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Formal Sensitivity Computation of Magnetic Moment method Applied to Linear Ferromagnetic Materials

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Abstract — This paper presents a methodology for computing formally sensitivities of the moment method in the linear case. This numerical method is known to be very accurate and light to compute force and torque in radiated magnetic devices. We propose an approach allowing to compute formal derivatives with respect to the geometrical parameters of the device. Once the sensitivities are obtained, gradient based optimization algorithm can be used.

I. INTRODUCTION

Most of times, the modeling of devices containing ferromagnetic material (iron, nickel, cobalt and their alloys...) is achieved thank to numerical approaches like FEM, BEM, MoM (Method of Moments)... In a design process, the modeling must be coupled with an optimization process. Even if stochastic method (like genetic algorithm) can be applied, these techniques lead to a very important computation time increasing drastically with the number of parameters. The use of a surface response can partially solve this problem but the number of parameters is still limited. Gradient based methods (like Quasi-Newton algorithm) are very efficient solution. It is then necessary to compute the sensitivity of the objective function versus the parameters. Numerical sensitivities calculation using finite difference is fairly straightforward and an easy way to achieve it, but it's not a robust technique. Hence, we present an original method based on formal differentiation of the MoM approach.

II. THE METHOD OF MOMENTS (MoM)

The method of moments is an accurate and light numerical approach dedicated to the modeling of electromagnetic devices. In this method, only active regions (i.e. ferromagnetic parts of the device) are meshed. Thus it is particularly well adapted for the modeling of simple radiated devices with few ferromagnetic parts in comparison of the total air region. In this paper, we consider a simple linear law for the ferromagnetic material.

The MoM is based on the discretization of (1), associated with a point matching approach applied at the barycenter of each elements [1].

$$\mathbf{M}(P) = (\mu_r - 1) \mathbf{H}_{\text{ext}}(P) + \frac{1}{4\pi} \iiint_V \left(\frac{3(\mathbf{M} \cdot \mathbf{r})}{r^5} \mathbf{r} - \frac{1}{r^3} \mathbf{M} \right) dv \quad (1)$$

where M is the magnetization of the material, μ_r is the relative permeability, H_{ext} is the external field generated by the sources (conductors, permanent magnet,...), P , the

point where equation is written and r is the vector linking the integration point to P .

In our approach, the ferromagnetic volume is meshed into N elements and magnetization is supposed to be uniform on each of them. Then we get a full linear system:

$$\mathbf{A} \cdot \mathbf{M} = (\mu_r - 1) \mathbf{H}_{\text{ext}} \quad (2)$$

A is the interaction matrix between elements. It depends on the geometrical parameters of the system and on the mesh.

Classically, it remains to solve (2) to get the magnetizations. Once it is obtained, forces and torques acting in the devices can be computed [1]. For example, the expression of the torque applied on a ferromagnetic volume can be expressed by (3):

$$\Gamma = \mu_0 \sum_{i=1}^N (\mathbf{M}_i \times \mathbf{H}_{\text{ext}_i}) \cdot \mathbf{v}_i \quad (3)$$

Where M_i is the magnetization of the element i , H_{ext_i} the field created by all the external sources on the barycenter of the element i , and v_i its volume.

III. FORMAL DERIVATION OF THE METHOD

Let's introduce a function Φ which represents, for example, the torque or the force applied on the ferromagnetic material or the field at an observation point. Φ depends on the parameter p and the magnetization M .

$$\Phi = \Phi(\mathbf{M}(p), p) \quad (4)$$

A. Basic expression

The sensitivity of Φ versus p can be expressed:

$$\frac{\partial \Phi}{\partial p} = \frac{\partial \Phi}{\partial \mathbf{M}} \Big|_p \cdot \frac{\partial \mathbf{M}}{\partial p} + \frac{d\Phi}{dp} \Big|_{\mathbf{M}} \quad (5)$$

The evaluation of $\frac{\partial \Phi}{\partial \mathbf{M}} \Big|_p$ and $\frac{d\Phi}{dp} \Big|_{\mathbf{M}}$ depend on how Φ is expressed, the most important operation is the computation of $\frac{\partial \mathbf{M}}{\partial p}$, this is obtained by solving the linear system (6):

$$\mathbf{A} \cdot \frac{\partial \mathbf{M}}{\partial p} = \left((\mu_r - 1) \frac{\partial \mathbf{H}_{\text{ext}}}{\partial p} - \frac{\partial \mathbf{A}}{\partial p} \mathbf{M} \right) \quad (6)$$

This expression is obtained by differentiating and arranging (2).

