Maximal wild monodromy in unequal characteristic

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June 2, 2006

Abstract

Let R be a complete discrete valuation ring of mixed characteristic $(0, p)$ with field of fractions K containing the p-th roots of unity and $C \longrightarrow \mathbb{P}^1_K$ a p-cyclic cover of the projective line. In this paper we study the finite monodromy extension of the curve C, i.e. the minimal finite extension K'/K over which C has a stable model. In particular we are interested in the wild part of this extension. In various examples we have shown that the finite monodromy can be maximal, i.e. attain certain bounds that were given in previous work by the authors.

1 Introduction

Let R be a complete discrete valuation ring of mixed characteristic $(0, p)$ with field of fractions K containing the p-th roots of unity. This work is about semi-stable models of p-cyclic covers of the projective line $C \longrightarrow \mathbb{P}^1_K$. It is a continuation of our paper [Le-Ma2]. Denote by K' the minimal extension of K over which C has a stable model. Using the techniques developed in that paper we study the wild finite monodromy extension, i.e. the extension corresponding to the Sylow p-subgroup of $\mathrm{Gal}(K'/K)$.

In section 2, we review some basic definitions and the results from [Le-Ma2] that are essential to this paper. Among these are the stably marked models and the monodromy polynomial. Associated with C/K are the finite monodromy group and related Galois groups which will be recalled briefly.

The Galois group of the finite monodromy extension acts on the special fiber $C_{R'} \otimes_{R'} k$ of the stable model:

$$\mathrm{Gal}(K'/K) \hookrightarrow \mathrm{Aut}(C_{R'} \otimes_{R'} k)$$

The bounds derived from [Le-Ma1] Theorem 1.1 for the right hand side also limit the possible size of $\mathrm{Gal}(K'/K)$. Hence one can ask if $v_p(|\mathrm{Gal}(K'/K)|)$ can be maximal? (Here v_p is the p-adic valuation with $v_p(p) = 1$). This is one central question of the paper and we study it in section 3 for covers that are defined over $\mathbb{Q}_p^{\mathrm{tame}}$. The reason for working over this ground field is that we are mostly interested in the wild part of the monodromy. We encounter various situations where the above can be affirmatively answered. (cf. Prop.3.2b, Rem.3.4, Prop.3.6, Exa.3.8, Prop.3.10 and Exa.3.12).

*Work supported in part by the European Community's Sixth Framework Programme under Contract MRTN-CT2003-504917

Section 3.1 provides families of curves with potentially good reduction and a criterion for when there is maximal finite monodromy. In this case the finite monodromy group turns out to be an extraspecial p-group.

Section 3.2 deals more specifically with the case of genus 2 curves and residue characteristic 2. Here we have shown that any type of degeneration with maximal wild monodromy is possible.

Section 3.3 relates our work to results by Silverberg and Zarhin on the monodromy of abelian surfaces (cf. [Si-Za1], [Si-Za2]). In particular we give an example of non-trivial monodromy in unequal characteristic; something they were asking for.

The Magma code we have used to construct the examples is available from the authors upon request. To give an idea we have included it here only in one example (cf. Exa. 3.8).

2 Background

This is a brief review of the results proven in [Le-Ma2]. The reader is encouraged to take a look at that paper for more details and motivation. Let R be a complete mixed characteristic $(0,p)$ discrete valuation ring with field of fractions K and algebraically closed residue field k. Denote by v the valuation defined by R on K and by π a uniformizer. We will always assume that R contains a primitive p-th root of unity ζ and define $\lambda = \zeta - 1$. Then $v(\lambda^{p-1}) = v(p)$. Let $C \longrightarrow \mathbb{P}^1_K$ be a p-cyclic cover of smooth projective geometrically irreducible K-curves with genus$(C) \geq 2$. We assume that the branch locus B of the cover consists of K-points, write $m = |B| - 1$, and assume that B has equidistant geometry in the sense of the following definition.

Definition 2.1. We say that B has *equidistant geometry* if there exists a smooth R-model Proj$(R[u,w])$ for $\mathbb{P}^1_K$ such that the points of B have distinct specializations on the special fiber of the model.

We can assume that, with respect to the coordinate $X_0 := u/w$, the cover is given birationally by the equation $Z_0^p = f(X_0)$ with $f(X_0) \in R[X_0]$ monic, $n = \deg(f(X_0))$, $(n,p) = 1$ and $n \leq m(p-1)$. Then any two distinct zeros of $f(X_0)$ specialize to distinct elements in k. We denote by $K(C)/K(X_0)$ the function field extension corresponding to the cover $C \longrightarrow \mathbb{P}^1_K$.

With the above notation, by Deligne and Mumford (cf. [De-Mu] Cor.2.7 and [Des] Prop.5.7, Lemme 5.16), there exists a minimal extension K'/K such that $C_{K'}$ has a stable model $C^{\circ}_{R'}$ over the integral closure R' of R in K'. Further K'/K is Galois and acts faithfully on the special fiber:

$$\mathrm{Gal}(K'/K) \hookrightarrow \mathrm{Aut}_k(C^{\circ}_{R'} \otimes_{R'} k) \tag{1}$$

Definition 2.2. Let K'/K be the minimal extension over which C has a stable model. We call Gal(K'/K) the *finite monodromy group* associated to C. The extension K'/K is called the *finite monodromy extension*, Gal(K'/K) has a unique p-Sylow subgroup which we call the *wild monodromy group* Gal$(K'/K)_w$. The field extension $K'/K'^{\mathrm{Gal}(K'/K)_w}$ is called the *wild monodromy extension*.

Let K'/K be as above. Then by [Le-Ma2] Prop.2.4 there exists a semi-stable model model $C_{R'}$, defined over the integral closure R' of R in K minimal with the property that the points in the ramification locus of $C_{K'} \longrightarrow \mathbb{P}^1_{K'}$ specialize to distinct smooth points on the special fiber. Further the finite monodromy group operates faithfully on its special fiber:

$$\mathrm{Gal}(K'/K) \hookrightarrow \mathrm{Aut}_k(C_{R'} \otimes_{R'} k) \tag{2}$$

Definition 2.3. The semi-stable model $C_{R'}$, given above, is called *the stably marked model*. We refer to the component to which the ramification points specialize as *original component*. Any model obtained from the stably marked model by a base change to a ring $R'' \supset R'$ we refer to as *a stably marked model*.

In [Ra1] Thm.2 Raynaud shows that the dual graph of the special fiber of a stably marked model of C is an oriented tree whose origin corresponds to the original component.

We will frequently use the following result which was shown in [Le-Ma2] Prop.2.7.

Proposition 2.4. *Let $C \longrightarrow \mathbb{P}^1_K$ be as above. Assume that for each irreducible component E of genus > 0 in the stable reduction there exists a flat quasi-projective R-model $\mathcal{C}$ for C, such that $\mathcal{C} \otimes_R k$ is birational to E. Then C has a stable model over R.*

As stated before, we shall always assume that the branch locus of $C \longrightarrow \mathbb{P}^1_K$ has equidistant geometry and consists of rational points. This has consequences for the image of the injection (2). Namely, any element of its image will have trivial action on the original component of the stably marked model. The group of such automorphisms we denote by $\mathrm{Aut}_k(C_{R'} \otimes_{R'} k)^{\#}$. Hence we have

$$\mathrm{Gal}(K'/K) \hookrightarrow \mathrm{Aut}_k(C_{R'} \otimes_{R'} k)^{\#} \tag{3}$$

Definition 2.5. If the injection (3) is surjective we say that C has *maximal monodromy*. If $v_p(|\mathrm{Gal}(K'/K)|) = v_p(|\mathrm{Aut}_k(C_{R'} \otimes_{R'} k)^{\#}|)$ we say that C has *maximal wild monodromy*.

The rest of this section introduces the monodromy polynomial and its main properties. (cf. [Le-Ma2] section 3). Let m be the number of distinct zeros of $f(X_0)$ in an algebraic closure of $\mathrm{frac}(R)$ and r the greatest integer such that $rp < n$. For $X_0 = X + Y$ Taylor expansion yields

$$f(X_0) = f(X+Y) = s_0(Y) + s_1(Y)X + s_2(Y)X^2 + \cdots + s_n(Y)X^n \in R[X,Y] \tag{4}$$

which we view as a polynomial in two variables.

Definition 2.6. If $f(X_0)$ as above can be written

$$f(X+Y) = s_0(Y)\left(H(X,Y)^p - \sum_{i=r+1}^{n} A_i(Y)X^i\right) \tag{5}$$

with $H(X,Y) = 1 + a_1(Y)X + a_2(Y)X^2 + \cdots + a_r(Y)X^r \in K(Y)[X]$, $A_i(Y) \in K(Y)$ and r as introduced earlier, we call this a *special decomposition* of $f(X_0)$.

The existence of such a decomposition is shown in [Le-Ma2] Lemma 3.3. We set $S_1(Y) = s_1(Y)/\gcd(s_0(Y), s_1(Y))$ and $S_0 = s_0(Y)/\gcd(s_0(Y), s_1(Y))$. Then

$S_1(Y), S_0(Y) \in R[Y]$, $f'(Y)/f(Y) = S_1(Y)/S_0(Y)$, $\deg(S_1(Y)) = m-1$, $\deg(S_0(Y)) = m$ and $(S_0(Y), S_1(Y)) = 1$. Also, for $t \in \mathbb{N}$, we will use the following notation

$$\binom{\frac{1}{p}}{t} = \frac{\prod_{i=0}^{t-1}\left(\frac{1}{p} - i\right)}{t!}.$$

For later use we note that the p-adic value of these binomial coefficients is given by

$$-v_p(\binom{\frac{1}{p}}{t}) = t + [t/p] + [t/p^2] + \dots \tag{6}$$

where $v_p(p) = 1$.

