta}} \left[ \frac{\partial}{\partial \alpha} (h_{\beta} h_{\zeta} \delta j_{\alpha}) + \frac{\partial}{\partial \beta} (h_{\alpha} h_{\zeta} \delta j_{\beta}) \right] - B_0 \frac{\partial}{\partial s} \left( \frac{\mathbf{j}_0 \cdot \delta \mathbf{B}_{\perp}}{B_0} \right). \quad (23)$$

It is easily shown that the second term on the right-hand side is of higher order in the small parameter  $\varepsilon$  and can therefore be neglected. Hence to leading order in  $\varepsilon$ , a divergence in perpendicular currents involves only the  $\alpha$  and  $\beta$  components of the perturbed current density, whose expressions are given by the perturbed Ampère's equation:

$$\delta j_{\alpha} = \frac{1}{\mu_0} \left[ -\frac{1}{h_{\beta}} \frac{\partial}{\partial s} (h_{\beta} \delta B_{\beta}) + \frac{1}{h_{\zeta}} \frac{\partial}{\partial r_{\beta}} (h_{\zeta} \delta B) \right] \quad (24)$$

$$\delta j_{\beta} = \frac{1}{\mu_0} \left[ \frac{1}{h_{\alpha}} \frac{\partial}{\partial s} (h_{\alpha} \delta B_{\alpha}) - \frac{1}{h_{\zeta}} \frac{\partial}{\partial r_{\alpha}} (h_{\zeta} \delta B) \right]. \quad (25)$$

The force associated with the first contribution to both components is the magnetic tension force resulting from field line bending (fourth term on the right-hand side of equation (1)), while the force associated with the second contribution is the magnetic pressure gradient force  $-\nabla_{\perp} \delta P_M$  (first term on the right-hand side of equation (1) with  $P = 0$ ) plus two forces that are of higher order in  $\varepsilon$  and can be neglected in the case of a cold plasma. Clearly, the second contribution has zero perpendicular divergence. Therefore, by using Vasyliunas' approximation for the electric currents, Pontius automatically omitted the magnetic pressure gradient force in the magnetospheric disk.

[38] However, when the magnetic pressure gradient force is neglected, motions cannot satisfy the perpendicular quasi-incompressibility condition. Indeed, in the absence of magnetic pressure gradient, no force balance can be reached in the direction of  $\mathbf{k}_{\perp}$ , as the Coriolis and buoyancy forces,

which act independently, have no reason to exactly counteract each other. The plasma is then accelerated in the direction of  $\mathbf{k}_{\perp}$ , and it becomes compressed together with the attached field lines, in contradiction with the perpendicular quasi-incompressibility condition ( $\mathbf{k}_{\perp} \cdot \delta \mathbf{r}_{\perp} = 0$ ). What happens next is that the compression of field lines gives rise to a magnetic pressure gradient restoring force, which, on an Alfvénic timescale (much shorter than the timescale of the interchange mode; see section 2), restores force balance along  $\mathbf{k}_{\perp}$ .

[39] The careful reader may have noticed an apparent conflict between the perpendicular quasi-incompressibility condition (which implies  $\delta P_M = 0$  to leading order in the small parameter  $\varepsilon$ ; see beginning of section 2.2) and the presence of a nonnegligible magnetic pressure gradient force. To resolve this apparent conflict, it suffices to realize that the magnetic pressure gradient force is second order in  $\varepsilon$  (just like  $\delta P_M$ ) but that so too are the driving buoyancy force (since both  $g_{\alpha}$  and  $\delta \rho$  are first order) and the Coriolis force (since both  $\Omega$  and  $\omega$  are also first order). In other words, the magnetic pressure gradient force is very small (compared to what it would be in a standard MHD wave), yet nonnegligible (compared to the other forces at play in the interchange mode).

[40] Returning to our main reasoning, we conclude that by ignoring the magnetic pressure gradient force, Pontius violated the perpendicular quasi-incompressibility condition. This is the reason why the Coriolis term did not drop out from his dispersion relation. It is indeed straightforward to verify that motions satisfying the perpendicular quasi-incompressibility condition in the simplified magnetospheric configuration considered by Pontius are unaffected by the Coriolis force. Such motions, by essence, take place in planes normal to  $\mathbf{k}_{\perp}$ , and the component of the Coriolis force in these planes is  $\propto \mathbf{k}_{\perp} \times (2\Omega \times \delta \mathbf{r}) = (\mathbf{k}_{\perp} \cdot \delta \mathbf{r}_{\perp}) 2\Omega - (\mathbf{k}_{\perp} \cdot 2\Omega_{\perp}) \delta \mathbf{r}$ , which vanishes for quasi-incompressible motions ( $\mathbf{k}_{\perp} \cdot \delta \mathbf{r}_{\perp} = 0$ ) in the thin magnetospheric disk (where, by symmetry,  $\Omega_{\perp} = 0$ ).