The quantity $\frac{\partial \mathbf{H}_{\text{ext}}}{\partial p}$ could be expressed formally by applying formal derivation on the semi-analytical model of the magnetic field sources [3]. $\frac{\partial \mathbf{A}}{\partial p}$ is also computed symbolically, depending on the mesh.

To achieve the Jacobian computation (i.e. vector of partial derivative of Φ for the k parameters), the linear system (6) has to be solved k times. A well known method to overpass this problem is the use of the adjoint state as in [2].

B. Enhanced solution by using the adjoint state

Now, let's introduce the adjoint state λ , which has the same dimension as M , and satisfies (7):

$$\mathbf{A}^t \cdot \lambda = \left(\frac{\partial \Phi}{\partial \mathbf{M}} \Big|_p \right)^t \quad (7)$$

By solving the linear system (7) and introducing its transposed in (6), the sensitivity formula (5) can be also written with the following equation:

$$\frac{\partial \Phi}{\partial p} = \lambda^t \cdot \left((\mu_r - 1) \frac{\partial \mathbf{H}_{\text{ext}}}{\partial p} - \frac{\partial \mathbf{A}}{\partial p} \mathbf{M} \right) + \frac{d\Phi}{dp} \Big|_M \quad (8)$$

So, only one linear system (7) resolution per function Φ is required to compute the sensitivity versus k parameters. This approach is more efficient than (6) if the number of function Φ is less than k , that is often the case.

IV. APPLICATION

The application studied was an iron plate placed in a uniform magnetic field (Figure 1), as described in [4], the orientation of the field is adjusted by an angle α .

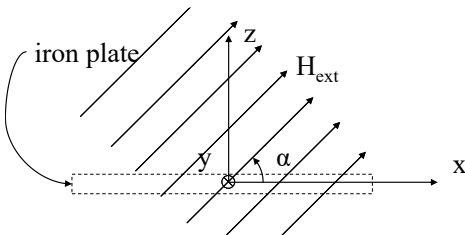


Fig. 1. An iron plate placed in a uniform swiveling magnetic field

The magnetic torque Γ applied on the plate according α is given by (3). The forward finite difference (9) will be compared to the formal derivative.

$$\frac{\partial \Gamma}{\partial p} = \frac{\Gamma(p+h) - \Gamma(p)}{h} \quad (9)$$

If we rotate the external field, by increasing the angle α (Fig 1) from 0 to $\pi/2$, the sensitivity of the torque is given in the figure 2.

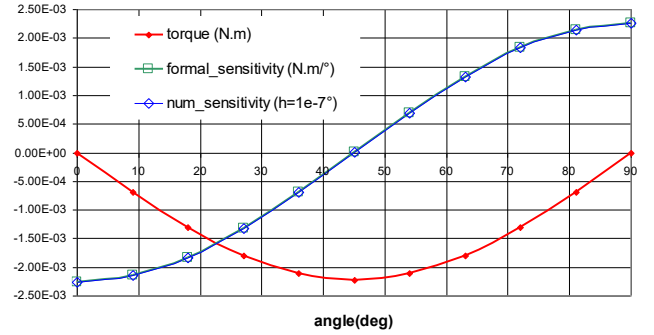


Fig. 2. The torque and its sensitivity versus the field orientation, the plate dimensions and orientation remains fixed. Curves presenting formal and numerical sensitivities computation perfectly matches.

The main application of sensitivities computation is design optimization. But it can be used to solve an equilibrium equation. Indeed, our soft material is attached to a flexible non magnetic part with a resistive torque (10). Then, equilibrium of magnetic torque and resistive torque can be found for α_{eq} .

$$\Gamma_r(\alpha) = -k \cdot \alpha \quad (10)$$

The soft material movement can be easily modeled with MoM, since the air region is not meshed. Moreover, thanks to derivatives, a fast convergence algorithm can be used like Newton-Raphson. This last was compared with a non gradient based algorithm using a combination of bisection, secant, and inverse quadratic interpolation methods [5]. With the same initial point, with a 1% precision required on α_{eq} , Newton-Raphson finds equilibrium in 2 iterations and 5 seconds, when non gradient [5] gives less accurate result in 11 iterations and 17 seconds.

V. CONCLUSION

The approach presented here shows the feasibility of the formal sensitivity calculation applied on the Magnetic Moment Method. The use of the adjoint state is an efficient way to reduce its computation time. In the full paper, more details about the use of the adjoint state technique and the application will be given.

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