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Fluctuations of energy flux in a simple dissipative out-of-equilibrium system

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We report the statistical properties of the fluctuations of the energy flux in an electronic RC circuit driven with a stochastic voltage. The fluctuations of the power injected in the circuit are measured as a function of the damping rate and the forcing parameters. We show that its distribution exhibits a cusp close to zero and two asymmetric exponential tails, the asymmetry being driven by the mean dissipation. This simple experiment allows to capture the qualitative features of the energy flux distribution observed in more complex dissipative systems. We also show that the large fluctuations of injected power averaged on a time lag do not verify the Fluctuation Theorem even for long averaging time. This is in contrast with the findings of previous experiments due to their small range of explored fluctuation amplitude. The injected power of an ensemble of N circuits is also studied to mimic systems with large number of particles either correlated or not, such as in a dilute granular gas.

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I. INTRODUCTION

The fluctuation theorem (FT) is of fundamental importance for microscopic systems far from equilibrium in a stationary state. It was first introduced numerically for a fluid under an external shear [1], then mathematical proof was given [2]. For a nonequilibrium dissipative system, this theorem describes the asymmetry of distribution of a fluctuating global quantity I_τ (energy flux, entropy production rate,...) averaged over a time τ much larger than its typical correlation time τ_c . For systems close to equilibrium or for macroscopic ones, the FT gives a generalisation of the second law of thermodynamics, and also implies the Green-Kubo relations for linear transport coefficients when combined with the central limit theorem [3]. Moreover, it can be applied to nonequilibrium transitions between two different equilibrium states leading to the so-called Jarzynski equality [4]. Its derivation requires the assumption of time reversibility of the system dynamics, ergodic consistency, and a certain initial distribution of particle states. Finally, it does not require or imply that the distribution of time averaged fluctuating quantity I_τ is Gaussian.

Experimental tests of the fluctuation theorem relation have been reported in various systems: in granular gases [5], in turbulent flows (thermal convection [6, 7], swirling flows [8]), in liquid crystals [9], with an electric dipole [10] or a mechanical oscillator [11], in a two-level atomic system [12], and by means of a colloid particle [13] or an RNA molecule [14] in an optical trap. In all these experiments, the fluctuation theorem is found to be verified with good accuracy despite the non-reversibility in time of the systems. Such a good agreement has been also reported in numerical simulations of granular gases [15, 16], turbulence [15, 17], and earthquakes [15]. The reasons of this apparent verification of the FT are two-fold: either due to the small range of explored fluctuation amplitude

$\epsilon \equiv I_\tau / \langle I \rangle$ [15, 16, 17], or due to the long averaging time τ needed [15, 16, 18]. Only small relative fluctuation amplitudes ($\epsilon \leq 0.8$ for $\tau \leq 20\tau_c$) have been reached in the above experiments [5, 6, 7, 8]. Very recently, large range of ϵ has been attained, even for $\tau \gg \tau_c$, by measuring the fluctuating injected power in an experiment of wave turbulence on a fluid surface [19]. This experiment then shows that the FT is not satisfied for high enough ϵ . Such a disagreement was also predicted theoretically in a system described by a Langevin equation [20]. Note that the breakdown of FT has been recently reported numerically [17] or theoretically [21] in other systems.

In this paper, the fluctuations of energy flux in an electronic circuit are measured to test the fluctuation theorem within a large range of accessible value of fluctuation amplitude ($\epsilon \simeq 3$) even for long averaging time ($\tau/\tau_c \simeq 20$). The electronic circuit is a resistor of resistance R in series with a capacitor of capacitance C driven with a stochastic voltage. This circuit can be viewed as an electronic analogue of the Langevin equation [22] which describes usually the brownian motion of a particle [23]. It is important to notice that in our experiment the dissipation is selected by the system itself. No *ad-hoc* dissipation or thermostat is introduced to ensure the FT hypothesis (i.e. the time-reversibility of the system). The study of the statistical properties of the injected power in such a circuit point out three important results:

First, the probability density function (PDF) of the fluctuations of the injected power in the circuit is studied as a function of the control parameters (damping rate, amplitude of the stochastic forcing). The asymmetry is driven by the damping rate: The more the mean dissipation increases, the less the negative events of injected power occur. This electronic circuit is one of the simplest system to understand the properties of the energy flux fluctuations shared by other dissipative out-of-equilibrium systems (such as in granular gases [5], wave

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turbulence [19] and convection [7, 25]).

Second, we show that the fluctuations of injected power averaged on a time τ do not verify the fluctuation theorem at large values of ϵ , even for $\tau \gg \tau_c$. This occurs for values of ϵ larger than the most probable value of the injected power PDF. This electronic circuit thus appears to be a very useful tool to test fluctuation theorem in the different limits of the averaging time and of the fluctuation amplitude.

Third, the injected power of an ensemble of N uncorrelated RC circuits is then studied. This mimics a dissipative multi-component system driven out-of-equilibrium without spatial correlation between them. The fluctuations of the time averaged injected power of the N circuits then verifies the fluctuation theorem for finite time. This bridges the gap between results about the test of the FT for systems with low particle number (such as the ones described by the Langevin equation), and systems with large number of uncorrelated particles (such as in a dilute granular gas). The link between them can be understood as a consequence of the central limit theorem.

It is well known that electronic circuits are very useful analogue experiments to study stochastic nonlinear problems [26]. However, one could wonder their relevance with respect to numerical simulations. Analogue circuits get the advantages that any naturally occurring noise necessarily has a finite correlation time, and thus avoid to pre-select a correlated noise type (Ito-Stratanovich dilemma) in writing the numerical code [26]. Moreover, the simulation leads to the accumulation of truncation errors, and it takes a longer time to implement and to compute [26].

The paper is organized as follows. Section II explains the experimental setup of the RC circuit. Section III contains the results about the statistical properties of the injected power in the circuit. Section IV contains the experimental test of the Fluctuation Theorem for the energy flux in a RC circuit. Finally, Section V is devoted to the experimental study of the injected power in a set of N uncorrelated RC circuits, as well as the test of the Fluctuation Theorem for its energy flux.