[41] As already noted, our corrective factor in the ionospheric drag term (third term in equation (20)) has no impact on the stability of the interchange mode, in accordance with the usual effect of a friction force. Likewise, the Coriolis term appearing in Pontius' dispersion relation (fourth term in equation (22)) does not modify the stability condition of the interchange mode, although it can substantially reduce the instability growth rate.

[42] If the ionosphere is absent or ineffective at controlling the magnetospheric plasma flow (weak ionospheric/magnetospheric coupling, with  $\Sigma_P \rightarrow 0$  and  $\tau_a \rightarrow \infty$ ), our dispersion relation (20) indicates that one mode grows at the rate

$$\gamma = \left| \frac{k_{\beta,e}}{k_{\perp,e}} \right| \sqrt{\nu} \Omega, \quad (26)$$

while the other two modes decay at rates  $\gamma = -|k_{\beta,e}/k_{\perp,e}| \sqrt{\nu} \Omega$  and  $\gamma \rightarrow -\infty$ , respectively. Hence the instability growth rate approaches a constant value, proportional to the planetary rotation rate. In this limit, our growth rate corresponds exactly to the growth rate derived by Vasyliunas [1994]. This result was physically expected, as (1) the Coriolis force, omitted by Vasyliunas, was shown to



have no effect on the interchange mode, and (2) when the ionospheric conductivity vanishes, no electric current can flow along field lines between the magnetospheric disk and the ionosphere, with the result that the transverse displacement  $\delta\alpha$  is constant along field lines, as implicitly assumed by Vasyliunas.

[43] If the ionosphere is very efficient at controlling the magnetospheric plasma flow (strong ionospheric/magnetospheric coupling, with  $\Sigma_P \rightarrow \infty$  and  $\tau_a \rightarrow 0$ ), one mode grows at the vanishingly small rate

$$\gamma = \frac{k_{\beta,e}^2}{\frac{1}{\xi^2} k_{\alpha,e}^2 + k_{\beta,e}^2 \left(1 - \frac{\nu \Omega^2}{\xi} \tau_a (\mu_0 \Sigma_P \mathcal{L}_i)\right)} \frac{\nu \Omega^2}{\xi} \tau_a, \quad (27)$$

while the other two modes are oscillatory. By using equation (10) and the perpendicular equilibrium condition, one can show that the second term inside the parentheses in the denominator of equation (27) is  $\lesssim (H_{\parallel}/H_{\perp}) (\mathcal{L}_i/R) \ll 1$  for a cold plasma, so that

$$\gamma \simeq \frac{k_{\beta,e}^2}{\frac{1}{\xi^2} k_{\alpha,e}^2 + k_{\beta,e}^2} \frac{\nu \Omega^2}{\xi} \tau_a. \quad (28)$$

Here, the instability growth rate is proportional to the ionospheric acceleration time,  $\tau_a \rightarrow 0$ . Physically, the interchange mode may not grow faster than allowed by the large ionospheric conductivity. When  $\mathbf{k}_{\perp,e} \parallel \hat{\mathbf{e}}_{\beta}$ , equation (28) reduces to  $\gamma \simeq (\nu \Omega^2/\xi) \tau_a$ , and we recover the growth rate (more exactly, the inverse time scale for the overturning of centrifugally driven turbulent eddies) obtained by *Siscoe and Summers* [1981] as well as the growth rate obtained by *Huang and Hill* [1991], both in the case of a dipolar magnetic field with no Coriolis force. Once again this was expected, insofar as the effect of the Coriolis force on the interchange mode was shown to vanish.

#### 4. Conclusion

[44] The present paper was motivated by the renewed interest in the interchange instability and by the problems posed by the effects of the planetary ionosphere and of the Coriolis force on the interchange mode. Our objective was by no means to solve the whole problem of the interchange instability in a completely realistic manner but only to point out and correct a number of erroneous conclusions published in the literature. We used a simplified geometry and assumed an axisymmetric, rapidly rotating magnetosphere, with no field-aligned electric currents at equilibrium. Moreover, we worked under the thin-disk approximation and restricted our attention to short-wavelength perturbations.

[45] Clearly, our restrictive prescription for the perturbations misses some important aspects of the interchange instability in realistic rotating magnetospheres, such as the inner and outer boundary conditions, the radial dependence of the perturbations, the nonlinear development of the instability... Alternative linear studies exist [e.g., *Huang and Hill*, 1991], which start from perturbations with large azimuthal wave number and solve a boundary value problem in order to obtain the radial eigenfunctions of the

perturbations. In the nonlinear regime, numerical simulations have shown that the dominant spatial scale of the interchange instability in the Io torus is determined by the radial density gradient in the initial torus [*Yang et al.*, 1994]. Although our analysis does not include all realistic magnetospheric conditions, it contains enough physics to fulfill our stated objective.

