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# An ab-initio evaluation of the local effective interactions in the $\mathrm{Na}_{0.5} \mathrm{CoO}_{2}$ 

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#### Abstract

We used quantum chemical ab initio methods to determine the effective parameters of Hubbard and $t-J$ models for the $\mathrm{Na}_{0.5} \mathrm{CoO}_{2}$ compound. As for the superconducting compound we found the $a_{1 g}$ cobalt orbitals $260 \mathrm{meV}(\mathrm{Co}(1))$ and $320 \mathrm{meV}(\mathrm{Co}(2))$ above the $e_{g}^{\prime}$ ones. These orbital order is explained by the $e_{g}^{\prime}-e_{g}$ hybridization of the cobalt $3 d$ orbitals. The effective exchange integrals are found strongly reduced compared to the superconducting system with values ranging from -11 to -27 meV according to the bonds. In fact, surprisingly, the magnetic exchanges in the $\mathrm{CoO}_{2}$ layers, are found strongly dependant of the weak local structural distortions.


## I. INTRODUCTION

Since the discovery of the large thermoelectric power and then of superconductivity in the $\mathrm{Na}_{\mathrm{x}} \mathrm{CoO}_{2}$ layered cobaltate family, these systems have attracted a lot of attention. Indeed, they present a very rich phases diagram as a function of the sodium content $x$ and the possible water intercalation. For small $x(0<x<0.5$ ) the system is a paramagnetic metal, while for larger $x$ $(0.5<x<0.75)$ it is a Curie-Weiss metal. Over 0.75 , the system can be seen as a weakly ferromagnetic metall however neutron diffraction measurements $A$ rather sees A-type antiferromagnetism, that is ferromagnetic $\mathrm{CoO}_{2}$ layers, antiferromagnetically coupled. Fpr special values of the sodium content, $x=1 / 4, x=1 / 26$ and $x=1$, the system is insulating. A band insulator for $x=1$ and a charge/spin ordered state for $x=0.5$.

The $\mathrm{Na}_{\mathrm{x}} \mathrm{CoO}_{2}$ compounds are composed of $\mathrm{CoO}_{2}$ layers in the $(\vec{a}, \vec{b})$ plane, between which the sodium ions are located. The $\mathrm{CoO}_{2}$ layers are formed by $\mathrm{CoO}_{6}$ edgesharing octahedra forming a two-dimensional triangular lattice of cobalt ions. The $\mathrm{CoO}_{6}$ octahedra present a trigonal distortion along the crystallographic $\vec{c}$ axis corresponding to a compression of the oxygen planes toward the cobalt one. Additional distortions take place, according to the sodium ordering. In the $\mathrm{Na}_{0.5} \mathrm{CoO}_{2}$ compound (P2 structure), subject of the present work, the sodiums ordering is commensurate and presents two types of alternated stripes along the $\vec{b}$ direction ${ }^{6}$. In the first stripes, the sodium atoms are located on-top of (below) the cobalts atoms, and in the second type of stripes the sodium ions are located on-top of (below) the center of the cobalt triangles (see figure 11). It results two types of cobalt atoms organized in stripes along the $\vec{b}$ direction, namely the $\mathrm{Co}(1)$ atoms (located on top of a sodium ion) and the $\mathrm{Co}(2)$ ones. Compared to the superconducting compound $\mathrm{Na}_{0.3} \mathrm{CoO}_{2}-1.3 \mathrm{H}_{2} \mathrm{O}$, the present system exhibit a enlarged $\mathrm{CoO}_{2}$ layer. Indeed, the average $\mathrm{Co}-\mathrm{O}$ distances are $1.887 \AA$ for $\mathrm{Co}(1)$ and $1.900 \AA$ for $\mathrm{Co}(2)$ to be compared to the $1.855 \AA$ in the superconductor. Similarly, the average trigonal distortions of the $\mathrm{CoO}_{6}$ octahedra are weaker, with 58.9 degree between the $\vec{c}$ axis and the $\mathrm{Co}(1)-\mathrm{O}$ directions, and 59.4 degrees $(\mathrm{Co}(2))$, to be compared to 61.5 degrees for $\mathrm{Na}_{0.3} \mathrm{CoO}_{2}-1.3 \mathrm{H}_{2} \mathrm{O}$.


FIG. 1: Schematic structure of the $\mathrm{Na}_{0.5} \mathrm{CoO}_{2}$ compound. The black lines connect the $\mathrm{Na}^{+}$ions with their nearest cobalt neighbors.

One can thus expect a lowering of the cobalt $3 d$ orbital splitting due to the crystalline field.

