Scalar Potential Formulation and Inverse Problem Applied to Thin Magnetic Sheets

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Our goal is to identify sheet steel magnetization with near field measurements. Indeed, direct calculation of the whole magnetization is impossible because the remanent part of the magnetization is nondeterminist. Consequently, our strategy is to obtain a magnetostatic formulation able to compute magnetic field as close as possible to the sheet and which is adapted to solve an inverse problem. In this paper, a scalar potential integral formulation is introduced and compared to a magnetization formulation. We are especially interested in the magnetic anomaly created by ferromagnetic ships.

Index Terms—**Inverse problem, magnetization identification, scalar magnetic potential, thin magnetic sheet.**

I. INTRODUCTION

A FERROMAGNETIC hull ship, placed in the earth's magnetic field and under mechanical constraints, gets a magnetization which creates a local anomaly of the field. It makes the ship vulnerable to detection and mines. Therefore, for decades, marines worldwide are looking for reducing magnetic anomaly. Before achieving this, it is necessary for the ship to evaluate its own magnetic anomaly. The key point of such system is the identification of the magnetization of the ship's ferromagnetic sheets steel.

The magnetization can be divided in two parts: an induced one, due to the reversible reaction of the material in the inductor field, and a remanent one due to the magnetic history of the material (which depends on hysteresis, mechanical, and thermal constraints). The computation of the induced magnetization is now well known [1], [2]. However, the remanent part is impossible to evaluate with a deterministic calculation because we have no access to the magnetic past of the material. Moreover, even if we had such knowledge, existing models would be too complex to be applied to 3-D geometries. It is then necessary to use magnetic measurements to determine the total magnetization of the hull. Thus, the main goal is to solve an inverse problem (i.e., determination of the sources by knowing the effects) with magnetic sensors placed in the air region inside the hull.

Among the different magnetostatic formulations, few fit with our problem. The first major need is a correct computation of the magnetic field close to the hull. Indeed, measurements are done near the hull for two reasons: the first one is that technically sensors cannot be placed outside the ship and the second one is because the magnetic information quickly decreases with distance. It means that it is impossible to identify local anomaly with distant measurements. The second constraint consists in an explicit mathematical link between field sources and the field itself.

Each formulation has advantages and drawbacks which must be clearly examined. For example, since the computation

problem of thin magnetic sheets has been overcome [1], the finite element method could be a solution: indeed magnetic field calculation near the hull is easy in this case; whereas it is critical in the case of integral methods. But it is important to notice that solving the inverse problem by the finite element method is really difficult and usually impossible. This is due to the lack of an explicit mathematical link between causes and effects. From a general point of view, this link exists in the integral method.

Magnetization identification based on near field measurements has already been achieved [3]. The main results of this problem are recalled in this paper. Now, we expect to find another formulation more accurate near the sheet steel.

II. INDUCED MAGNETIZATION COMPUTATION

In this section, we are interested in the calculation of the induced magnetization $\mathbf{M^{ind}}$ of a thin magnetic sheet in the external low field $\mathbf{H_0}$. The geometry and the physics are perfectly known (relative permeability μ_r especially).

A. Magnetic Sheet's Behavior

The induced magnetization creates a local perturbation $\mathbf{H_{red}}$ of the external field, so the total field $\mathbf{H}$ is

$$\mathbf{H} = \mathbf{H_0} + \mathbf{H_{red}} \tag{1}$$

The sheet's behavior, with no remanent magnetization, is given by

$$\mathbf{M} = \mathbf{M^{ind}} = \chi_r \mathbf{H} \tag{2}$$

where χ_r is the magnetic susceptibility.