[46] From section 2.2 onward, we focused on the particular case of a cold plasma, and in section 3, we compared in a critical way our results with those obtained earlier by *Vasyliunas* [1994] and *Pontius* [1997]. Their studies had the merit of pointing out for the first time the role of the ionospheric acceleration time in the dynamics of rapidly rotating magnetospheres, but they constituted only partial (or even partly incorrect) approaches to the problem. First, they were based on a simplified system of electric currents coupling the magnetosphere to the ionosphere. Second, *Pontius* [1997], who specifically inquired into the influence of the Coriolis force on the interchange instability, mistakenly ignored the divergence-free perpendicular electric currents responsible for the magnetic pressure gradient force in the magnetospheric disk. This led him to violate the perpendicular quasi-incompressibility condition that applies to interchange motions and to retain in his final dispersion relation a Coriolis term which should in fact be cancelled out by the other forces at play (magnetic pressure and centrifugal buoyancy).

[47] We reiterated *Ferrière et al.*'s [2001] conclusion that the Coriolis force does not affect the short-wavelength interchange mode in an axisymmetric, rapidly rotating magnetosphere. We also confirmed that the planetary ionosphere has no influence on the interchange instability criterion, but that it controls the instability growth rate, especially when the ionospheric Pedersen conductivity is large (strong ionosphere/magnetosphere coupling). In the opposite limit, the magnetospheric dynamics remain local, and we recover *Vasyliunas*' [1994] result.

[48] Finally, *Pontius* [1997] noticed that giant planet magnetospheres exhibit persistent departures from corotation, which vary in magnitude with radial distance from the planet. Our efforts to understand the exact conditions for the development of the centrifugal instability and the properties of the ensuing interchange motions rely on the assumption that the magnetospheric plasma is in rigid corotation. This restrictive assumption calls for future careful treatments that will allow for differential rotation in the equilibrium state. Anticipating this future direction, let us point out that the upcoming treatments will presumably bear some resemblance to studies of the magnetorotational instability (MRI) in the astrophysical context.

#### Appendix A: Important Properties of the Interchange Mode

[49] To study the interchange mode in the framework of the WKB approximation, we require that the perpendicular wave number of the perturbation,  $k_{\perp}$ , be much greater than the inverse scale height of the perpendicular equilibrium,  $1/H_{\perp}$ , and we introduce the small expansion parameter

$$\varepsilon \equiv \frac{1}{k_{\perp} H_{\perp}} \ll 1. \quad (\text{A1})$$



[50] The fact that the interchange mode is driven by buoyancy implies that in the perturbed momentum equation (1) the buoyancy forces must be at least comparable to the standard MHD forces. In particular, in the perpendicular direction, the total pressure gradient force (first term on the right-hand side of equation (1)) must be smaller than or of the same order as the gravitational buoyancy force (sixth term on the right-hand side):

$$k_{\perp} \delta P_T \lesssim \delta \rho g_{\perp} ; \quad (\text{A2})$$

since these two forces are comparable in the equilibrium state ( $P_{T0}/H_{\perp} \sim \rho_0 g_{\perp}$ ), equation (A2) can be rewritten as

$$\frac{\delta P_T}{P_{T0}} \lesssim \varepsilon \frac{\delta \rho}{\rho_0} . \quad (\text{A3})$$

Likewise for the magnetic tension force (fourth term on the right-hand side of equation (1), with  $\delta B_{\perp} \sim B_0 (\partial \delta r_{\perp} / \partial s)$ , as implied by the perturbed induction equation):

$$\mathcal{T}_{M0} \frac{\partial^2}{\partial s^2} \delta r_{\perp} \lesssim \delta \rho g_{\perp} \quad (\text{A4})$$

or, equivalently (assuming  $\mathcal{T}_{M0} \ll P_{T0}$ ),

$$\varepsilon H_{\perp}^2 \frac{\partial^2}{\partial s^2} (k_{\perp} \delta r_{\perp}) \lesssim \frac{\delta \rho}{\rho_0} . \quad (\text{A5})$$

In the perturbed continuity equation,

$$\delta \rho = -\nabla \rho_0 \cdot \delta \mathbf{r} - \rho_0 \nabla \cdot \delta \mathbf{r} , \quad (\text{A6})$$

the contribution from perpendicular advection to the density perturbation (included in the first term of the right-hand side) must be at least comparable to the contribution from pure compression (second term on the right-hand side):

$$\nabla \cdot \delta \mathbf{r} \lesssim \varepsilon (k_{\perp} \delta r_{\perp}) \quad (\text{A7})$$

and hence

$$\frac{\delta \rho}{\rho_0} \sim \varepsilon (k_{\perp} \delta r_{\perp}) . \quad (\text{A8})$$

Inserting equation (A8) into equations (A3) and (A5) immediately leads to

$$\frac{\delta P_T}{P_{T0}} \lesssim \varepsilon^2 (k_{\perp} \delta r_{\perp}) \quad (\text{A9})$$

and

$$\frac{\partial}{\partial s} \lesssim \frac{1}{H_{\perp}} \ll k_{\perp} , \quad (\text{A10})$$

respectively.