Lemma 2.7. *There exist $c_i \in K$, $T(Y) \in R[Y]$ and $N_i(Y) \in R[Y]$ monic such that*

$$A_i(Y) = c_i \frac{N_i(Y)}{s_0(Y)^i} \quad \textit{for } r+1 \le i \le n \quad \textit{and}$$

$$A_{p^\alpha}(Y) = -\binom{\frac{1}{p}}{p^{\alpha-1}}^p \frac{S_1(Y)^{p^\alpha} + pT(Y)}{S_0(Y)^{p^\alpha}} \quad \textit{with} \quad v_p(c_{p^\alpha}) = v_p(\binom{\frac{1}{p}}{p^{\alpha-1}}^p) \le 0$$

where p^α is the power determined by $p^\alpha < n < p^{\alpha+1}$.

Definition 2.8. In the situation of Lemma 2.7 we define $\mathcal{L}(Y) = S_1(Y)^{p^\alpha} + pT(Y)$. It is a polynomial of degree $p^\alpha(m-1)$ and we call it the *monodromy polynomial* of $f(Y)$.

From the definition one immediately obtains $\mathcal{L}(Y) \equiv S_1(Y)^{p^\alpha} \bmod p$. Notice that $f'(Y)/f(Y) = S_1(Y)/S_0(Y)$.

Definition 2.9. The valuation on $K(X_0)$ corresponding to the DVR $R[X_0]_{(\pi)}$ is called the *Gauss valuation* v_{X_0} with respect to X_0.

$$\text{We then have} \quad v_{X_0}\left(\sum_{i=0}^{m} a_i X_0^i\right) = \min\{v(a_i) | 0 \le i \le m\}.$$

Remark 2.10. a) In the above situation, consider the flat projective R-model $\mathcal{C}_R$ for C obtained by normalizing $\mathrm{Proj}(R[u,w])$ in the function field $K(C)$. From $\mathcal{C}_R$ we obtain the stably marked model $C_{R'}$ for C by a series of blowups, after passing to a finite extension R'. The uniqueness of $C_{R'}$ implies that the p-cyclic group action on C extends to $C_{R'}$ and the quotient by this action is a semi-stable model for $\mathbb{P}^1_{K'}$, where $K' = \mathrm{frac}(R')$ (cf. [Ra1] appendice). This semi-stable model is obtained from $\mathrm{Proj}(R'[u,w])$ by a series of blowups. By construction, these are all centered on the affine patch $\mathrm{Spec}(R'[X_0])$ and hence correspond to ideals of the form $(X_0 - y, \rho) \subset R'[X_0]$. We call y the center and ρ the radius, and often speak of the Gauss valuation v_X with $X = (X_0 - y)/\rho$ instead. All blowups we will consider are of this type and on $\mathrm{Proj}(R^{alg}[u,w])$. Blowing up $(X_0 - y, \rho) \subset R'[X_0]$ gives an exceptional divisor which in turn, by normalization in $K'(C)$, yields irreducible components of the stably marked model. The DVRs at their generic points are the ones that dominate the DVR of v_X.

A local computation shows that the centers specialize to points below the singularities (cusps) of $\mathcal{C}_R \otimes_R k$ and are outside the specialization of the branch locus. Their X_0-coordinates are zeros of $\bar{f}'(X_0)$, hence also zeros of $\bar{S}_1(X_0) = \bar{f}'(X_0)/\gcd(\bar{f}'(X_0), \bar{f}(X_0))$. In particular, if $X_0 = y$ is such a center, then $y \in R^{alg}$ and $\bar{f}(\bar{y}) \in k - \{0\}$. A schematic picture of this situation is given in figure 1.

b) It is immediately derived from the equation that the cover of special fibers $\mathcal{C}_R \otimes_R k \longrightarrow \mathbb{P}^1_k$ is purely inseparable. Therefore, to yield a component of genus > 0 in the stable reduction of C, a blowup (on $\mathrm{Proj}(R[u, w])$) must have a radius ρ with $v(\rho) > 0$.

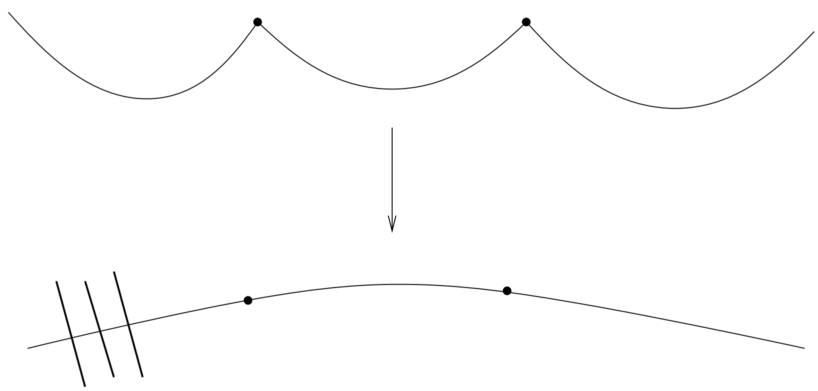


Figure 1: $\mathcal{C}_{R'} \otimes_{R'} k \longrightarrow \mathbb{P}^1_k$ with singularities and branch locus

Now we can state the main theorem of [Le-Ma2], describing a stably marked model. Suppose that the cover is given birationally by $Z_0^p = f(X_0)$ with $f(X_0)$ monic of degree n prime to p and such that, with respect to X_0, the branch points have distinct specializations. Let r be the greatest integer such that $rp < n$ and let α be determined by $p^\alpha < n < p^{\alpha+1}$. We write $|B| - 1 = m$ and can assume that $f(X_0)$ has been chosen such that $m(p-1) \geq n$. Choosing $f(X_0)$ this way limits the size of $\mathcal{L}(Y)$. We point out that, due to the freedom in choosing $f(X_0)$, the polynomial $\mathcal{L}(Y)$ is not uniquely determined by the cover $C \longrightarrow \mathbb{P}^1_K$.

Theorem 2.11. *The components of genus > 0 of the stably marked model of C correspond bijectively to the Gauss valuations v_{X_j} with $\rho_j X_j = X_0 - y_j$ where y_j is a zero of the monodromy polynomial $\mathcal{L}(Y)$ and $\rho_j \in R^{\mathrm{alg}}$ is such that*

$$v(\rho_j) = \max\{\frac{1}{i} v\left(\frac{\lambda^p}{A_i(y_j)}\right) \textit{ for } r+1 \leq i \leq n\}.$$

The dual graph of the special fiber of the stably marked model of C is an oriented tree whose ends are in bijection with the components of genus > 0.

Corollary 2.12. *Let K'/K be the minimal extension such that $C_{K'}$ has a stable model over the integral closure R' of R in K' and denote by F the splitting field of $\mathcal{L}(Y)$ over K. Let E be the field obtained from F by adjoining ρ_i as well $(s_0(y_i))^{1/p}$ for each i. Then $K' \subseteq E$. In particular C has a stable model over the integral closure of R in E.*

3 Maximal wild monodromy over $\mathbb{Q}_p^{\text{tame}}$

In this section, we illustrate the effectiveness of our methods in order to construct covers with maximal wild monodromy over a local field $K \subset \mathbb{Q}_p^{\text{tame}}$, i.e. a finite tamely ramified extension of $\mathbb{Q}_p$. We work under the hypotheses of section 2. In particular $C \longrightarrow \mathbb{P}^1_K$ is a p-cyclic cover of the projective line with branch locus B. Recall that there exists a minimal extension K'/K such that C has a stably marked model $C_{R'}$ over the integral closure R' of R in K. The group $\text{Gal}(K'/K)$ injects into $\text{Aut}_k(C_{R'} \otimes_{R'} k)^{\#}$ and, if $v_p(\text{Gal}(K'/K)) = v_p(\text{Aut}_k(C_{R'} \otimes_{R'} k)^{\#})$, we say that the cover has *maximal wild monodromy* (cf. Definition 2.5).

3.1 Potentially good reduction with $m = 1 + p^s$

In this section we analyze the finite monodromy for a family of curves with potentially good reduction. We will see that the wild finite monodromy group can, but needs not to be maximal with respect to the injection (3). We denote by $\mathfrak{m}$ the maximal ideal of R^{alg} and assume that $C \longrightarrow \mathbb{P}^1_K$ is given birationally by the equation

$$Z_0^p = f(X_0) = 1 + cX_0^q + X_0^{q+1} \quad \text{for } c \in R, \quad q = p^n \quad \text{and } n \geq 1.$$

Clearly the branch locus has equidistant geometry and $m = 1+q$. In the following, when considering $\alpha \in R^{\text{alg}}$, we will write $\alpha^{1/p}$ for a p-th root of α. The formulas we get will be valid independently of the choice of the p-th roots.

Definition 3.1. For $p > 2$ we define the two groups $E(p^3)$ and $M(p^3)$ by $E(p^3) = \langle x, y | x^p = y^p = [x,y]^p = [x,[x,y]] = [y,[x,y]] = 1 \rangle$ and $M(p^3) = \langle x, y | x^{p^2} = y^p = 1, y^{-1}xy = x^{1+p} \rangle$. These are the two non-abelian p-groups of order p^3. The group $E(p^3)$ has exponent p and $M(p^3)$ has exponent p^2. We write Q_8 and D_8 for the quaternion and dihedral groups of order 8. If G is an extraspecial p-group then $|G| = p^{2n+1}$ for some $n > 0$ and the following four types occur:
I.a) If $p > 2$ and exponent$(G) = p$ then G is isomorphic to the central product of n copies of $E(p^3)$.
I.b) If $p > 2$ and exponent$(G) = p^2$ then G is isomorphic to the central product of $M(p^3)$ with $n-1$ copies of $E(p^3)$.
If $p = 2$ then either
II.a) $E = D_8 * D_8 * \cdots * D_8$ is the central product of n factors D_8.
Or II.b) $E = Q_8 * D_8 * \cdots * D_8$ is the central product with $n-1$ factors D_8.

For the purposes of [Le-Ma1] and this paper we call this groups *extraspecial group of type* I.a, I.b, II.a and II.b.

For a complete account on extraspecial p-groups we refer to [Su] chapter 4.