Finally, the field created by the ferromagnetic material can be calculated thanks to (see also Fig. 1) [3]

$$\mathbf{H_{red}} = -\frac{1}{4\pi} \mathbf{grad} \iiint\limits_{\Omega} \mathbf{M} \cdot \frac{\mathbf{r} - \mathbf{r'}}{|\mathbf{r} - \mathbf{r'}|^3} d\Omega. \tag{3}$$

The magnetic field sources can be represented by different models. Two are described below: the magnetization formulation (M-formulation) and the scalar potential formulation (U-formulation).
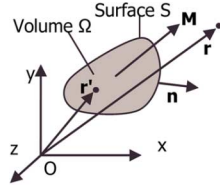
Fig. 1. Symbols definition.

B. Magnetization Formulation

This is the most common way to describe magnetic sources.

It is possible to compute the magnetization in the ferromagnetic material thanks to (1), (2), and (3)

$$\mathbf{M} + \frac{\chi_r}{4\pi} \mathbf{grad} \iiint_\Omega \mathbf{M}. \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|^3} d\Omega = \chi_r \mathbf{H_0}. \quad (4)$$

The sheet is thin enough to be considered as a surface and is now meshed in ne surface elements and nn nodes. Moreover, the magnetization is approximately tangential to the sheet [1].

The integral in the previous equation is obviously convergent [4] but its computation is complex when $\mathbf{r} = \mathbf{r}'$. To overcome this problem, a collocation method at the barycentre of each element has been applied [3]. Finally, discretization and simplification of (4) give at barycentre j

$$\mathbf{M_j} - \frac{e\chi_r}{4\pi} \sum_{k=1}^{\mathbf{ne}} \int_{L_k} \mathbf{M_k} \cdot \mathbf{n_k} \cdot \frac{\mathbf{r_j} - \mathbf{r_k}}{|\mathbf{r_j} - \mathbf{r_k}|^3} dL = \chi_r \mathbf{H_{0j}} \quad (5)$$

where L_k is the edge of the kth element, $\mathbf{n_k}$ is the external normal of L_k, and e is the thickness of the sheet.

Written at each barycentre, this equation leads to a linear system whose solution gives the induced magnetization of each surface element. The magnetic interaction matrix is called C and the vector of the external field is called K

$$C \cdot M = \mathrm{K}. \quad (6)$$

The total field at any point $\mathbf{r_c}$ is calculated by

$$\mathbf{H}(\mathbf{r_c}) = \frac{e}{4\pi} \sum_{k=1}^{\mathbf{ne}} \int_{L_k} \mathbf{M_k} \cdot \mathbf{n_k} \cdot \frac{\mathbf{r_c} - \mathbf{r_k}}{|\mathbf{r_c} - \mathbf{r_k}|^3} dL + \mathbf{H_0}(\mathbf{r_c}). \quad (7)$$

Our goal is the magnetization identification based on near field measurements and it is already possible to estimate what will be the results of this formulation for the near field computation. Clearly, they will not be precise near the ferromagnetic sheet steel because magnetization is constant on each element. In term of charge density, it means that the sources of the magnetic field are only on the elements' edges. It is not a problem far from the elements but near them singularities appear. The use of linear magnetization in this formulation has been studied but the gradient computation of the integral in (4) is very complex.

C. Scalar Potential Formulation

The total scalar potential U is the sum of the source potential U_0 and the reduced potential U_{red}

$$U = U_0 + U_{red}. \quad (8)$$

Naturally

$$\mathbf{M} = -\chi_r \mathbf{grad}U \quad (9)$$

and thanks to (3)

$$U_{red} = \frac{1}{4\pi} \iiint_\Omega \mathbf{M} \cdot \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|^3} d\Omega. \quad (10)$$

Using (8), (9), and (10), the induced potentials are given by [5]

$$U + \frac{\chi_r}{4\pi} \iiint_\Omega \mathbf{grad}U \cdot \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|^3} d\Omega = U_0. \quad (11)$$

Now, as normal magnetization is neglected, the potential can be considered constant in the thickness of the sheet. Inside each element, the scalar potential is approximated by linear or quadratic shape functions φ

$$U(\boldsymbol{\tau_1}, \boldsymbol{\tau_2}) = \sum_{i=1}^{3} U_i \varphi_i^{(k)}(\boldsymbol{\tau_1}, \boldsymbol{\tau_2}) \quad (12)$$

where $\boldsymbol{\tau_1}, \boldsymbol{\tau_2}$ are a local base of the kth element.