II. EXPERIMENTAL SETUP

The experimental setup consists of a resistor, R , in series with a capacitor, C , driven with an external stochastic voltage $\zeta(t)$ as displayed in Fig. 1. The equation of continuity for the voltage V across the resistor R leads to

$$\gamma^{-1} \frac{dV(t)}{dt} + V(t) = \zeta(t), \quad (1)$$

where $\gamma^{-1} = RC$. As it will be shown below, the injected power in the RC circuit is

$$I(t) \equiv \gamma V(t)\zeta(t). \quad (2)$$

The zero mean gaussian random noise $\zeta(t)$ is generated by a Spectrum Analyzer (Hewlett-Packard HP 35670A). This noise is low-pass filtered at a cut-off frequency λ fixed to 5 kHz, unless specified otherwise. The control parameter is the noise amplitude D defined by the constant value of its power spectral density, as an analogy to the white noise limit. C is fixed to $1 \mu F$, and R can be varied between 200Ω and $10 \text{ k}\Omega$ leading to values of γ between 50 Hz and 10 kHz. The output $V(t)$ of the RC circuit is multiplied by the random forcing $\zeta(t)$ by means of an analog multiplier (Analog Devices AD540). The resulting voltage $V(t)\zeta(t)$ is proportional to the injected power (see below) and is acquired with a Digital-to-Analog Acquisition card (AT-MIO-16X) at 100 kHz sampling frequency for 10 s, with a precision of 0.3 mV.

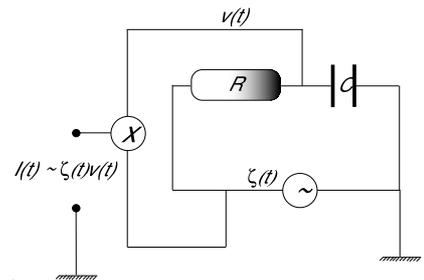


FIG. 1: Scheme of the electronic circuit as an analogue of the Langevin equation.

Equation (1) is the analogue of the Langevin equation that usually describes the dynamics of a Brownian particle of velocity v as [23]

$$\frac{dv(t)}{dt} + \tilde{\gamma}v(t) = f(t), \quad (3)$$

where $\tilde{\gamma}$ is the inverse of a damping time. f is an external random forcing: a gaussian white noise with zero mean and autocorrelation function $\langle f(t)f(t') \rangle = f_0\delta(t-t')$, f_0 being the noise amplitude. Multiplying Eq. (3) by v gives

$$\frac{d}{dt} \left[\frac{v(t)^2}{2} \right] = f(t)v(t) - \tilde{\gamma}v(t)^2, \quad (4)$$

meaning that the energy budget of the system is driven by the injected power, $f(t)v(t)$, and the dissipative one, $\tilde{\gamma}v(t)^2$. This analogy thus shows easily that Eq. (2) is the injected power in the electronic circuit.

The aim is now to study the probability distribution function (PDF) of the injected power in the RC circuit, described by a Langevin equation as the simplest dissipative system driven out of equilibrium by an external force. The objective is to probe the out-of-equilibrium statistical properties of the injected power and its relation with the fluctuation theorem. It is noteworthy to mention that in this simple system the forcing $f(t)$ is not in any case a thermal bath: it forces the system strongly out of equilibrium, where the Fluctuation-Dissipation theorem

does not hold [27]. This is mainly due to the non-gaussian shape of the injected power distribution, in contrast with other experimental devices where the injected power fluctuations are quasi-normal [10, 11].

III. STATISTICAL PROPERTIES OF THE INJECTED POWER

The probability density function of the injected power, $I(t)$, is shown in Fig. 2 for different values of the noise amplitude D , and the damping rate γ . For all values of D and γ , the PDFs exhibit two asymmetric exponential tails and a cusp near $I \simeq 0$. Note that this typical PDF shape has been also observed in various more complex systems (granular gases [5], wave turbulence [19] and convection [7, 25]). As shown in Fig. 2, the PDF skewness increases strongly with γ at a fixed D . Moreover, the extremal fluctuations increase strongly with D at a fixed γ .

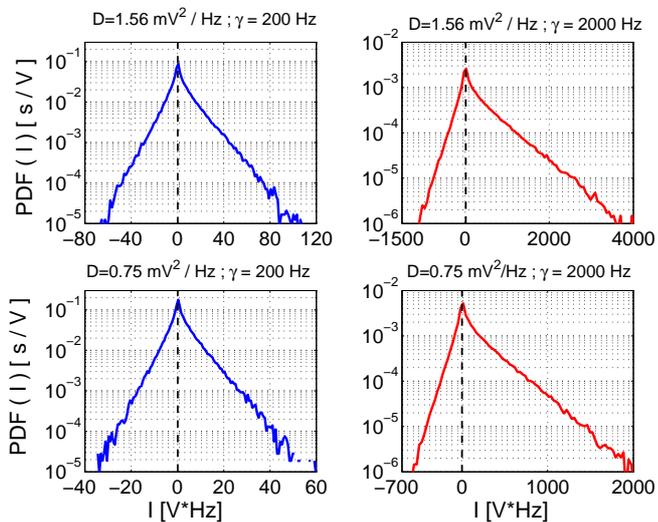


FIG. 2: Probability density functions of the injected power I for two different noise amplitudes ($D = 7.55 \times 10^{-4}$, $1.56 \times 10^{-3} \text{ V}_{\text{rms}}^2/\text{Hz}$), and damping rates ($\gamma = 200, 2000 \text{ Hz}$).

At a fixed value of γ , the PDFs of I are plotted in Fig. 3 for 9 different noise amplitudes. As shown in the inset of Fig. 3, all these PDFs collapse on the same curve when plotted in the centered-reduced variable, $(I - \langle I \rangle)/\sigma_I$, where σ_I is the rms value of I , and $\langle I \rangle$ its mean value. Such a collapse means that all the moments of I scale as σ_I . As shown in Fig. 4, σ_I (as well as $\langle I \rangle$) scales linearly with D . This linear dependence with D of the PDF of I can be recovered by dimensional analysis. Thus, since the slopes of the exponential tails scale as D^{-1} , when the noise amplitude D is doubled, the largest injected power fluctuation reached is doubled.