Proposition 3.2. *a) If $v(c) \geq v(\lambda^{p/(q+1)})$ then C has good reduction over K and the wild monodromy is trivial.*
b) If $v(c) < v(\lambda^{p/(q+1)})$ let $s_i(Y) = f^{(i)}(Y)/i!$ and define $L(Y) = s_1(Y)^q - a_n s_0(Y)^{q-1} s_q(Y)$ where $a_n = (-1)^q(-p)^{p+p^2+\cdots+p^n}$. Let y be a root of $L(Y)$. Then C has good reduction over $M := K(y, f(y)^{1/p})$. Further, if $L(Y)$ is irreducible over K, then M/K is Galois with the extraspecial group of order pq^2 and exponent p if $p > 2$, and type II.b. if $p = 2$. In this case M/K is the finite monodromy extension denoted K'/K in section 2 and the finite monodromy is maximal in the sense of Definition 2.5.

If $v(c) = (a/b)v(p) < v(\lambda^{p/(q+1)})$ with a, b both prime to p then $L(Y)$ is irreducible. (This is the case e.g. for $c = p^{p/(q+1)}$).

c) If $L(Y)$ is irreducible over K so is $\mathcal{L}(Y)$, and the finite monodromy extension M/K is obtained from the splitting field of $\mathcal{L}(Y)$ by adjoining $f(y)^{1/p}$ for y any zero of $\mathcal{L}(Y)$.

Remark 3.3. The polynomial $L(Y)$ appearing in the proposition is closely related to $\mathcal{L}(Y)$. In fact it turns out that for this family all terms $s_i(Y)$ for $2 \leq i \leq q-1$ yield p-adically small contributions in $\mathcal{L}(Y)$ and hence are negligible (cf. Lemma 2.7 and Def.2.8). Consequently in this case it is sufficient to work with $L(Y)$. Notice though that $\deg(L(Y)) = \deg(\mathcal{L}(Y))$.

Proof. a) $v(c) \geq v(\lambda^{p/(q+1)})$. In this case consider $X = \lambda^{p/(q+1)}T$ and $Z_0 = \lambda Z_1 + 1$. With T and Z_1 the defining equation of the cover becomes

$$Z_0^p = (\lambda Z_1 + 1)^p = \lambda^p Z_1^p + p\lambda^{p-1}Z_1^{p-1} + \cdots + p\lambda Z_1 + 1 = c\lambda^{pq/(q+1)}T^q + \lambda^p T^{q+1}.$$

The assumption made about c implies that this equation is divisible by λ^p and after reducing it to k we get $Z_1^p - Z_1 = aT^q + T^{q+1}$ for some $a \in k$. Using Artin-Schreier Theory, we see that this equation has the same geometric genus as the generic fiber. By Proposition 2.4 we conclude that C has good reduction over K. Therefore, in this case the finite monodromy is trivial.

b) The claim is that the cover has potentially good reduction. We therefore look for $y \in R^{\text{alg}}$ such that T defined by $X_0 = y + \lambda^{p/m}T$, with $m = q+1$, induces the good reduction. The proof is divided into several steps:

I. If y is a root of $L(Y)$ we claim that $v(y) = v(a_n c)/q^2$. One has $s_0(Y) = 1 + cY^q + Y^{q+1}$, $s_1(Y) = qcY^{q-1} + (q+1)Y^q$ and $s_q(Y) = c + (q+1)Y$. Then $L(Y) = [(q+1)^q - a_n(q+1)]Y^{q^2} + \cdots - a_n(q+1)Y - a_n c$ and we look at its Newton Polygon. The coefficient of Y^{q^2} is a unit, the coefficients of the linear and the constant term have valuation $v(a_n)$ and $v(ca_n)$, respectively. Moreover for $0 < i \leq q$ the coefficient of Y^{q^2-i} has valuation $\geq \min\{v((cq)^i), v(a_n)\}$ and for $q < i < q^2$ it has valuation $\geq v(a_n)$. By assumption $v(c) < v(\lambda^{p/(q+1)}) = v(a_n)/(q^2-1)$, hence $v(a_n c)/q^2 \leq v(a_n)/(q^2-1)$ and it follows that the Newton Polygon is the line joining the points $(0,0)$ and $(q^2, v(a_n c)/q^2)$. We conclude that $v(y) = v(a_n c)/q^2$.

II. We show the irreducibility of $L(Y)$ if $v(c) = (a/b)v(p)$ with a, b prime to p. Under these hypothesis we have $v(y) = ((p + ... + p^n) + (a/b))/q^2 v(p)$. Therefore the ramification index in L/K is q^2 and so $L(Y)$ is irreducible in this case.

III. For y a root of $L(Y)$ and X, T defined by $X_0 = X + y$ and $X = \lambda^{p/m}T$ we claim that

$$f(X+y) = s_0(y) + s_1(y)X + s_q(y)X^q + X^{q+1} + r \quad \text{with } r \in \lambda^p\mathfrak{m}[T].$$

Using the general formula

$$(A+B)^{p^n} = (A^{p^{n-1}} + B^{p^{n-1}})^p \mod(p^2)$$

we compute $f(y + \lambda^{p/m}T)$ modulo $(\lambda^p\mathfrak{m}[T])$:

$$\begin{aligned} f(\lambda^{p/m}T + y) &= 1 + c(y + \lambda^{p/m}T)^q + (y + \lambda^{p/m}T)^{q+1} \\ = 1 + c(y^{p^{m-1}} &+ \lambda^{p^n/m}T^{p^{n-1}})^p = (y + \lambda^{p/m}T)(Y^{p^{n-1}} + \lambda^{p^n/m}Y^{p^{n-1}})^p \\ &= f(y) + y^{p^n}X + py^{p^{n-1}(p-1)}(c+y)X^{p^{n-1}} + X^{p^n}(y + c + X) \end{aligned}$$

$$= s_0(y) + s_1(y)X + s_q(y)X^q + X^{q+1} \mod (\lambda^p \mathfrak{m}[T]).$$

IV. Again let y be a root of $L(Y)$. We claim that for $0 \leq i \leq n$ there exist $A_i(y, X) \in \mathfrak{m}[T]$ and $B_i(y) \in R^{\text{alg}}$ such that

$$f(X + y) = (s_0(y)^{1/p} + XA_i(y, X))^p + s_1(y)X + B_i(y)X^{q/p^i} + X^{q+1} + r_i \tag{7}$$

with $r_i \in \lambda^p \mathfrak{m}[T]$ and

$$v(B_i(y)) \geq (1 + \frac{1}{p} + ... + \frac{1}{p^{i-1}})v(p) \quad \text{for } i \geq 1 \tag{8}$$

The proof works by induction. Clearly $A_0(y, X) = 0$ and $B_0(y) = s_q(y)$ work. For $i = 1$ one can take $XA_1(y, X) = B_0(y)^{1/p}X^{q/p}$ and $B_1(y) = -ps_0(y)^{(p-1)/p}B_0(y)^{1/p}$. Then (7) follows as $pX^{2q/p} \in \lambda^p \mathfrak{m}[T]$ (one can assume $q > 2$) and (8) is trivially satisfied.

Assume that for some i with $1 \leq i < n$ conditions (7) and (8) are both satisfied. We define A_{i+1} and B_{i+1} by $XA_{i+1}(y, X) = XA_i(y, X) + B_i(y)^{1/p}X^{q/p^{i+1}}$ and $B_{i+1}(y) = -ps_0(y)^{(p-1)/p}B_i(y)^{1/p}$. Observe that $v(pB_i(y)^{1/p}) \geq (1 + \frac{1}{p} + ... + \frac{1}{p^i})v(p)$. Hence it follows that

$$v(pB_i(y)^{1/p}\lambda^{\frac{pq}{(q+1)p^{i+1}}+1}) \geq ((1 + \frac{1}{p} + ... + \frac{1}{p^i}) + ((\frac{pq}{(q+1)p^{i+1}} + 1)\frac{1}{p-1}))v(p) > v(\lambda^p)$$

and so $pB_i(y)^{1/p}X^{q/p^{i+1}+1} \in \lambda^p \mathfrak{m}[T]$. This proves (7) and (8) for $i + 1$. Note that (7) does not depend on the choice of the p-th root $B_i(y)^{1/p}$, since for ζ a p-th root of 1 one has $v(p(\zeta - 1)) = v(\lambda^p)$.

V. Still denoting by y a root of $L(Y)$ we claim that C has good reduction over $M := K(y, f(y)^{1/p})$. We apply IV. formula (7) for $i = n$. Then we get

$$f(X + y) = (s_0(y)^{1/p} + XA_n(y, X))^p + (s_1(y) + B_n(y))X + X^{q+1} + r_n \tag{9}$$

with $r_n \in \lambda^p \mathfrak{m}[T]$. Note that in (9) one can replace $B_n(y)$ by $\zeta_n B_n(y)$ where $\zeta_n^q = 1$ as $v((\zeta_n - 1)B_n(y)) \geq v(\lambda^p)$. Recall that $B_0(y) = s_q(y)$ and $B_{i+1}(y) = -ps_0(y)^{(p-1)/p}B_i(y)^{1/p}$. It follows that

$$B_n(y) = (-p)^{1+\frac{1}{p}+...+\frac{1}{p^{n-1}}} s_0(y)^{1-\frac{1}{q}} s_q(y)^{\frac{1}{q}}.$$

By definition of $L(Y)$ we have $s_1(y)^q = (-B_n(y))^q$ and it follows that (9) can be written in the following way:

$$f(X + y) = (s_0(y)^{1/p} + XA_n(y, X))^p + X^{q+1} + \tilde{r}_n \quad \text{with } \tilde{r}_n \in \lambda^p \mathfrak{m}[T] \tag{10}$$

It follows that the Gauss valuation for $T = (X_0 - y)/\lambda^{p/(q+1)}$ induces at the special fiber the cover $Z_1^p - Z_1 - \overline{T}^{1+q}$ whose genus is $g(C)$ and the corresponding model is defined over $K(y, f(y)^{1/p})$.