Notice that the use of quadratic shape functions needs nq new nodes in the middle of each edge of the mesh elements. Only the quadratic functions will be used in the following. Thus, the magnetization is linear along each element. In addition, the use of quadratic shape functions allows having a tangential continuity of the magnetization between two elements.

As in (4), the integral computation of (11) is not easy and because of the shape functions no collocation method at the barycentres is possible. Obviously, no analytical solution exists for this integral and consequently a numerical computation is needed. However, a direct numerical computation is not recommended because of the high singularity of the $1/r^2$ term. Indeed, the integration is done on the whole volume, we cannot avoid the case $\mathbf{r} = \mathbf{r}'$ as in the M-formulation. But mathematically, the field created by the volume distribution is equivalent to a surface charge density and a volume charge density [6]

$$U_{red} = \frac{1}{4\pi} \iint_S \frac{\mathbf{M} \cdot \mathbf{n}}{r} dS - \frac{1}{4\pi} \iiint_\Omega \frac{1}{r} \mathrm{div}\mathbf{M} d\Omega. \quad (13)$$

In this equation, the singularity is reduced ($1/r$ instead of $1/r^2$) and $\mathrm{div}\mathbf{M}$ is a constant. A good solution to compute these two integrals is to integrate analytically with respect to the normal direction of the element and then to use numerical integration method.

As a consequence, the discretization of (11) at node j is

$$U_{0j} = U_j$$
$$+ \frac{\chi_r}{4\pi} \sum_{i=1}^{nn+nq} U_i \sum_{k=1}^{ne} \left(\iint_{S_k} \mathbf{G} \cdot \frac{\mathbf{n_k}}{R} dS - \iiint_{\Omega_k} \mathrm{div}\mathbf{G} \cdot \frac{1}{R} d\Omega \right) \quad (14)$$

where

$$\mathbf{G} = \frac{\partial \varphi_i^k}{\partial \tau_1} \boldsymbol{\tau_1} + \frac{\partial \varphi_i^k}{\partial \tau_2} \boldsymbol{\tau_2} \quad (15)$$

$$R = |\mathbf{r_j} - \mathbf{r_i}|. \quad (16)$$

Equation (14) is written at each node of the mesh to obtain a linear matrix of nn+nq rows and columns and called Ψ. The
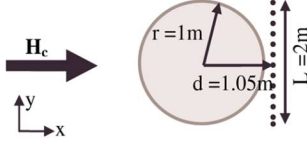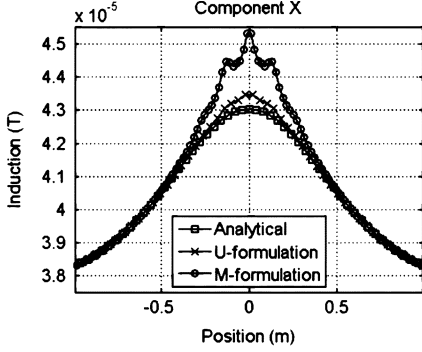
Fig. 2.   System definition.

Fig. 3.   Magnetic field along the defined line: analytical versus M-formulation versus U-formulation.

source vector is called SU. The total potential at each node is obtained by solving the linear system

$$\Psi \cdot U = SU. \tag{17}$$

The total field at any point $\mathbf{r_c}$ is calculated thanks to

$$\mathbf{H}(\mathbf{r_c}) = \frac{\chi_r}{4\pi}\mathbf{grad}\iiint_{\Omega}\mathbf{grad}U\frac{\mathbf{r_c}-\mathbf{r}}{|\mathbf{r_c}-\mathbf{r}|^3}d\Omega+\mathbf{H_0}(\mathbf{r_c}). \tag{18}$$

In that case, an accurate near field computation is expected. Indeed, the magnetic sources are far less singular than in M formulation because it is now the sum of a surface charge density and a volume charge density instead of a linear charge density located on the edges of the elements.