[51] Finally, in the perturbed equation for the total pressure,

$$\delta P_T = -\nabla P_{T0} \cdot \delta \mathbf{r} - \gamma P_0 \nabla \cdot \delta \mathbf{r} - 2P_{M0} (\nabla \cdot \delta \mathbf{r})_{\perp} , \quad (\text{A11})$$

the contribution from perpendicular advection to the total pressure perturbation (included in the first term of the right-hand side) must be at least comparable to the two contributions from pure compression (last two terms). Consequently, in addition to equation (A7), one must also have

$$(\nabla \cdot \delta \mathbf{r})_{\perp} \lesssim \varepsilon (k_{\perp} \delta r_{\perp}) \quad (\text{A12})$$

and hence

$$(\nabla \cdot \delta \mathbf{r})_{\parallel} \lesssim \varepsilon (k_{\perp} \delta r_{\perp}) . \quad (\text{A13})$$

[52] By comparison with the classical compressible MHD waves (for which  $(\delta \rho / \rho_0)$ ,  $(\delta P_T / P_{T0})$ ,  $(\nabla \cdot \delta \mathbf{r})_{\perp}$ ,  $(\nabla \cdot \delta \mathbf{r})_{\parallel}$  are all  $\sim (k_{\perp} \delta r_{\perp})$ ), equations (A12) and (A13) indicate that the interchange mode is compressible perpendicular and parallel to field lines only to first order in  $\varepsilon$ , and equation (A9) indicates that the total pressure is perturbed only to second order in  $\varepsilon$ . Thus to zeroth order in  $\varepsilon$ , the interchange mode is incompressible both perpendicular and parallel to the magnetic field:

$$(\nabla \cdot \delta \mathbf{r})_{\perp} = (\nabla \cdot \delta \mathbf{r})_{\parallel} = 0 , \quad (\text{A14})$$

and it leaves the total pressure unperturbed:

$$\delta P_T = 0 . \quad (\text{A15})$$

## Appendix B: Neglect of the Coriolis Term

[53] For a thin magnetospheric disk, where the equilibrium scale height is much shorter along field lines than across them ( $H_{\parallel} \ll H_{\perp}$ ), the perturbed momentum equation (2) is such that the Coriolis force (last term on the right-hand side) is negligible compared to the inertial force (term on the left-hand side). To see this, we first note that the ratio of both forces is on the order of

$$\frac{F_{\text{Coriolis}}}{F_{\text{inertial}}} \sim \frac{\Omega_{\alpha}}{\omega} \frac{k_{\alpha} k_{\beta}}{k_{\perp}^2} \frac{\delta s}{\delta r_{\alpha}} \simeq \frac{\Omega_{\alpha}}{\omega} \frac{k_{\alpha}}{k_{\perp}} \frac{\delta s}{\delta r_{\perp}} , \quad (\text{B1})$$

where the perpendicular quasi-incompressibility condition,  $\mathbf{k}_{\perp} \cdot \delta \mathbf{r}_{\perp} \simeq 0$ , was used to write the second relation. The first factor on the right-hand side is  $\sim (\Omega_{\alpha} / \Omega) (\Omega / \omega)$ , with  $(\Omega / \omega) \lesssim 1$  and  $(\Omega_{\alpha} / \Omega) \simeq c_{\alpha} s \lesssim (H_{\parallel} / H_{\perp}) \ll 1$  (as a reminder,  $c_{\alpha}$  is the magnetic curvature and  $s$  the field-aligned coordinate). The second factor is always  $< 1$ . The third factor is  $\sim (\partial \delta s / \partial s) (s / \delta r_{\perp})$ , with  $(\partial \delta s / \partial s) \lesssim (\delta r_{\perp} / H_{\perp})$  (from equation (A13) together with equation (A1)), i.e.,  $\lesssim (s / H_{\perp}) \lesssim (H_{\parallel} / H_{\perp}) \ll 1$ . Altogether

$$\frac{F_{\text{Coriolis}}}{F_{\text{inertial}}} \lesssim \left( \frac{H_{\parallel}}{H_{\perp}} \right)^2 \ll 1 . \quad (\text{B2})$$

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