VI. Let y_i, y_j be two distinct roots of $L(Y)$. We claim that $v(y_i - y_j) = v(\lambda^{p/(q+1)})$.

One has $L'(Y) = qs_1(Y)^{q-1}s_1'(Y) - a_n(s_0(Y)^{q-1}s_q'(Y) + (q-1)s_0(Y)^{q-2}s_1(Y)s_q(Y))$. For y a root of $L(Y)$ we remark that $s_q'(y) = q + 1$ and $v(s_1(y)) > 0$ so we get $v(L'(y)) = v(a_n)$. Further the uniqueness of the model yielding good reduction implies that for all the roots y of $L(Y)$, $(X_0 - y)/\lambda^{p/(q+1)}$ defines the same Gauss valuation. It follows that if y_i, y_j are two such roots then $v(y_i - y_j) \geq v(\lambda^{p/(q+1)})$. Finally as $v(L'(y_i)) = v(a_n) = (q^2 - 1)v(\lambda^{p/(q+1)})$ it follows that we have equality $v(y_i - y_j) = v(\lambda^{p/(q+1)})$ for any choice of two distinct roots y_i, y_j of $L(Y)$.

VII. Assuming that $L(Y) \in K[Y]$ is irreducible, we will show that the extension M/K is the finite monodromy. Further, the extension is Galois with the extraspecial group of order pq^2 and exponent p if $p > 2$ and type II. b. if $p = 2$.

Denote by y_i, for $0 \leq i < q^2$, the q^2 roots of L. As $L(Y)$ is assumed to be irreducible over K, we can consider elements $\sigma_i \in \mathrm{Gal}(K^{alg}/K)$ for which $\sigma_i(y_0) = y_i$ for $0 < i < q^2$. Then $\sigma_i \neq \sigma_j$ for $i \neq j$. Consider the equation $Z_0^p = f(X_0)$ for $C \longrightarrow \mathbb{P}^1_K$. By [Le-Ma2] Proposition 4.4 we know that the change of coordinates

$$X_0 = \lambda^{p/(q+1)} X + y_0 \quad \text{and } Z_0/(s_0(y_0)^{1/p}) = \lambda Z_1 + H(X\lambda^{p/(q+1)}, y_0) \qquad (11)$$

induces the component of genus > 0 in the stably marked model. We now look at the action of σ_i on this component.

$$\sigma_i(X) = \frac{X_0 - y_i}{\lambda^{p/(q+1)}} \quad \text{hence} \quad X - \sigma_i(X) = \frac{y_i - y_0}{\lambda^{p/(q+1)}}.$$

We conclude that the σ_i act non-trivially on the special fiber and so $K(y_i, 0 \leq i < q^2) \subset K'$ where K' is the minimal extension of K such that C has a stably marked model.

By V. we know that $K' \subseteq M := K(y_0, s_0(y_0)^{1/p})$ and so we also have $K(y_i, 0 \leq i < q^2) \subseteq M$. In V. we have seen that the curve $C_{R'} \otimes_{R'} k$ is isomorphic to the curve $C_f : w^p - w = f(x)$ where $f(x) = x^{q+1}$. We denote by G_∞ the inertia group of C_f at infinity, and by $G_{\infty,1}$ its p-Sylow subgroup. The image of (3) fixes ∞ and is a p-group. Therefore we have

$$\mathrm{Gal}(K'/K) \hookrightarrow \mathrm{G}_{\infty,1} \subset \mathrm{Aut}_k(C_{R'} \otimes_{R'} k)^{\#}.$$

We will use a group theoretical argument in order to prove that the injection is actually surjective onto $G_{\infty,1}$ and hence $[K' : K] = pq^2$. In [Le-Ma1] we have shown that the group $G_{\infty,1}$ is an extension of $\mathbb{Z}/p\mathbb{Z} = \langle \tau \rangle$ (normal subgroup) by $(\mathbb{Z}/p\mathbb{Z})^{2n}$ where $\tau(x) = x$, and $\tau(w) = w + 1$. Further τ generates the center $Z(G_{\infty,1})$. The quotient $(\mathbb{Z}/p\mathbb{Z})^{2n}$ is the group of translations $x \to x + y$ which extend to elements of $G_{\infty,1}$. The above injection therefore induces a homomorphism

$$\mathrm{Gal}(K'/K) \hookrightarrow \mathrm{G}_{\infty,1}/\langle \tau \rangle$$

whose image contains the translations by the q^2 elements $\overline{(y_i - y_0)/\lambda^{p/(q+1)}}$. It follows that this map is surjective, and $\mathrm{Gal}(K'/K)$ is a subgroup of $G_{\infty,1}$ of index p or 1. Then it is a maximal subgroup (strict or not) of $G_{\infty,1}$ and so it contains the Frattini subgroup which in this case is the center. This implies the surjectivity of (3). As $[K' : K] = pq^2$ and $K(y_i, 0 \leq i < q^2) \subset K' \subset M = K(y_0, s_0(y_0)^{1/p})$ it follows that $K' = M$.

c) Let y be a root of $\mathcal{L}(Y)$ and $E = K(y, f(y)^{1/p})$. By b) we already know that we have potentially good reduction with $M = K'$. Hence by Theorem 2.11 E contains K' and $[E : K] \leq pq^2$. We conclude that $\mathcal{L}(Y)$ is irreducible and $E = M = K'$. Observe that then E also contains the splitting field of $\mathcal{L}(Y)$ and the construction is independent of the choice of the zero y of $\mathcal{L}(Y)$.

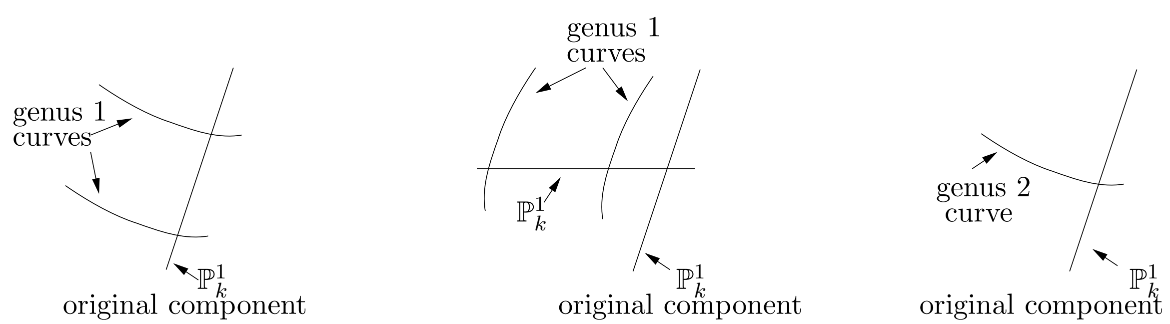
□

Remark 3.4. The main interest of Proposition 3.2 is that it works uniformly for all p. The proof is facilitated by the fact that all roots of $f(X_0)$ have multiplicity 1. This implies $n = m$ which is the smallest possible value for n for a given genus.

Nevertheless, we can show similar results as well in the case where there are zeros of $f(X_0)$ of multiplicity > 1: for example take $p = 3$, $K \subset \mathbb{Q}_p^{\text{tame}}$ and $f(X_0) = (1 + X_0 + X_0^3)(1 + X_0)^2$ (resp. $f(X_0) = (1 + X_0 + X_0^9)(1 + X_0)^2$). Then $\frac{f'(X_0)}{f(X_0)} = \frac{2X_0^3}{(1+X_0+X_0^3)(1+X_0)} \mod 3$ (resp. $\frac{f'(X_0)}{f(X_0)} = \frac{2X_0^9}{(1+X_0+X_0^9)(1+X_0)} \mod 3$) and, in the notation of Remark 2.10, it follows that $\mathcal{C} \otimes_R k$ has only one cusp and hence is a good candidate for having potentially good reduction. Note that $m = 4, n = 5$ and so $p^\alpha = 3$ (resp. $m = 10, n = 11$ and so $p^\alpha = 9$). In fact, following the proof of Proposition 3.2, one can show that both examples have potentially good reduction, and using Magma we check that the monodromy polynomials are irreducible over $\mathbb{Q}_3^{\text{tame}}$. Then the p-adic valuation of the discriminant of $L(Y)$ shows that the wild monodromy group is maximal, isomorphic to the extraspecial group of order 3^3 and exponent 3 (resp. of order 3^5 and exponent 3). Note that the two curves $Z^3 = 1 + X_0^3 + X_0^4$ (appearing in Prop.3.2) and $Z^3 = (1 + X_0 + X_0^3)(1 + X_0)^2$ both have genus 3, maximal wild monodromy over $\mathbb{Q}_3^{\text{tame}}$, and the same type of reduction. Yet, as covers of $\mathbb{P}_K^1$, these two curves have different branch cycle descriptions originating from the multiplicities in their defining equations. This suggests that we can refine the problem of realizing maximal wild monodromy groups over $\mathbb{Q}_p^{\text{tame}}$ and also prescribe the branch cycle description.

3.2 Genus 2-curves and wild monodromy

Now we consider the case of p-cyclic covers where $p = 2$ and $m = 5$, i.e. genus 2 curves over a 2-adic field in Q_2^{tame}. In this case, there are three possible types for the degeneration of the stably marked model (cf. figure 2). We assume that the cover $C \longrightarrow \mathbb{P}_K^1$ is given birationally by $Z_0^p = f(X_0)$ and can choose $f(X_0)$ of the form $f(X_0) = 1 + b_2X_0^2 + b_3X_0^3 + b_4X_0^4 + X_0^5 \in R[X_0]$. In the following we shall also write C_k for the special fiber of the stably marked model. In this case we obtain for the monodromy polynomial $\mathcal{L}(Y) = (s_1(Y)^2 - 4s_0(Y)s_2(Y))^2 - 64s_0(Y)^3s_4(Y)$, a polynomial of degree 16. The condition on the branch locus to have equidistant geometry translates to $\text{disc}(\bar{f}(X_0)) \neq 0$.