D. Numerical Example

The better accuracy of the U formulation is illustrated thanks to the following example: a 1 m radius hollow sphere, with the thickness 1.4 mm and with the relative permeability $\mu_r = 96$, is meshed in 576 quadrangular elements. It is placed in a 30 A·m^{-1} external field along the x axis. The quadratic shape functions are used.

The magnetic field is calculated along the dotted line in Fig. 2.

The results of the two formulations are compared to the analytical result (see Fig. 3).

At a short distance of the sheet steel, the scalar potential formulation results are excellent compared to M-formulation's ones.

Finally, the U-formulation seems to be better adapted than the M-formulation to identify magnetization based on near field measurements.

III. REMANENT MAGNETIZATION IDENTIFICATION

The problem is to determine the remanent magnetization of sheet steel from a few number of magnetic field measurements.

The starting point of the inverse problem is (18), for the U-formulation, which links sources and effects. In (18), H is the measurement and the unknowns are U.

A. Addition of Physical Information

Among the infinite number of solutions of (18), it is necessary to choose the most physical one. For this purpose, the following equations, describing the shell magnetic behavior with remanent magnetization are added to (18), [3]:

$$\mathbf{M^{ind}} = \chi_r\mathbf{H} \tag{19}$$

$$\mathbf{M} = \mathbf{M^{ind}} + \mathbf{M^{rem}}. \tag{20}$$

B. Inverse Problem: M-Formulation

Introducing (19) and (20) in (5) and (7) leads to the following system [3] which is solved by pseudo-inverse:

$$\begin{pmatrix} C & C - Id \\ Am & Am \end{pmatrix}\begin{pmatrix} M^{\text{ind}} \\ M^{\text{per}} \end{pmatrix} = \begin{pmatrix} K \\ b \end{pmatrix} \tag{21}$$

where Id is the identity matrix, b the vector of measurements, and Am the matrix expression of (7) at sensors' position.

C. Inverse Problem: U-Formulation

The choice of the unknown that will represent the remanent part of the magnetization is not obvious because the existence of a remanent scalar potential is not clearly established. In particular, the relation below imposes strong constraints on the remanent magnetization

$$\mathbf{curl}\mathbf{M^{rem}} = \mathbf{0}. \tag{22}$$

Nevertheless, this formulation is written and will be tested with real measurements. Consequently, the total potential is considered as the sum of the induced and remanent potentials

$$U = U^{\text{ind}} + U^{\text{rem}}. \tag{23}$$

The regularized inverse problem is obtained by adding (19) and (20) to (18)

$$\begin{pmatrix} \Psi & \Psi - Id \\ Um & Um \end{pmatrix}\begin{pmatrix} U^{\text{ind}} \\ U^{\text{rem}} \end{pmatrix} = \begin{pmatrix} SU \\ b \end{pmatrix} \tag{24}$$

where Um is the matrix expression of (18).

D. Inverse Problem: U/M-Formulation

As the inverse U-formulation is of doubtful validity, a more rigorous formulation is introduced: a coupled induced potential and remanent magnetization formulation.

Using (9), (19), and (20)

$$\mathbf{M} = -\chi_r\mathbf{grad}U + \mathbf{M^{rem}}. \tag{25}$$

And (11) becomes

$$U + \frac{\chi_r}{4\pi}\iiint_{\Omega}\mathbf{grad}U\cdot\frac{\mathbf{r}-\mathbf{r'}}{|\mathbf{r}-\mathbf{r'}|^3}d\Omega$$
$$-\frac{1}{4\pi}\iiint_{\Omega}\mathbf{M^{rem}}\cdot\frac{\mathbf{r}-\mathbf{r'}}{|\mathbf{r}-\mathbf{r'}|^3}d\Omega = U_0. \tag{26}$$

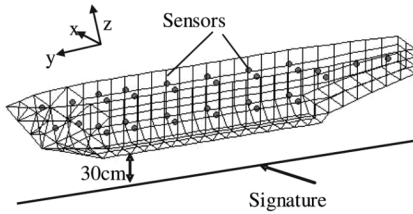Fig. 4. Mock-up in the field simulator.