The noise amplitude D is now fixed in order to take into account the effect of the damping rate γ on the injected power fluctuations. For different values of γ , $\langle I \rangle$

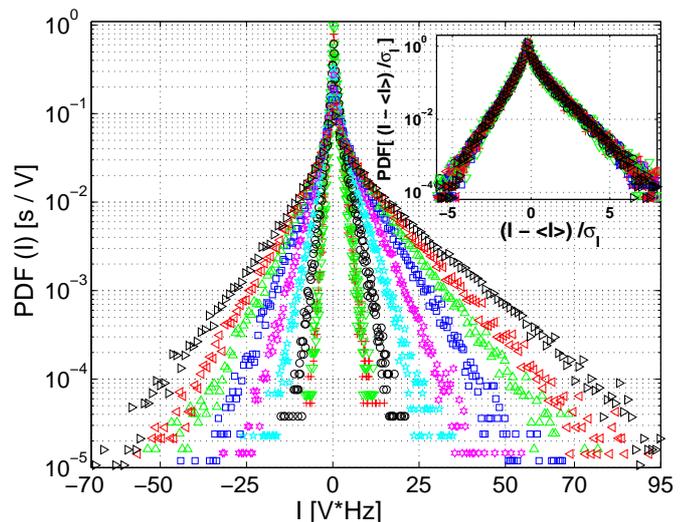


FIG. 3: Probability density functions of injected power, I , for $D = 0.06$ (+) to 1.56 (\triangleright) $\times 10^{-3} \text{ V}_{\text{rms}}^2/\text{Hz}$, for $\gamma = 200 \text{ Hz}$. Inset: Probability density functions in the re-scaled variable $(I - \langle I \rangle)/\sigma_I$.

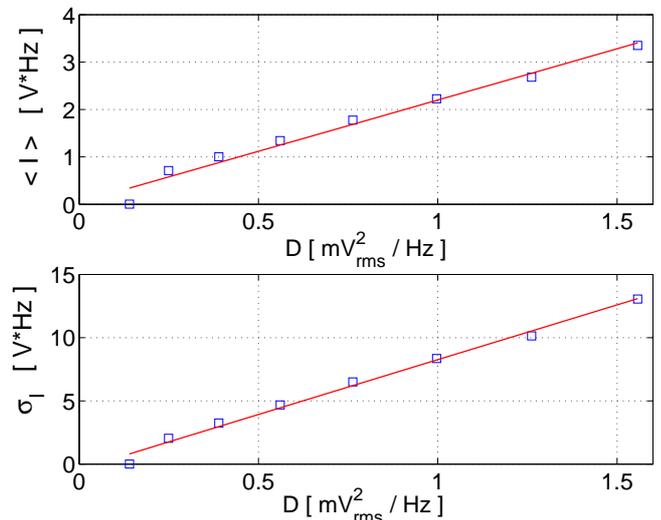


FIG. 4: Mean $\langle I \rangle$ and standard deviation σ_I of the injected power as a function of the noise amplitude D . $\gamma = 200 \text{ Hz}$.

and σ_I are plotted in Fig. 5. Both moments scale as a power law of γ with two different exponents. Therefore no collapse occurs when the PDFs of I are plotted in the centered-reduced variable. However, as displayed in Fig. 6, both the exponential tails of positive and negative values of I show power law dependences with γ . The slope of the positive exponential tails scales as $\sim \gamma^{-1.65 \pm 0.05}$, whereas the negative one scales as $\sim \gamma^{-1.33 \pm 0.05}$. This means that the probability of having negative values of injected power decreases faster than the probability of having positive ones as the system becomes more and more dissipative.

Taking into account both the effect of D and γ , the

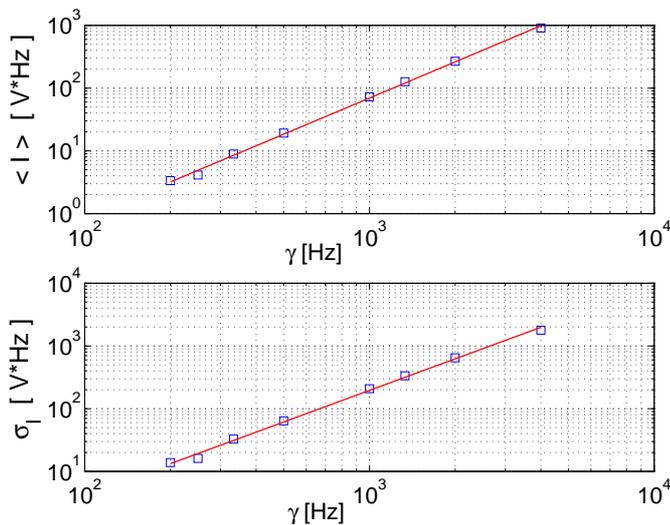


FIG. 5: Mean $\langle I \rangle$ and standard deviation σ_I of the injected power as a function of the damping rate γ . $D = 0.75 \times 10^{-3} \text{V}^2/\text{Hz}$. (—): linear best fits of slopes 1.9 V and 1.59 V, respectively.

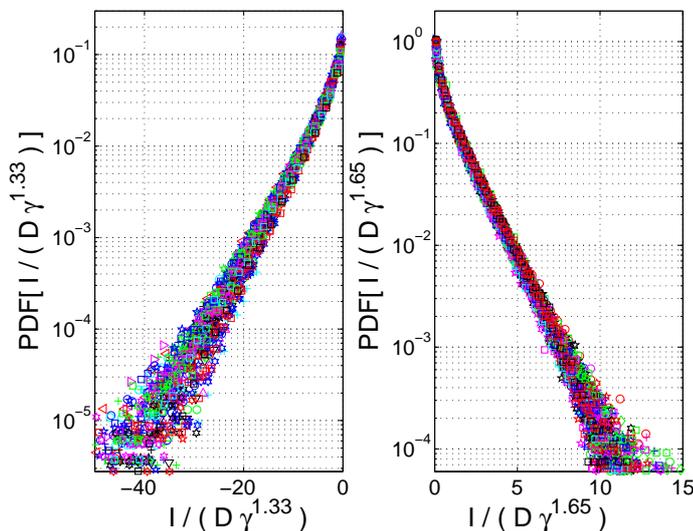


FIG. 6: Scaling of the PDFs of the negative values (left) and the positive values (right) of injected power I , for 9 values of D , and 10 values of γ .

PDF of the positive values of I behaves, far from the cusp at $I \simeq 0$, as

$$P_+(I) \sim \exp\left(-\alpha_+ \frac{I}{D\gamma^{1.65}}\right). \quad (5)$$

Similarly, the PDF of the negative values of I behaves as

$$P_-(I) \sim \exp\left(\alpha_- \frac{I}{D\gamma^{1.33}}\right) \quad (6)$$

where α_{\pm} are two constants.