Type 1 $\text{Gal}(K'/K)_w \hookrightarrow Q_8 \times Q_8$ Type 2 $\text{Gal}(K'/K)_w \hookrightarrow (Q_8 \times Q_8) \rtimes \mathbb{Z}/2\mathbb{Z}$ Type 3 $\text{Gal}(K'/K)_w \hookrightarrow Q_8 * D_8$

Figure 2: The three types of degeneration

Consider the normal R-model $\mathcal{C}$ for C given by $Z_0^p = f(X_0)$. Its cusps are above the points with $\bar{f}'(X_0) = \bar{b}_3X_0^2 + X_0^4 = 0$. (See Remark 2.10 and figure 1). Therefore we have two cusps iff $\bar{b}_3 \neq 0$. This case we call type 1, and the stably marked reduction consists of the original component intersected by two elliptic curves E_1 and

E_2 which are isomorphic to the unique supersingular curve E in characteristic 2 given by $w^2 + w = x^3$. The p-Sylow of the group of automorphisms of E leaving ∞ fixed is Q_8 and $Q_8/Z(Q_8) = (\mathbb{Z}/2\mathbb{Z})^2$ is the group of induced translations $x \mapsto x + y$ for y a constant. We have $\mathrm{Syl}_p(\mathrm{Aut}_k(C_k)^{\#}) = Q_8 \times Q_8$.

Type 2 is the case where $\mathcal{C}$ has one cusp and the stably marked reduction C_k has two components of genus zero and two of genus one, isomorphic to E. In this case $\mathrm{Syl}_p(\mathrm{Aut}(C_k)^{\#}) = (Q_8 \times Q_8) \rtimes \mathbb{Z}/2\mathbb{Z}$ (here $\mathbb{Z}/2\mathbb{Z}$ acts on $Q_8 \times Q_8$ in exchanging the factors) because the crossing points with the rational component and the original component are left fixed.

Type 3 is the case of potentially good reduction with group $Q_8 * D_8$ and has been treated in more generality in section 3.1.

The following is a group theoretical lemma we shall need at various places.

Lemma 3.5. *i) Let $H \subseteq Q_8$ be a subgroup of index smaller or equal to 2 and assume that the natural projection $H \longrightarrow Q_8/Z(Q_8)$ is onto. Then $H = Q_8$.*
ii) Let $H \subseteq Q_8 \times Q_8$ be a subgroup of index smaller or equal to 2 and assume that the projection $H \longrightarrow Q_8/Z(Q_8) \times Q_8/Z(Q_8)$ is onto. Then $H = Q_8 \times Q_8$.
iii) Same statement for $(Q_8 \times Q_8) \rtimes \mathbb{Z}/2\mathbb{Z} \longrightarrow ((Q_8 \times Q_8) \rtimes \mathbb{Z}/2\mathbb{Z})/(Z(Q_8) \times Z(Q_8))$. (Here the action of $\mathbb{Z}/2\mathbb{Z}$ is the one exchanging the factors Q_8).

Proof. i) If the index is equal to 1, there is nothing to show. We hence assume that H has index 2. Then H is a maximal subgroup and therefore contains the Frattini subgroup $\Phi(Q_8)$ which is $Z(Q_8)$. This proves the claim.
ii) We point out that the Frattini subgroup $\Phi(Q_8 \times Q_8) = Z(Q_8) \times Z(Q_8)$ and the same proof as in i) works.
iii) By ii) H contains $Q_8 \times Q_8$. On the other hand it has to be strictly bigger. □

3.2.1 Degeneration of type 1

We show that the wild monodromy can be maximal in the case of degeneration of type 1.

Proposition 3.6. *Let $C \longrightarrow \mathbb{P}^1_K$ be given by the Kummer equation*

$$Z_0^2 = f(X_0) = 1 + b_2 X_0^2 + b_3 X_0^3 + b_4 X_0^4 + X_0^5$$

where $v(b_i) \geq 0$, $v(b_3) = 0$ and $K = \mathbb{Q}_2(\rho, b_2, b_3, b_4)$ with $\rho = 2^{2/3}$. We write u for a uniformizer of K and assume $1 + b_3 b_2 + b_3^2 b_4 \not\equiv 0 \bmod u$. Then C has stably marked reduction of type 1. Let $T(Y) = s_1(Y)^2 - 4 s_0(Y) s_2(Y)$. Then $T(Y)$ factorizes over K in $T(Y) = T_1(Y) T_2(Y)$ with $\bar{T}_1(Y) = Y^4$ and $\bar{T}_2(Y) = Y^4 + \bar{b}_3^2$. For $1 \leq i \leq 4$ denote by y_i the zeros of $T_1(Y)$ and for $5 \leq i \leq 8$ the zeros of $T_2(Y)$. If $K(y_1)$ and $K(y_5)$ are linearly disjoint over K then the finite monodromy extension is maximal, equal to the splitting field of $T(Y) \prod_{i=1}^{8} (Y^2 - y_i)$. Its Galois group is $Q_8 \times Q_8$.

Remark 3.7. The proof will show that the centers for the blowups to obtain the stably marked model are given by the zeros of $T(Y)$. It turns out that in this case $T(Y)$, which is of degree 8, is easier to handle than the polynomial $\mathcal{L}(Y)$, which has degree 16. Observe, though, that $T(Y)^2$ and $\mathcal{L}(Y)$ are congruent modulo 2^6.

We point out that the fact that here we can use the polynomial $T(Y)$ of degree strictly smaller than $\deg(\mathcal{L}(Y))$ is not in contradiction to what was said in [Le-Ma2] Remark 5.4. The reason for this is that in the present situation the type of degeneration has been fixed. This implies that the possible maximum size of the right hand side group in (3) is smaller than in the general situation, i.e. the situation where no supplementary assumptions on the special fiber are made. Hence following the same reasoning as in [Le-Ma2] Remark 5.4 one expects to be able to find polynomials that have smaller degree than $\mathcal{L}(Y)$ but can function in the same way in Theorem 2.11.

Proof. Choose $y \in \bar{K}$ with $T(y) = 0$. Then for $X_0 = X_1 + y$ we get

$$f(X_1 + y) = s_0(y) + s_1(y)X_1 + \cdots + X_1^5$$
$$= (s_0(y)^{1/2} + s_2(y)^{1/2}X_1)^2 + s_3(y)X_1^3 + s_4(y)X_1^4 + X_1^5$$

where $s_3(y) = b_3 + 4b_4y + 10y^2$, $s_4(y) = b_4 + 5y$ and we have chosen the roots $s_0(y)^{1/2}$ and $s_2(y)^{1/2}$ such that $2s_0(y)^{1/2}s_2(y)^{1/2} = s_1(y)$. We write $X = X_1/\rho$ and obtain

$$f(X_1 + y) = (s_0(y)^{1/2} + s_2(y)^{1/2}\rho X)^2 + s_3(y)\rho^3X^3 + s_4(y)\rho^4X^4 + \rho^5X^5.$$

Now define Z by $Z_0 = Z + s_0(y)^{1/2} + s_2(y)^{1/2}\rho X$. From $Z_0^2 = f(X_0)$ we get, after a small calculation,

$$Z^2 + 2(s_0(y)^{1/2} + s_2(y)^{1/2}\rho X)Z = s_3(y)\rho^3X^3 + s_4(y)\rho^4X^4 + \rho^5X^5.$$

With $W = Z/2$ we get

$$W^2 + (s_0(y)^{1/2} + s_2(y)^{1/2}\rho X)W = s_3(y)X^3 + s_4(y)\rho X^4 + \rho^2X^5. \tag{12}$$

This last equation defines a model over $K(y, s_0(y)^{1/2})$ with special fiber given by $W^2 + \bar{s}_0(y)^{1/2}W = \bar{b}_3X^3$. This equation has genus 1 and therefore also defines a component in the stable reduction. Now $\bar{T}_1(Y) = Y^4$ and using the uniqueness of the stable reduction we conclude that the four roots of $T_1(Y)$ induce the same ideal $(\rho, X_0 - y) \subset R^{\mathrm{alg}}[X_0]$. The corresponding statement for the roots of $\bar{T}_2(Y) = (\bar{b}_3 + Y^2)^2$ holds. Hence we conclude

$$v(y_i - y_j) \geq v(\rho) \quad \text{for } i \neq j \in \{1,2,3,4\} \text{ or } i \neq j \in \{5,6,7,8\} \tag{13}$$

This implies $\mathrm{disc}(T_i(Y)) \geq 12v(\rho) = 8v(2)$ for $i = 1, 2$ and we have equality iff (13) are all equalities. On the other hand, using Maple, $2^{-16}\mathrm{disc}(T(Y)) = b_3^8(1 + b_2b_3 + b_3^2b_4)^4 \mod 2$. Now observe that $\mathrm{disc}(T_1(Y)) + \mathrm{disc}(T_2(Y)) = \mathrm{disc}(T(Y))$ and hence $\mathrm{disc}(T_i(Y)) = 8v(2)$ and so (13) are all equalities. (This geometry is illustrated in figure 3). Therefore

$$0, \frac{y_2 - y_1}{\rho}, \frac{y_3 - y_1}{\rho}, \frac{y_4 - y_1}{\rho}$$

are all distinct modulo (u). Hensel's Lemma therefore shows that $K(y_1)/K$ is Galois, i.e. $T_1(Y)$ is totally split over $K(y_1)$. The same holds for $K(y_5)$ and $T_2(Y)$. By Proposition 2.4 a stably marked model is defined over

$$L := K(y_1, y_5, s_0(y_1)^{1/2}, s_0(y_2)^{1/2}, \ldots, s_0(y_8)^{1/2})$$

and we will show that L is equal to the finite monodromy extension. Let E_1 and E_2 be the two elliptic curves that are the components of genus > 0 in the stably marked model. Let i_1, i_2 be their hyperelliptic involutions and ∞_1, ∞_2 be the crossing points of E_1 and E_2 with the original component. We have the following diagram where j is given by the action on the special fiber of the stably marked model defined over L.