Fig. 5. Mock-up's mesh with sensors and signature line.

Inside each element, $\mathbf{M}^{\mathrm{rem}}$ is approximated by linear shape functions. Equation (18) is similarly modified and the regularized inverse problem is obtained by solving the linear system

$$\begin{pmatrix} \Psi & P \\ Um & \Delta \end{pmatrix} \begin{pmatrix} U^{\mathrm{ind}} \\ M^{\mathrm{rem}} \end{pmatrix} = \begin{pmatrix} SU \\ b \end{pmatrix} \quad (27)$$

where P and Δ are the matrix expressions of the integrals containing $\mathbf{M}^{\mathrm{rem}}$.

E. Example: Identification Based on Experimental Near Field Measurements

The mock-up is made of steel ($\mathrm{e} = 0.0014$ m, $\chi_{\mathbf{r}} = 95$) and is 4.4 m long (see Fig. 4). The external field is the earth's field ($H_x = -0.44$ A·m^{-1}, $H_y = -17.94$ A·m^{-1}, $H_z = -32.36$ A·m^{-1}). The signature is evaluated on a line below the mock-up (see Fig. 5). Measurements come from 32 magnetic tri-axial sensors which are 20 cm far from the hull. The magnetization is unknown and unspecified.

In the case of the M-formulation and U/M-formulation, there is a good agreement between the predicted and real field (on Figs. 6 and 7 only U/M-formulation's results are shown because they are very close to the results of M-formulation). So our new formulation is validated to identify magnetization based on near field measurements. The U-formulation is not. The constraint (22) is clearly too strong. It appears that the remanent scalar potential is not adapted to represent remanent magnetization. To go further, we can reasonably think that remanent scalar potential does not exist in general.

IV. CONCLUSION

A new formulation of the magnetostatic inverse problem has been proposed: a coupled induced potential and remanent magnetization formulation. It allows a better computation of field close to the sheets than a classical M-formulation. In addition,
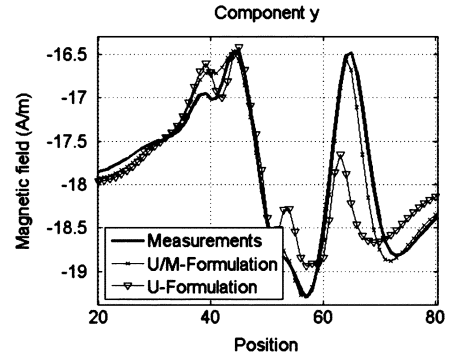

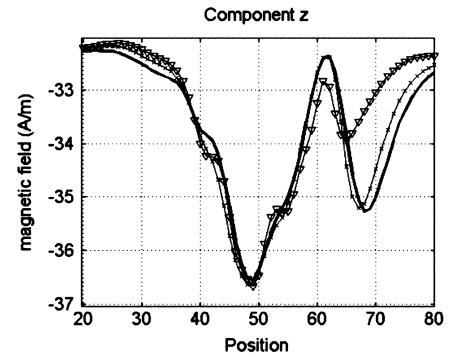Fig. 6. Comparison of predicted and real fields. Longitudinal component.

Fig. 7. Comparison of predicted and real fields. Vertical component.

it is able to identify magnetization based on near field measurements but shows no improvement compared to the M-formulation. Sensors seem to be far enough from the hull to avoid inaccuracy of the M-formulation.

Furthermore, it was proven thanks to real measurements that remanent scalar potential is not adapted to represent remanent magnetization.

To conclude, we expect to improve our results thanks to a statistical approach which may allow introducing new physical knowledge and take into account data that are not perfectly known (relative permeability for example).

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