A model of the shape of the injected power distribution has been recently presented in Ref. [19] and will be

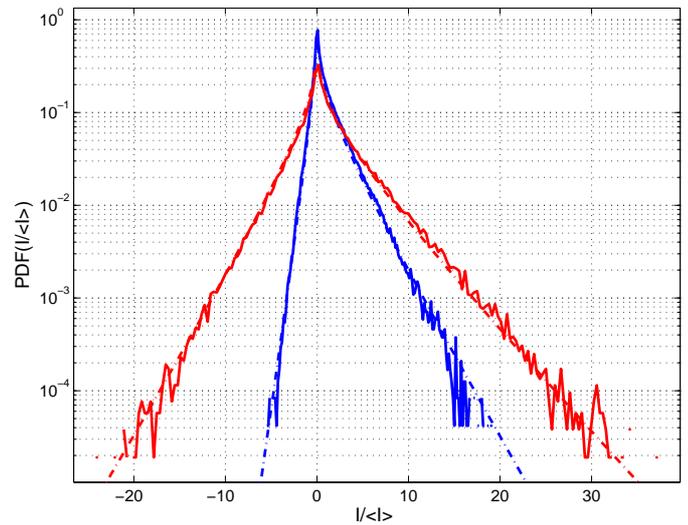


FIG. 7: PDFs of $I / \langle I \rangle$: Comparison between experiment (—) and theory [(—) from Ref. [19, 24]] for two different values of the damping rate $\gamma = 2000 \text{ Hz}$ ($\frac{\langle I \rangle}{\sigma_V \sigma_\zeta} = 0.45$) (blue) and $\gamma = 200 \text{ Hz}$ ($\frac{\langle I \rangle}{\sigma_V \sigma_\zeta} = 0.15$) (red). The cut-off frequency λ is fixed to 10 kHz.

discussed in details in another paper [24]. The experimental shape of the distribution of injected power can be compared with this prediction with no adjustable parameter. This is shown in Fig. 7 for two different values of γ . The computed PDFs display a cusp at $I = 0$ and exponential asymmetrical tails for large values of I in good agreement with the experimental shapes. As shown in Fig. 7, increasing the damping rate γ leads to PDF more and more asymmetrical with less and less negative events. The asymmetry then increases when the damping rate γ increases. The asymmetry or the skewness of the injected power distribution is then controlled by the damping parameter γ .

Note that for other dissipative out-of-equilibrium systems showing energy flux fluctuations, an analogue of the parameter γ can be found. For instance, in an experiment of wave turbulence on a fluid surface [19], the distribution shapes of the injected power I by the wavemaker resemble to the ones found here: When the fluid used is mercury, the $\text{PDF}(I)$ is strongly asymmetrical whereas with water, it is much more symmetrical. This is due to mean dissipation which is different for each fluid. The analogue of the γ parameter for wave turbulence experiment is indeed related to the inverse of a typical damping time of the wavemaker which is linear with the fluid density [19].

Let us now have a look on the scaling of first cumulants ($\langle I \rangle$ and σ_I) with the parameters D , γ and λ . Both D and γ are now fixed in order to study the effect of the random noise cut-off frequency λ on $\langle I \rangle$ and σ_I . As shown in Fig. 8, when λ is varied from 3 kHz to 40 kHz, the mean injected power slightly increases with λ , whereas σ_I scales as the square root of λ .

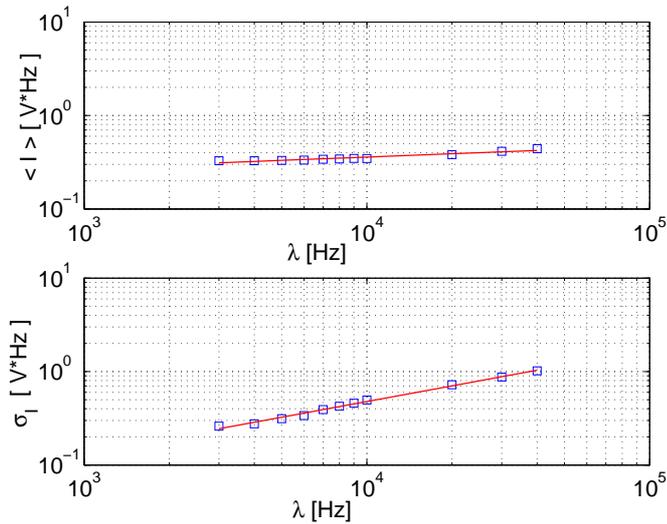


FIG. 8: Scaling of the mean $\langle I \rangle$ and standard deviation σ_I with the cut-off frequency λ . (—): linear best fit of slope 0.11 V and 0.56 V, respectively.

Finally, to summarize all the experimental results, the two first moments of injected power behave like

$$\langle I \rangle \sim D\gamma^{1.90}\lambda^{0.05} \quad \text{and} \quad \sigma_I \sim D\gamma^{1.59}\lambda^{0.50}. \quad (7)$$

Note that all the previous exponents are experimentally measured with a precision of ± 0.05 . Thus, the noise amplitude D is found to drive the scale of the injected power fluctuations whereas the damping rate γ controls the asymmetry of the PDF of I .

Based on the model described in Ref. [19] and discussed in more details in another paper [24], one can calculate all the cumulants of $I(t)$. The first cumulants of $I(t)$ in the stationary limit read [28]

$$\langle I \rangle = \gamma^2 \frac{D\lambda}{\lambda + \gamma}, \quad (8)$$

$$\sigma_I = \gamma^2 \frac{D\lambda}{\lambda^{1/2}\gamma^{1/2}}. \quad (9)$$

In the limit $\gamma/\lambda \ll 1$, Eq. (8) yields

$$\langle I \rangle \sim D\gamma^2\lambda^0, \quad (10)$$

which does not depend on the cut-off frequency λ , and Eq. (9) yields

$$\sigma_I \sim D\gamma^{3/2}\lambda^{1/2} \quad (11)$$

The range of γ used experimentally is between 50 and 2000 Hz, and the frequency cut-off λ is in the range from 3 kHz to 40 kHz. This leads to $\gamma/\lambda \sim 0.1$ in the worst case. The first two cumulants of Eqs. (10) and (11) thus are in good agreement with the experimental results of Eqs. (7).

IV. RELATION WITH THE FLUCTUATION THEOREM

The smoothing average of the injected power I_τ is computed from the previous data of I as

$$I_\tau(t) = \frac{1}{\tau} \int_t^{t+\tau} I(t') dt', \quad (12)$$

where τ stands for the time of average of the signal, which is several times the correlation time τ_c of the injected power I . For our experiment, the correlation time τ_c is the inverse of the cut-off frequency, $1/\lambda$, which is now fixed to 10^{-4} s.