$$\begin{array}{cccc} \mathrm{Gal}(L/K) & \stackrel{j}{\longrightarrow} & \mathrm{Syl}_2(\mathrm{Aut}(E_1)_\infty) \times \mathrm{Syl}_2(\mathrm{Aut}(E_2)_\infty) & \cong & Q_8 \times Q_8 \\ & & \downarrow \mathrm{pr} & & \\ & & \mathrm{Syl}_2(\mathrm{Aut}(E_1)_\infty)/\langle i_1 \rangle \times \mathrm{Syl}_2(\mathrm{Aut}(E_2)_\infty)/\langle i_2 \rangle & \cong & (\mathbb{Z}/2\mathbb{Z})^2 \times (\mathbb{Z}/2\mathbb{Z})^2 \end{array}$$

Here the index ∞ means that we consider only automorphisms that leave the infinite points ∞_1 and ∞_2 fixed. We claim that $\mathrm{pr} \circ j$ is onto. By assumption $K(y_1)$ and $K(y_5)$ are linearly disjoint. Hence there exist $\sigma_{i,j} \in \mathrm{Gal}(L/K)$ for $i \in \{1,2,3,4\}$ and $j \in \{5,6,7,8\}$ such that

$$\sigma_{i,j}(y_1) = y_i \quad \text{and } \sigma_{i,j}(y_5) = y_j.$$

Now $\mathrm{pr} \circ j$ operates on $E_1/\langle i_1 \rangle \times E_2/\langle i_2 \rangle$ in the following way.

$$\begin{aligned} \mathrm{pr} \circ j(\sigma_{i,j}) \left(\frac{X_0 - y_1}{\rho}\right)^- &= \left(\frac{X_0 - y_1}{\rho}\right)^- + \left(\frac{y_1 - y_i}{\rho}\right)^- \\ \mathrm{pr} \circ j(\sigma_{i,j}) \left(\frac{X_0 - y_5}{\rho}\right)^- &= \left(\frac{X_0 - y_5}{\rho}\right)^- + \left(\frac{y_5 - y_j}{\rho}\right)^- . \end{aligned}$$

Notice that $\left(\frac{X_0 - y_1}{\rho}\right)^-$ and $\left(\frac{X_0 - y_5}{\rho}\right)^-$ are local coordinates on $E_1/\langle i_1 \rangle$ and $E_2/\langle i_2 \rangle$ and that the action is non trivial. This shows the claim and in particular we have $2^4|\mathrm{Im}(j)$. Now consider the obvious maps

$$\mathrm{Syl}_2(\mathrm{Aut}(E_1)_\infty) \times \mathrm{Syl}_2(\mathrm{Aut}(E_2)_\infty) \stackrel{\mathrm{pr}_1}{\longrightarrow} \mathrm{Syl}_2(\mathrm{Aut}(E_1)_\infty) \longrightarrow \mathrm{Syl}_2(\mathrm{Aut}(E_1)_\infty)/\langle i_1 \rangle.$$

Their composition is surjective by what was said about the $\sigma_{i,j}$ above. Now applying Lemma 3.5 i) to the second arrow shows that pr_1 is onto. We conclude that $2^5|\mathrm{Im}(j)$. Further Lemma 3.5 ii) applied to pr implies that j is onto. This shows that the curve C has maximal monodromy.

We proceed to show that j is an isomorphism. Let $\sigma \in \ker(j)$. By what was said above about the $\sigma_{i,j}$ we know that $\sigma(y_i) = y_i$ for all $i \in \{1, \dots 8\}$. If $\sigma(s_0(y_i)^{1/2}) = -s_0(y_i)^{1/2}$ for some i, then from the equation of the model (12), we get

$$\sigma(W) = W + s_0(y_i)^{1/2} + s_2(y_i)^{1/2} \rho \frac{X_0 - y_i}{\rho}.$$

Note that the action on $s_2(y_i)^{1/2}$ is known via the above equality $2s_0(y)^{1/2}s_2(y)^{1/2} = s_1(y)$ which now holds for $y = y_i$. In reduction this yields

$$\sigma(\bar{W}) = \bar{W} + \overline{s_0(y_i)^{1/2}} \neq \bar{W}.$$

Hence such a σ would induce a non-trivial action on the special fiber of the stably marked model. We conclude that σ acts trivially on all $s_0(y_i)^{1/2}$, hence $\sigma = \mathrm{id}$ and so j is injective. □

We next give an example where the conditions of Proposition 3.6 hold.

Example 3.8. Let $f(X_0) = 1 + 2^{3/5}X_0^2 + X_0^3 + 2^{2/5}X_0^4 + X_0^5$. One checks, using Magma, that $T(Y)$ decomposes into two irreducible factors $T_1(Y), T_2(Y)$ of degree 4 over $K = \mathbb{Q}_2(2^{1/15})$. Then one checks that $T_2(Y)$ is irreducible over the splitting field of $T_1(Y)$. Hence we satisfy the linear disjointness condition of the proposition. Below we have included the Magma and Maple programs used in this example:

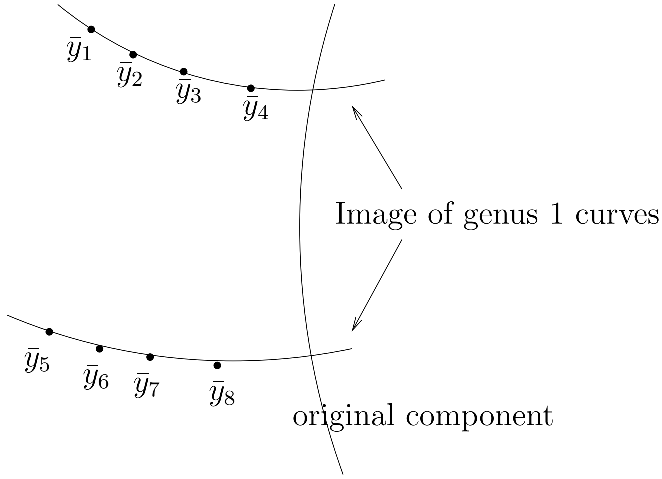


Figure 3: The geometry of the zeros of $T(Y)$ in Proposition 3.6

```
q2 := pAdicField(2,32);
q2x<x> := PolynomialAlgebra(q2);
k<pi> := TotallyRamifiedExtension(q2,x^15-2);
K<rho> := UnramifiedExtension(k,8);
Ky<y> :=  PolynomialAlgebra(K);
b2:=pi^9;
b4:=pi^6;
T:= 15*y^8+24*b4*y^7-(-22-8*b4^2)*y^6-(-24*b2-12*b4)*y^5
-(-12*b2*b4-3)*y^4-(-40-4*b2)*y^3+24*b4*y^2+12*y+4*b2;
F,s,C := Factorization(T:Certificates:= true, Extensions:= true);
L1 := C[1]`Extension;
DefiningPolynomial(L1);
L2 := C[2]`Extension;
DefiningPolynomial(L2);
L1 := C[1]`Extension;
> DefiningPolynomial(L1);
y^4 + ... + pi*(-7946432*pi^12 - 2670580*pi^9 + 5092944*pi^6
    + 7945696*pi^3 + 14027211) + O(pi^480)
> L2 := C[2]`Extension;
> DefiningPolynomial(L2);
y^4 + ... + pi*(-7766643*pi^12 - 13742317*pi^9 + 12031385*pi^6
    - 2800087*pi^3 - 1812931) + O(pi^480)
```

The two degree 4 factors obtained from the above program are Eisenstein polynomials for the extensions defined by the polynomials $T_1(Y)$ and $T_2(Y)$. We calculate the resultant $\mathrm{res}(Y)$ of the Eisenstein polynomial (see the polynomial $L1(Y)$ above) for the extension defined by $T_1(Y)$ and $\mathrm{pi}^{15}-2$ using Maple. This resultant gives an equation over $\mathbb{Q}_2$ for the compositum of the extensions defined by $T_1(Y)$ and $\mathrm{pi}^{15}-2$. This polynomial has too big coefficients to be worked with in Magma and so we use a theorem of Krasner (cf. [Pa-Ro] Theorem 5.2) in order to get an Eisenstein polynomial with smaller coefficients and defining the same extension. We first calculate the 2-adic valuation of the discriminant of $\mathrm{res}(Y)$ using Maple. In the notation of [Pa-Ro] we get $v_2(\mathrm{res}(Y)) = 62 = n + j - 1$ where n is the degree of the polynomial $\mathrm{res}(Y)$. This gives $j = 3$. Let c be the smallest integer such that $c > (n + 2j)/n = 66/60$, i.e.

$c = 2$. Then the polynomial $\mathrm{eis}(Y) := \mathrm{res}(Y) \bmod 2^c$ will define the same extension. Here we get:

```
eis:=x^60+2*x^45+2*x^27+2*x^24+2*x^21+2*x^12+2*x^9+2*x^3+2
```

Next we need an expression for $a := 2^{1/15}$ in the splitting field of the resultant.

```
q2 := pAdicField(2,32);
q2x<x> := PolynomialAlgebra(q2);
k<pi> := TotallyRamifiedExtension(q2,eis);
K<rho> := UnramifiedExtension(k,8);
Ky<y> :=  PolynomialAlgebra(K);
P:=y^15-2;
FP:=Factorization(P);
P=(y-a)...
a:=-( pi^4*(953699913*pi^57 - 862782732*pi^54 + 920312767*pi^51 - ...
```

The following program shows that $T_2(Y)$ is irreducible over the splitting field of $T_1(Y)$. Hence we satisfy the linear disjointness condition of the proposition.

```
q2 := pAdicField(2,32);
q2x<x> := PolynomialAlgebra(q2);
k<pi> := TotallyRamifiedExtension(q2,eis);
K<rho> := UnramifiedExtension(k,8);
Ky<y> :=  PolynomialAlgebra(K);
a:=-( pi^4*(953699913*pi^57 - 862782732*pi^54 + 920312767*pi^51 - ...
b2:=a^9;
b4:=a^6;
T:= 15*y^8+24*b4*y^7-(-22-8*b4^2)*y^6-(-24*b2-12*b4)*y^5
-(-12*b2*b4-3)*y^4-(-40-4*b2)*y^3+24*b4*y^2+12*y+4*b2;
F,s,C := Factorization(T:Certificates:= true, Extensions:= true);
L1 := C[1]`Extension;
DefiningPolynomial(L1);
L2 := C[2]`Extension;
DefiningPolynomial(L2);
L5 := C[5]`Extension;
DefiningPolynomial(L5);
y^4 + (pi^13*(994560434*pi^57 - 852077331*pi^54 + 792865022*pi^51 + ...
```

As a corollary we get the following result of Inverse Galois Theory:

Corollary 3.9. *Let $L/\mathbb{Q}_2^{\mathrm{tame}}$ be the splitting field of*

$$T(Y)\prod_{i=1}^{8}(Y^2 - f(y_i))$$

where $T(Y) = 15Y^8 + 24b_4Y^7 - (-22-8b_4^2)Y^6 - (-24b_2-12b_4)Y^5 - (-12b_2b_4-3)Y^4 - (-40-4b_2)Y^3 + 24b_4Y^2 + 12Y + 4b_2$, $b_2 = 2^{3/5}$, $b_4 = 2^{2/5}$, *the* y_i *are the zeros of* $T(Y)$ *and* $f(X_0) = 1 + 2^{3/5}X_0^2 + X_0^3 + 2^{2/5}X_0^4 + X_0^5$. *Then this is a Galois extension with group* $Q_8 \times Q_8$.

3.2.2 Degeneration of type 2

This case is more subtle and so we only give an example of a cover that has maximal finite monodromy.

Proposition 3.10. *Let* $K = \mathbb{Q}_2(a)$ *with* $a^9 = 2$. *Consider the cover* $C \longrightarrow \mathbb{P}^1_K$ *given by*

$$Z_0^2 = f(X_0) = 1 + a^3X_0^2 + a^6X_0^3 + X_0^5.$$

This cover has stably marked reduction of type 2 and maximal finite monodromy. Let $T(Y) = s_1(Y)^2 - 4s_0(Y)s_2(Y)$ *and denote by* y_i, $0 \leq i \leq 8$ *its zeros in some algebraic closure of* K. *Then the finite monodromy is the splitting field* M *of*

$$T(Y)\prod_{i=1}^{8}(Y^2 - f_0(y_i)).$$

Its Galois group is isomorphic to $(Q_8 \times Q_8) \rtimes \mathbb{Z}/2\mathbb{Z}$.

Proof. We first show that the model is of type 2. Consider

$$f(X + Y) = s_0(Y) + s_1(Y)X + \cdots + X^5.$$

Then $s_3(Y) = a^6 + 10y^2, s_4(y) = 5y$ and $T(Y) = 15Y^8 + 22a^6Y^6 + 24a^3Y^5 + 6a^3Y^4 + 48Y^3 + 12a^6Y + 4a^3$. In this case the valuation of $\mathrm{disc}(T(Y))$ has not enough information to determine the relative geometry of the zeros of $T(Y)$. We therefore proceed as follows. A look at the Newton polygon of $T(Y)$ shows that the 8 roots all have valuation equal to $(1/8)v(4a^3) = (7/24)v(2)$. Next we consider the Newton polygon of $S(Z) = (T(Y + Z) - T(Y))/Z$ with $Y = y$ a zero of $T(Y)$. One obtains that $S(Z)$ has 3 roots with valuation $v(\rho) = (4/9)v(2)$ and 4 roots with valuation $v(2)/3$. Figure 4 shows the geometry of the zeros of $T(Y)$.

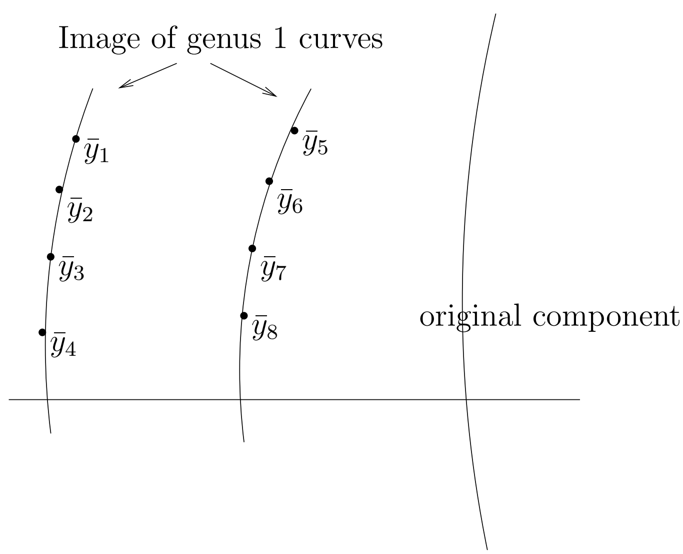


Figure 4: The geometry of the zeros of $T(Y)$ in Proposition 3.10

Still for y a zero of $T(Y)$ and $X_0 = X_1 + y$ we have

$$f(X_1 + y) = (s_0(y)^{1/2} + s_2(y)^{1/2}X_1)^2 + s_3(y)X_1^3 + s_4(y)^4X_1^4 + X_1^5$$

where $v(s_3(y)\rho^3) = v(4)$, $v(s_4(y)\rho^4) > v(4)$ and $v(\rho^5) > v(4)$. As in the case of degeneration of type 1 we set $Z_0 = 2Z + s_0(y)^{1/2} + s_2(y)^{1/2}X_1$ and $X = X_1/\rho$. With these coordinates the model has the equation:

$$Z^2 + 2(s_0(y)^{1/2} + s_2(y)^{1/2}\rho X)Z = s_3(y)\rho^3 X^3 + s_4(y)^4\rho^4 X^4 + \rho^5 X^5.$$

With $W = Z/2$ we get

$$W^2 + (s_0(y)^{1/2} + s_2(y)^{1/2}\rho X)W = \frac{\rho^3}{4}s_3(y)X^3 + \frac{\rho^4}{4}s_4(y)^4 X^4 + \frac{\rho^5}{4}X^5. \qquad (14)$$

In reduction we obtain a curve of genus 1 and hence the curve C has stably marked reduction of type 2.

We proceed to compute its finite monodromy using Magma. We first check that $T(Y)$ is irreducible over K, then we show that if y_1 is a zero of $T(Y)$ then $T(Y)$ has the following decomposition into irreducible factors over $K(y_1)$.

$$T(Y) = \prod_{i=1}^{4}(Y - y_i)T_2(Y) \qquad (15)$$

From (14) and Proposition 2.4 we know that a stably marked model is defined over M and we proceed to show that M is the finite monodromy. As in the case of the degeneration of type 1 we have

$$\mathrm{Gal}(M/K) \xrightarrow{j} \mathrm{Syl}_2(\mathrm{Aut}(C_k)^{\#}) \cong (\mathrm{Syl}_2(\mathrm{Aut}(E_1)_\infty) \times \mathrm{Syl}_2(\mathrm{Aut}(E_2)_\infty)) \rtimes \mathbb{Z}/2\mathbb{Z}.$$

Here E_1 and E_2 are the two elliptic curves, i_1 and i_2 their hyperelliptic involutions, and $\mathrm{Aut}(C_k)^{\#}$ are the automorphisms that leave the initial component fixed. The index ∞ means that the crossing point of E_i, with the rational component that is not the original component in the stably marked reduction, is left fixed.

Now $\mathrm{Syl}_2(\mathrm{Aut}(E_i)_\infty) \cong Q_8$ and we have a projection map pr obtained by modding out by the subgroup generated by the involutions i_1 and i_2.

$$\begin{array}{ccc} \mathrm{Gal}(M/K) & \xrightarrow{j} & (Q_8 \times Q_8) \rtimes \mathbb{Z}/2\mathbb{Z} \\ & & \downarrow \mathrm{pr} \\ & & ((\mathbb{Z}/2\mathbb{Z})^2 \times (\mathbb{Z}/2\mathbb{Z})^2) \rtimes \mathbb{Z}/2\mathbb{Z} \end{array}$$

From (15) it follows that there exist $\sigma_{i,j} \in \mathrm{Gal}(M/K)$ for $i \in \{1,2,3,4\}$ and $j \in \{5,6,7,8\}$ such that $\sigma_{i,j}(y_1) = y_i$ and $\sigma_{i,j}(y_5) = y_j$. Further there is $\tau \in \mathrm{Gal}(M/K)$ with $\tau(y_1) = y_5$. They act on the special fiber via:

$$\begin{aligned} j(\sigma_{i,j})\left(\frac{X_0 - y_1}{\rho}\right)^- &= \left(\frac{X_0 - y_1}{\rho}\right)^- + \left(\frac{y_1 - y_i}{\rho}\right)^- \\ j(\sigma_{i,j})\left(\frac{X_0 - y_5}{\rho}\right)^- &= \left(\frac{X_0 - y_5}{\rho}\right)^- + \left(\frac{y_5 - y_j}{\rho}\right)^- \\ j(\tau)\left(\frac{X_0 - y_1}{\rho}\right)^- &= \left(\frac{X_0 - y_1}{\rho}\right)^- + \left(\frac{y_1 - y_5}{\rho}\right)^- \end{aligned}$$

This shows that $\mathrm{pr} \circ j$ is onto and hence, by Lemma 3.5 iii), j is onto. To obtain the injectivity of j one follows the same route as in the case of degeneration of type 1.

□

As before, we have a corollary in Inverse Galois Theory.

Corollary 3.11. *Let $L/\mathbb{Q}_2^{\text{tame}}$ be the splitting field of*

$$T(Y)\prod_{i=1}^{8}(Y^2 - f(y_i))$$

where $T(Y) = 15Y^8 + 22a^6Y^6 + 24a^3Y^5 + 6a^3Y^4 + 48Y^3 + 12a^6Y + 4a^3$, $a^9 = 2$, the y_i are the zeros of $T(Y)$ and $f(X_0) = 1 + a^3X_0^2 + a^6X_0^3 + X_0^5$. Then this is a Galois extension with group $(Q_8 \times Q_8) \rtimes \mathbb{Z}/2\mathbb{Z}$.

3.2.3 Degeneration of type 3

This is the case of potentially good reduction and, as said above, we have treated it in some generality already in section 3.1. Here we add an example that has not been included earlier.