To describe the asymmetry of time-averaged injected power I_τ distribution, the quantity $\rho(\epsilon)$ is computed as

$$\rho(\epsilon) \equiv \lim_{\tau \rightarrow \infty} \frac{\tau_c}{\tau} \ln \left[\frac{P(\epsilon)}{P(-\epsilon)} \right], \quad (13)$$

where $P(\epsilon \equiv I_\tau/\langle I \rangle)$ is the probability to have a ϵ equal to a certain value $I_\tau/\langle I \rangle$. $\rho(\epsilon)$ is usually called the asymmetrical function [20]. The fluctuation theorem (also called the Gallavotti–Cohen relation) states that [1, 2, 3]

$$\rho(\epsilon) = \beta\epsilon \quad (14)$$

where β is a dimensionless constant. It means that the probability ratio to have a positive value of injected power (ϵ) with respect to its negative value ($-\epsilon$) increases exponentially with ϵ at large τ . The hypothesis for deriving Eq. (14) are three-fold: the system should be microscopically reversible, dissipative and the dynamics on the phase space should be chaotic [1, 2, 3]. For our dissipative system, the reversibility condition is obviously not fulfilled. However, let us try to test the relation of Eq. (14) with our experimental data of injected power.

Figure 9 displays the PDF of time-averaged injected power $I_\tau/\langle I \rangle$ when τ/τ_c is increased. Several features appear. First, the negative injected power events decrease with increasing τ until they disappear for $\tau \gtrsim 5\tau_c$. Second, when τ/τ_c is increased, the PDF shape for negative values of $I_\tau/\langle I \rangle$ change from an exponential shape to a Gaussian one, whereas the exponential shape of the positive part is quite robust. Only when $\tau \gg \tau_c$, the PDF shape close to the maximum tends towards a gaussian, as one would expect from the central limit theorem. In Fig. 9, when τ/τ_c increases, the PDF most probable value ϵ^* (i.e., where the PDF amplitude is maximum) increases slowly from $I_\tau/\langle I \rangle = 0$ to 1 (the mean value of the injected power). This dependence of ϵ^* is shown in Fig. 11 as a function of τ/τ_c . This dependence will be of fundamental importance when probing the FT relation (see below).

The Large Deviation Function (LDF) $f(\epsilon)$ is generally defined as

$$f(\epsilon) \equiv \lim_{\tau \rightarrow \infty} \frac{\tau_c}{\tau} \ln [P(\epsilon \equiv I_\tau/\langle I \rangle)], \quad (15)$$

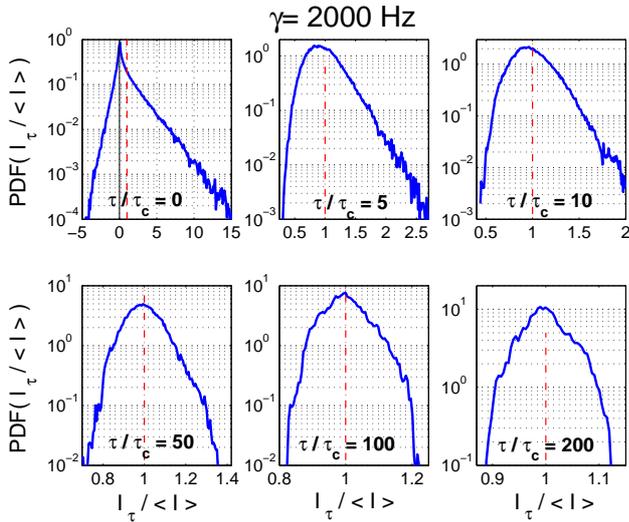


FIG. 9: PDF of $I_\tau/\langle I \rangle$ for various values of $\tau/\tau_c = 0, 5, 10, 50, 100$ and 200 at a fixed value of $\gamma = 2000$ Hz. The straight line (—) correspond to $I_\tau/\langle I \rangle = 0$ and the dashed line (---) to $I_\tau = \langle I \rangle$.

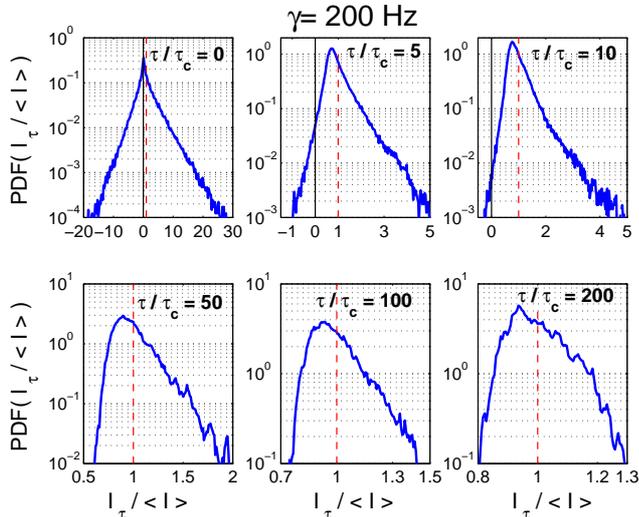


FIG. 10: Same as Fig. 9 for $\gamma = 200$ Hz.

and Eq. (13) thus leads to

$$\rho(\epsilon) = f(\epsilon) - f(-\epsilon). \quad (16)$$

Developing Eq. (16) up to first order in ϵ , that means, regarding only the terms $\epsilon \simeq 0$, thus leads easily to Gallavotti-Cohen relation of Eq. (14). This was first conjectured by Aumaitre et al. [15] and then predicted in a particular system by Farago [20]. But, what would happen if ϵ is far from zero? The experimental values of the asymmetrical function $\rho(\epsilon)$ are shown in Figs. 12 and 13 for two different values of γ , as a function of ϵ with $0 \leq \epsilon < 3$. For small ϵ , $\rho(\epsilon)$ increases linearly as expected, then $\rho(\epsilon)$ saturates when ϵ is increases further. For each value of τ/τ_c , the beginning of the saturation occurs for

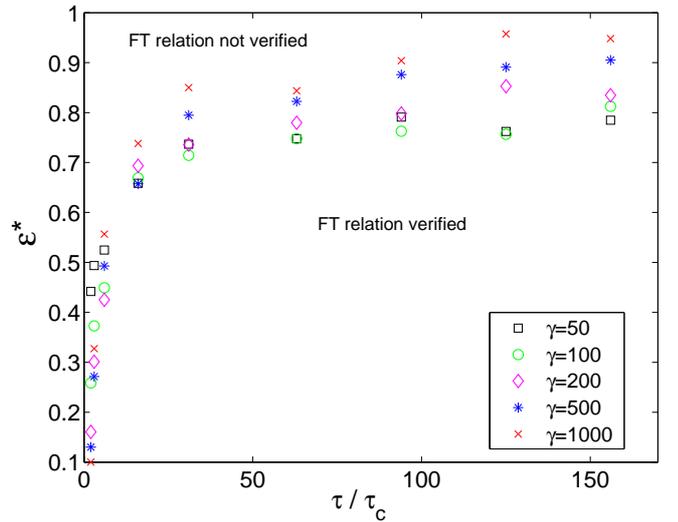


FIG. 11: Most probable value ϵ^* of PDF($I_\tau/\langle I \rangle$) as a function of τ/τ_c for $\gamma = 50, 100, 200, 500$ and 1000 Hz. For $\epsilon < \epsilon^*$, the FT relation is verified, whereas it does not hold for $\epsilon > \epsilon^*$ (see text).

a critical ϵ value, $\epsilon_c < 1$. This saturation value ϵ_c correspond to the maximum of the PDF, *i.e.* when the most probable value, ϵ^* , is reached (see Fig. 11). Moreover, increasing γ at fixed τ/τ_c leads decreasing available values of ϵ necessary to probe the FT theorem (see Figs. 12 and 13). It comes from the fact that when γ is increased, the number of negative injected power events, $\epsilon < 0$, decreases (γ controls the skewness of the PDF at a given $\tau_c \sim 1/\lambda$). We stress the fact that the damping rate γ , and therefore the mean dissipation, is not chosen in this simple experiment in an *ad-hoc* manner to satisfy time-reversibility. The smoothing of the signal around $\langle I \rangle$ also decreased the number of available negative events.

In most of the previous experimental test of the fluctuation theorem [5, 6, 7, 8], Eq. (14) is well followed, because of the small range of explored $\epsilon \leq 0.8$ at high $\tau/\tau_c \leq 20$. However, very recently, large range of ϵ has been reached experimentally [19] and does not satisfy the fluctuation theorem. In our experiment, large range of ϵ (up to 3) are also available even for high $\tau/\tau_c \simeq 20$. This, thus allow to test deeply the fluctuation theorem. As explained above, the FT works only for ϵ values smaller than the most probable value ϵ^* (see Fig. 11). Above this value, a saturation occurs, due to the different behavior of the PDF: for values larger than the most probable value, the PDF remains exponential, whereas for values smaller than ϵ^* it is smoother. Thus, large events of injected power are not well described by the FT, and lead to the observed saturation of $\rho(\epsilon)$.

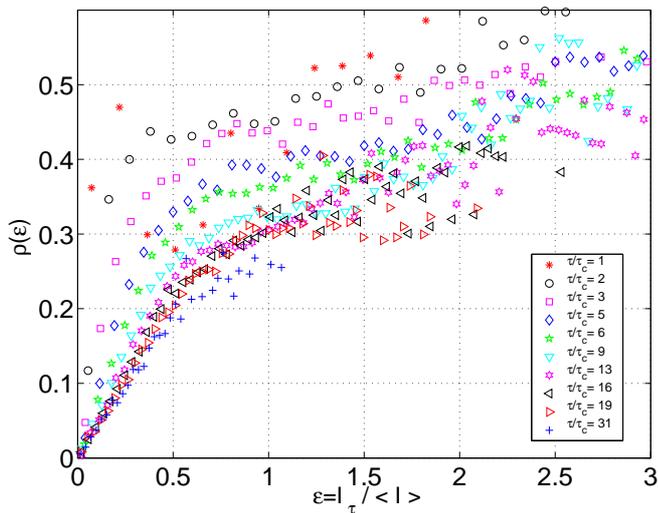


FIG. 12: Asymmetrical function $\rho(\epsilon) = \frac{\tau_c}{\tau} \ln \left[\frac{P(\epsilon)}{P(-\epsilon)} \right]$ as a function of ϵ for different integration times $\tau/\tau_c = 1$ (\star) to 31 ($+$) at fixed $\gamma = 100$ Hz and $D = 1.56$ mV $_{\text{rms}}^2$ /Hz.

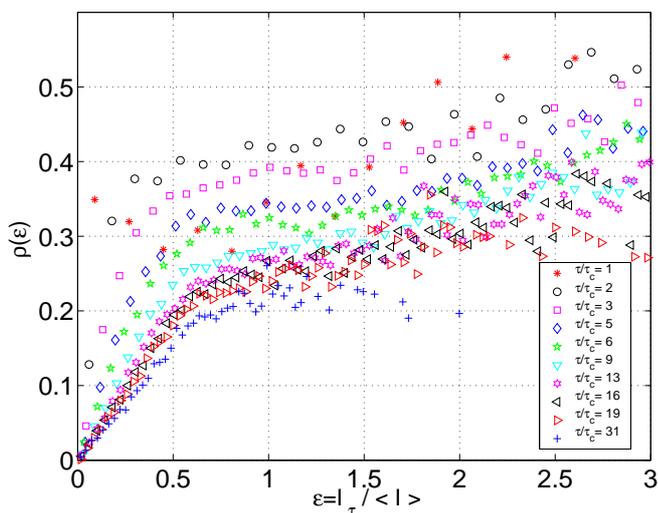


FIG. 13: Same as Fig. 12 for $\gamma = 50$ Hz.

V. INJECTED POWER IN A SET OF N CIRCUITS

Dissipative stochastic systems driven out of equilibrium are generally constituted of several components (e.g., in granular gases [5, 15]) that display correlations in space and time. One can wonder how these spatio-temporal correlations change the probability density function of the injected power fluctuations and their relevance in the fulfillment of the fluctuation theorem. In our simple experimental system, the correlations can be tuned as a control parameter. This can be achieved here by taking N electronic circuits and regarding them as a whole.

A. Correlated components

For a single electronic circuit, the smoothing average I_τ of the numerically sampled injected power $I(t)$, defined in Eq.(12) can be written as the discrete sum over N points,

$$I_\tau(t) = \frac{1}{N} \sum_{k=1}^N I(t + k\Delta t), \quad (17)$$

with $\tau \equiv N\Delta t$ and Δt the inverse of the sampling frequency. In our experiment, Δt is fixed at 10 μs . Since the correlation time of the injected power, $\tau_c \simeq 1/\lambda = 100$ μs , is greater than Δt , the elements of the sum above have a nonzero temporal correlation. This smoothing average can be also viewed as a sum of N *statistically dependent* components as

$$I_\tau(t) = \frac{1}{N} \sum_{k=1}^N I(t + k\Delta t) \equiv \frac{1}{N} \sum_{k=1}^N I_k(t), \quad (18)$$

where $I_k(t)$ corresponds to the injected power of the k th *correlated* circuit.