Example 3.12. Let $C \longrightarrow \mathbb{P}^1_K$ be given by the Kummer equation $Z_0^2 = f(X_0) = 1 + X_0^4 + X_0^5$ with $K = \mathbb{Q}_2^{\text{tame}}$ (i.e. $c = 1$ in Proposition 3.2). We claim that this cover has maximal wild monodromy with group $Q_8 * D_8$.

Let $a_n = (-1)^q(-p)^{p+p^2+\cdots+p^n} = 64$. Considering Proposition 3.2 it is sufficient to prove that the modified monodromy polynomial $L(Y) = s_1(Y)^q - a_n s_0(Y)^{q-1}s_q(Y) = s_1(Y)^4 - 64s_0(Y)^3s_4(Y)$ is irreducible over K. Note that the monodromy polynomial is $\mathcal{L}(Y) = (s_1(Y)^2 - 4s_0(Y)s_2(Y))^2 - 64s_0(Y)^3s_4(Y)$. We can check the irreducibility of $L(Y)$ using Magma. Let y be a root of $L(Y)$. Then C has good reduction over $M := K(y, f(y)^{1/2})$ and M/K is Galois with group $Q_8 * D_8$.

3.3 Related work and perspective

1. In [Si-Za1] and [Si-Za2] Silverberg and Zarhin study the finite monodromy groups for abelian surfaces. In particular, they classify the finite groups which can occur as finite monodromy groups for abelian surfaces. Their examples are mostly in equal characteristic, and they ask at the end of loc. cit. section 1 for examples in mixed characteristic. Our section 3 provides such examples, if one considers the Jacobians associated to the curves C. Note that realizing wild monodromy groups over $\mathbb{Q}_p^{\text{tame}}$ is a strong condition because $\mathbb{Q}_p^{\text{tame}}$ has trivial absolute wild ramification and this has no counterpart in the equal characteristic case. In particular, the fact that in equal characteristic a group is realized as soon as a bigger one is ([Si-Za2] Remark 1.9.) has no analog over $\mathbb{Q}_p^{\text{tame}}$.

2. Motivated by [Si-Za1] and [Si-Za2] , one can look at generalizations of type 2 degeneration (cf. section 3.2.2). Namely, we have the following:

Let C/k be a stable curve of genus $g \geq 2$, over an algebraically closed field of characteristic $p > 0$. Then $\text{Pic}^0(C)$ is an extension of an abelian variety by a torus. The group $\text{Aut}_k(C)$ is finite and injects into $\text{Aut}_k(\text{Pic}^0(C))$ (cf. [De-Mu] Thm.1.13). It can be described by considering its action on the torsion points of $\text{Pic}^0(C)$. Assume that the dual graph of C is a tree or, equivalently, that $\text{Pic}^0(C)$ is an abelian variety (this is the case for the special fibers of the stable models of the p-cyclic covers which appear in theorem 2.11). Then we can give a bound for a p-Sylow subgroup $\text{Syl}_p(C)$ of $\text{Aut}_k(C)$. Define $N_p(g) := a + [a/p] + [a/p^2] + \ldots$ where $a := [\frac{2g}{p-1}]$. We claim that

$$v_p(|\text{Syl}_p(C)|) \leq N_p(g) \tag{16}$$

and for $g \in p^{\mathbb{N}}(p-1)/2$ this bound is optimal. Note that these bounds are bad when C/k is smooth (compare with [Le-Ma1] Thm.1.1.II.(f)). Yet they can be sharp in the case where C is not assumed to be irreducible, as we now show.

Let $\ell > 2$ be a prime distinct from p and $[\ell]$ the multiplication by ℓ in $\mathrm{Pic}^0(C)$. Then, by [Se], $\mathrm{Aut}_k(C) \hookrightarrow \ker[\ell] \simeq \mathrm{GL}_{2g}(\mathbb{F}_\ell)$. The p-adic valuation $v_p(|\mathrm{GL}_{2g}(\mathbb{F}_\ell)|) = v_p(\prod_{1\leq i\leq 2g}(\ell^i - 1))$ and, varying ℓ, the minimal value for $p > 2$ is $N_p(g)$ (exercise using [We] §2). Hence we obtain (16) for $p > 2$.

For $p = 2$, the minimal value is

$$5g - t(2g) = g + N_2(g) > N_2(g) \tag{17}$$

where $t(2g)$ is the number of digits in the 2-adic expansion of $2g$ (see [Ca-Fo]).

Let us first assume that $p > 2$ and $k = \mathbb{F}_p^{\mathrm{alg}}$. We consider the marked curve (C_1, ∞_1), birational to the curve $y^p - y = x^2$, and ∞_1 is the point $x = \infty$. The curve has genus $(p-1)/2$. Let $\mathrm{Syl}_p(C_1) \subset \mathrm{Aut}_{\mathbb{F}_p}(C_1)$ be a (the) p-Sylow subgroup. Then $\mathrm{Syl}_p(C_1) = \mathbb{Z}/p\mathbb{Z}$ and it fixes ∞_1.

Now, for $j \geq 1$, we build inductively a semi-stable curve C_j (with a marked point ∞_j). The curve C_{j+1} is obtained from the projective line $\mathbb{P}^1_{\mathbb{F}_p}$ by choosing one rational point over $\mathbb{F}_p$, say ∞_{j+1} and patching transversally to each of the p other $\mathbb{F}_p$-rational points a copy of C_j at the point ∞_j. The genus is $g(C_j) = p^{j-1}(p-1)/2$. Let $\mathrm{Syl}_p(C_j) \subset \mathrm{Aut}_{\mathbb{F}_p}(C_j)$ be the p-Sylow subgroup fixing ∞_j. Then $\mathrm{Syl}_p(C_{j+1}) = \mathrm{Syl}_p(C_j) \wr \mathbb{Z}/p\mathbb{Z}$, the wreath product (i.e. the semidirect product of p copies of $\mathrm{Syl}_p(C_j)$ and $\mathbb{Z}/p\mathbb{Z}$ where this last group acts cyclically on the components). We have $v_p(|\mathrm{Syl}_p(C_{j+1})|) = pv_p(|\mathrm{Syl}_p(C_j)|) + 1$ and so $v_p(|\mathrm{Syl}_p(C_j)|) = 1 + p + p^2 + ... + p^{j-1}$. This gives equality in (16) for $p > 2$. Moreover, we remark that in this case $\mathrm{Syl}_p(C_j)$ is isomorphic to a p-Sylow subgroup of the symmetric group S_{p^j} of degree p^j.

If $p = 2$, (17) gives $v_2(|\mathrm{Syl}_2(C_j)|) \geq N_2(1) + 1 = 4$ (resp. $N_2(2) + 2 = 9$) and it is an exercise to see that the sharp bound for genus 1 (resp. 2) is 3 (resp. 7) as is stated in (16). In order to prove (16) for $p = 2$, we need to invoke more structure. We remark that one can endow the Tate-module $T_\ell(\mathrm{Pic}^0(C)) = \varprojlim \mathrm{Pic}^0(C)(\ell^n) = \mathbb{Z}_\ell^{2g}$ with the Weil pairing $e_\ell : T_\ell(\mathrm{Pic}^0(C)) \times T_\ell(\mathrm{Pic}^0(C)) \to \mathbb{Z}_\ell(1)$ (see [Mi] §16).

Here the polarization is the product of the canonical polarizations of the jacobians of the irreducible components of genus > 0 of C. It is skew symmetric and the automorphisms in $\mathrm{Aut}_k(C)$ respect this polarization. For $(\ell, |G|) = 1$ and $\ell > 2$ in the above setting we can then replace the linear group $\mathrm{GL}_{2g}(\mathbb{F}_\ell)$ by the symplectic group $\mathrm{Sp}_{2g}(\mathbb{F}_\ell)$.

This time we need to minimize $v_p(|\mathrm{Sp}_{2g}(\mathbb{F}_\ell)|) = v_p((\ell^2-1)(\ell^4-1)...(\ell^{2i}-1)...(\ell^{2g}-1))$ by varying ℓ. It is an exercise to see that this minimum is $N_p(g)$ for all p (when $p > 2$ see [We] §3 and for $p = 2$ this follows from [Ca-Fo]).

In order to show that for $p = 2$ this is sharp, in the above construction, we replace the curve $y^p - y = x^2$ (whose genus is 0 when $p = 2$) by the curve $C_2 :\ y^2 - y = x^3$ whose genus is 1. (Note that there is a shift of the indices.) We proceed in the same way for the patching. The curve C_j will have genus $g = 2^{j-2} = 2^{j-1}(2-1)/2$. The Sylow subgroup $\mathrm{Syl}_2(C_j)$ is isomorphic to the wreath product $\mathrm{Syl}_2(C_j) \wr \mathbb{Z}/2\mathbb{Z}$. It follows inductively that $v_2(\mathrm{Syl}_2(C_j)| = 2^j - 1 = N_2(g)$. Note that this time $\mathrm{Syl}_2(C_j)$ is not isomorphic to a 2-Sylow subgroup of the symmetric group S_{2^j} of degree 2^j although it has the same cardinal.

We finish this paper with a question. The curves C_j (for all p) look like the special fibers of p-cyclic covers of the projective line (see Thm. 2.11); moreover by [Mau] Cor.

5.6. of Th. 5.4 we know that these curves lift as a p-cyclic cover of the projective line over some finite extension of $\mathbb{Q}_p^{\mathrm{tame}}$. Hence one can ask if they have a lift to a p-cyclic cover of the projective line over $\mathbb{Q}_p^{\mathrm{tame}}$ with maximal monodromy $\mathrm{Syl}_p(\mathrm{Aut}_k(C_j))$? (The case $p = 2$ and genus 2 corresponds to type 2 above).

Thanks. We would like to thank Qing Liu for illuminating discussions, Michel Raynaud for support concerning section 3.3, and Sebastian Pauli for his unfailing help in using the Magma package on factorization over p-adic fields. Part of this paper was written while the authors were guests of the Max-Planck-Institut in Bonn.

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