B. Uncorrelated components

Let us now focus on the case where correlations between components are neglected. That is to say each component loses its memory of the effect of the rest of the system faster than its internal dynamics, such as the case of a dilute granular gas where every particle dissipates its energy by collisions. After each collision, due to the low density of the gas, the particle loses its memory of its initial conditions decorrelating the injected power events in time. We take \mathcal{N} *statistically independent RC* circuits. For each time t , each injected power $I_i(t)$ of the i th *non-correlated* circuit is summed to obtain the ensemble average of the injected power, $I_{\mathcal{N}}(t)$ defined as

$$I_{\mathcal{N}}(t) = \frac{1}{\mathcal{N}} \sum_{i=1}^{\mathcal{N}} I_i(t). \quad (19)$$

This ensemble average $I_{\mathcal{N}}(t)$ should have different statistical properties than the smoothing one $I_\tau(t)$. Indeed, $I_{\mathcal{N}}(t)$ results from the sum over \mathcal{N} independent *RC* circuits (see Eq. (19)), whereas $I_\tau(t)$ comes from the sum over N correlated *RC* circuits (see Eq. (18)).

C. Results

The statistical properties of the injected power of the correlated and the uncorrelated systems display striking differences. The PDF of $I_\tau(t)$ is always asymmetric with exponential tails whatever $\tau \gg \tau_c$, whereas the PDF of $I_{\mathcal{N}}(t)$ tends towards a Gaussian when \mathcal{N} increases.

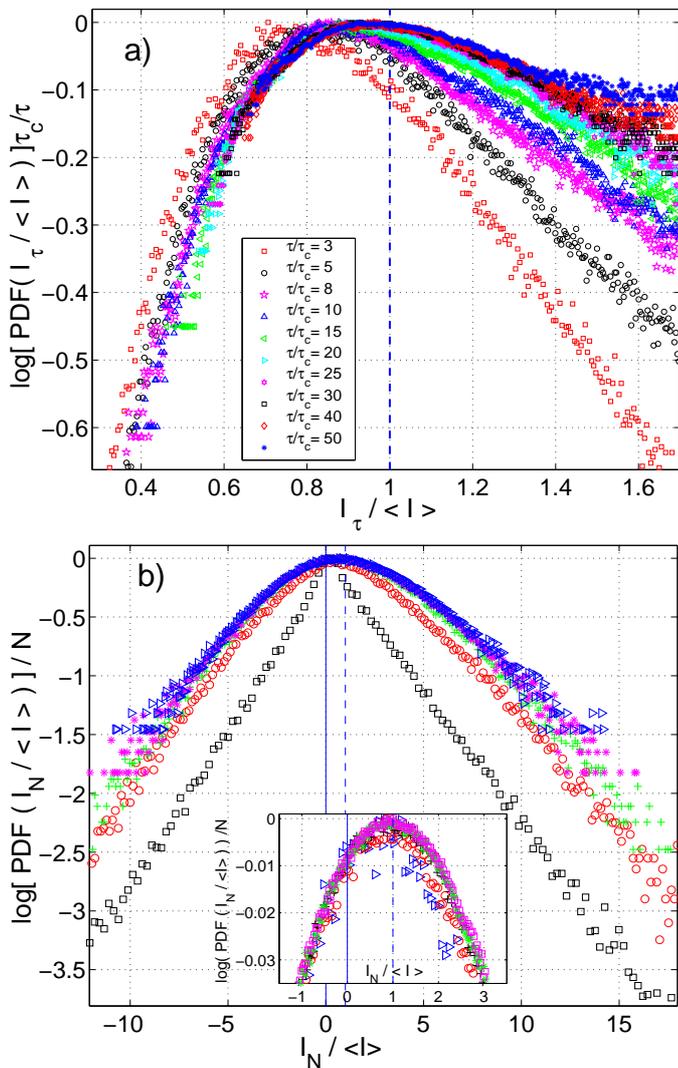


FIG. 14: **a)** Large deviation functions $\frac{\tau_c}{\tau} \ln [P(I_\tau / \langle I \rangle)]$ of τ/τ_c correlated circuits with $\tau/\tau_c = 3$ (\square) to 50 (\star) for $\gamma = 100$ Hz. **b)** Large deviation functions $\frac{1}{N} \ln [P(I_N / \langle I \rangle)]$ of N uncorrelated circuits with $N = 2$ (\square), 4 (\circ), 6 ($+$), 8 (\ast) and 10 (\triangleright), 100 for $\gamma = 100$ Hz. Inset: same with $N = 10$ (\triangleright), 20 (\circ), 30 ($+$), 40 (\ast) and 50 (\square). The dashed-lines show the mean injected power $\langle I \rangle$.

These features are shown in Fig. 14 by displaying the large deviation functions (LDF) of I_τ and I_N respectively defined by $\frac{\tau_c}{\tau} \ln [P(I_\tau / \langle I \rangle)]$ (see Eq. (15)) and by $\frac{1}{N} \ln [P(I_N / \langle I \rangle)]$. These LDFs describe how the fluctuations of both averages with respect to the mean $\langle I \rangle$ behave when the number of variables taken into account in the each sum, N or τ/τ_c , becomes larger and larger.

For the statistically dependent systems, the LDF of the injected power is not parabolic (as it should be if its PDF was a Gaussian) as shown in Fig. 14a. The convergence to its asymptotic shape is slow, depending strongly on the number of components of the system (i.e., of the durations of the time averaging, τ/τ_c). Moreover, when τ/τ_c increases, Fig. 14a shows also that the most

probable value of the PDF (the PDF's maximum) slowly tends towards the mean value $\langle I \rangle$, as already noticed in the previous section (Fig. 11). As it has been already shown in Fig. 12, for such a system with correlation, the fluctuation theorem is not satisfied.

For the N uncorrelated or statistically independent systems, when N is increased from 2 to 10, the LDF becomes more and more parabolic (its PDF becomes more and more Gaussian) as shown in Fig. 14b. For $N > 10$, the LDF shape is a parabola centered on $\langle I \rangle$, and independent on N (see inset of Fig. 14b). This is a consequence of the central limit theorem. As soon as $N > 10$, the most probable value of the PDF (the PDF's maximum) is equal to 1, that is the mean value (see Fig. 14b). Following the discussion of the previous section, this means that, for such a system without spatial or temporal correlations, the fluctuation theorem is satisfied as soon as $N > 10$ (see below and Fig. 15).

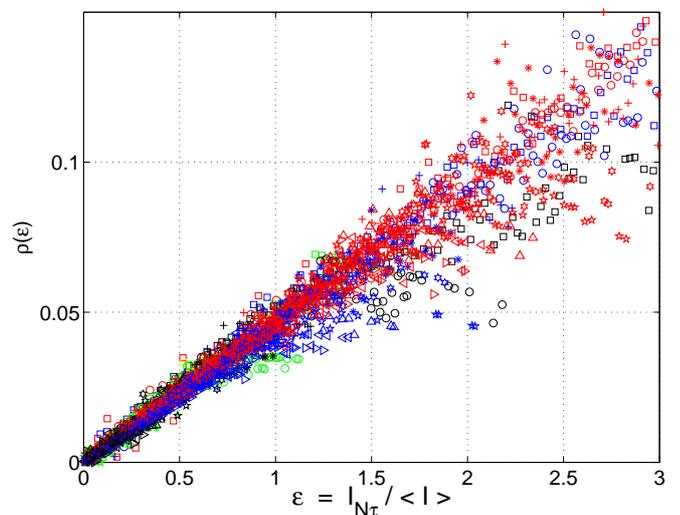


FIG. 15: Asymmetrical function $\rho(I_{N\tau} / \langle I \rangle)$ of N independent circuits, for different integration times $\tau/\tau_c = 1$ (\star) to 31 ($+$). $N = 10$ to 100 with a 10 step.

Let us finally test the FT for an ensemble of N independent circuits. The smoothing average of $I_N(t)$ over a time τ is defined as

$$I_{N\tau}(t) = \frac{1}{N\tau} \sum_{i=1}^N \int_t^{t+\tau} I_i(t') dt'. \quad (20)$$

The asymmetrical function $\rho(I_{N\tau} / \langle I \rangle)$ of the N independent circuits ($10 \leq N \leq 100$) is plotted in Fig. 15 for 10 different integration times τ/τ_c . Whatever the value of N and τ/τ_c , $\rho(I_{N\tau} / \langle I \rangle)$ increases linearly with $I_{N\tau} / \langle I \rangle$. Therefore, this means that the system of N uncorrelated circuits verify the FT relation (see Eq. (13)), as expected above. This could be also explained as following. When one plots its Large Deviation Function, that is $\frac{\tau_c}{N\tau} \ln [P(I_{N\tau} / \langle I \rangle)]$, the LDF is well fitted (not shown here) by the quadratic function $(I_{N\tau} / \langle I \rangle - 1)^2 / \sigma_0^2$,

σ_0^2 being the integral of the normalized autocorrelation function of $I(t)$ [15]. The quadratic shape of the LDF of $I_{\mathcal{N}\tau}(t)$ is a consequence of the Gaussian shape of the PDF of $I_{\mathcal{N}}(t)$. Therefore, using Eq. (16) and the LDF quadratic fit, the FT relation of Eq. (13) is satisfied with $\beta = 2/(\langle I \rangle \sigma_0^2)$. Note however that when $\mathcal{N}\tau_c/\tau \simeq 1$ (whatever the limits in \mathcal{N} and τ/τ_c), the FT relation is a straightforward consequence of the central limit theorem. The slight departure from linearity for large ϵ shown in Fig. 15 is also a consequence of the central limit theorem in the sense that we have not reach yet the asymptotic shape of the LDF, therefore the exponential tails of the instantaneous injected power PDF remain. Taking more elements in $I_{\mathcal{N}\tau}$, the values of ϵ are smaller but the convergence to the gaussian type of LDF is faster, making the assymetrical function $\rho(\epsilon)$ a linear function of ϵ .

VI. CONCLUSION

In conclusion, we have studied the statistical properties of the instantaneous injected power $I(t)$ in one of the simplest dissipative out-of-equilibrium system: an electronic RC circuit submitted to a stochastic voltage. The probability distribution function (PDF) of $I(t)$ is measured for different values of the forcing amplitude and of the damping rate γ . It displays a cusp near $I \simeq 0$ and asymmetric exponential tails. This typical PDF shape has been observed in more complex dissipative systems (such as in granular gases [5], wave turbulence [19] and convection [7, 25]). The relevant parameters of the system can be easily changed in our simple experiment. For instance, the damping rate appears as the only control parameter driving the asymmetry of the distribution of $I(t)$: The larger the damping rate γ , the larger the asymmetry of the PDF. The scalings of the first moments of the injected power ($\langle I \rangle$ and σ_I) are also measured.

The fluctuation theorem (FT) has then been probed by measuring the asymmetrical function $\rho(\epsilon)$ with $\epsilon = I_\tau/\langle I \rangle$, and I_τ the smoothing average on a time lag τ . Contrarily to previous experiments, the range of available fluctuation amplitude is large ($\epsilon \simeq 3$) even for long

averaging time ($\tau/\tau_c \simeq 20$). This experiment thus allow to probe the FT in the limit of large ϵ and large τ/τ_c . We have found out that the FT is only satisfied for values of ϵ smaller than the most probable value, ϵ^* (i.e. the maximum of the PDF of ϵ). For values larger than ϵ^* , the asymmetrical function is no more linear with ϵ but saturates. Thus, the FT does not hold for the large available values of ϵ even at large τ/τ_c . This disagreement is not particular of this electronic system, but seems to be generic to other systems. It has been also recently observed with a wave turbulence experiment [19]. This model experiment thus appears as a useful tools to probe the FT in various limit of averaging time and fluctuations amplitude.

Finally, this electronic experiment can be extend to mimic the behavior of a more complex out-of-equilibrium stochastic system with higher degree of freedom. To wit, we have studied the injected power fluctuations in i) a system of N correlated circuits and ii) a system of \mathcal{N} uncorrelated RC circuits. This latter can be viewed as an archetype of a dilute granular gas of uncorrelated particles. The fluctuation theorem (FT) for the time-averaged injected power has then been tested for the case of the correlated and uncorrelated systems. We have found out that the FT is not verify in the correlated system, whereas it is satisfied for the uncorrelated system at finite time. In this last case, the fulfillment of the relation is just a consequence of the central limit theorem. Finally, this work also points out that the agreement with the FT relation is dependent on how the averaging process is performed (non overlapping bins of duration $\tau > \tau_c$ [5] or overlapping ones are two different processes related to respectively uncorrelated or correlated system).

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