Out of all the $\mathrm{Na}_{\mathrm{x}} \mathrm{CoO}_{2}$ systems, the $x=0.5$ compound presents a very rich but not well understood phase diagram. Indeed, the $\mathrm{Na}^{+}$ions order at very large temperature, namely slightly above 300 KJ , inducing a small charge order in the cobalt layer. Despite this small charge order the system remains conducting. At 88 K a magnetic phase transition occurs, associated with a structural distortion, and the onset of a long range magnetic order. Again, despite this transition, the system remains conducting up tgr 53 K where a small charge gap ( 14 meV ) finally opens 5 .

The understanding of this complex phase diagram is still under process. Different hypotheses have been advanced such as successive Fermi surface nestinge, polarons formation 10 , one band versus three bands crossover ${ }^{11}$.

In order to understand the low energy physics of this compound it is thus necessary to have accurate determination of the pertinent degrees of freedom and of their coupling, as a function of the exact crystallographic structure. Indeed, all authors agree on the importance of the sodium ordering and the induced cobalt local envi-
ronment distortions, on the low energy properties. The aim of this paper is to provide such informations using state of the art quantum chemical spectroscopy methods. The next section will shortly recall the methods. Section III will presents our results and finally the last section will sum up our conclusions.

## II. METHOD AND COMPUTATIONAL DETAILS

The method used in this work (CAS + DDCI ${ }^{12}$ ) is a configurations interaction method, that is an exact diagonalization method within a selected set of Slater determinants, on embedded crystal fragments. This method has been specifically designed to accurately treat strongly correlated systems, for which there is no singledeterminant description. The main point is to treat exactly all correlation effects and exchange effects within a selected set of orbitals (here the $3 d$ shell of the cobalt atoms) as well as the excitations responsible for the screening effects on the exchange, repulsion, hopping, etc. integrals.

The CAS+DDCI method has proved very efficient to compute, within experimental accuracy, the local interactions (orbital energies, atomic excitations, exchange and transfer integrals, coulomb repulsion etc.) of a large family of stropoly correlated systems such as high $T_{c}$ copper oxidest3, vanadium oxides ${ }^{14}$, nickel and cuprate fluorides ${ }^{15}$, spin chains and ladders ${ }^{16}$, etc.

The clusters used in this work involve one and two cobalt atoms $\left(\mathrm{CoO}_{6}\right.$ and $\left.\mathrm{Co}_{2} \mathrm{O}_{10}\right)$ and their oxygen first coordination shell (see figure (2). These fragments are embedded in a bath designed so that to reproduce on them the main effects of the rest of the crystal ; that is the Madelung potential and the exclusion effects of the electrons of the other atoms of the crystal on the clusters electrons.

The electrostatic potential is reproduced by a set of point charges located at the atomic positions. The charges are renormalized next to the bath borders in order to obtain an exponential convergence of the Madelung potential ${ }^{17}$. The convergence accuracy was set in the present work to the mili-electron-Volt. The nominal atomic charges used in this work are the formal charges, that is +3.5 for the cobalt atoms, -2 for the oxygen atoms and +1 for the sodium atoms. The exclusion effects are treated using total ions pseudo-potentials 18 (TIP) on the first shell of atomic sites surrounding the clusters.

The falculations were done using the MOLCAS ${ }^{19}$ and CASDI ${ }^{20}$ suite of pregrams. The basis sets used can be found in reference 2 . The structural parameter were taken from the 10 K data of reference 6 .



FIG. 2: a) $\mathrm{CoO}_{6}$ and b) $\mathrm{Co}_{2} \mathrm{O}_{10}$ clusters used in the present calculations.

## III. RESULTS

## A. On site orbital energy splitting and charge order

A strong controversy has been going on in the literature about the cobalt 3d orbital splitting. Indeed, authors did not agree on the relative order of the $t_{2 g}$ orbitals under the trigonal distortion of the regular octahedron $\left(T_{2 g} \longrightarrow A_{1 g} \oplus E_{g}\right)$. The origin of the controversy was the fact that the crystalline field theory 22 as well as some LDA calculations ${ }^{23}$ found the $a_{1 g}$ orbital of lower energy than the $e_{g}^{\prime}$ ones, while other density functional calculations 24 , as well as quantum-chemical calculations 25 or ARPES experimental results 26 found them of reverse order. We recently showed 27 that the origin of the controversy was an incorrect treatment of the exchange and correlation integrals within the 3 d shell. Indeed, only when these effects are taken into account with their whole complexity, the relative order between the $a_{1 g}$ and $e_{g}^{\prime}$ orbitals issued from the $t_{2 g}$ is correctly predicted. This splitting is one of the crucial parameter for the determination of the nature of Fermi level states as recently shown by Marianetti et at
We thus computed the $t_{2 g}$ orbitals splitting in the $\mathrm{Na}_{0.5} \mathrm{CoO}_{2}$ compound, for the two different cobalt sites. These values can be extracted from the spectroscopy of the $\mathrm{CoO}_{6}$ embedded fragments. Indeed, the excitation energy between the three cobalt states, where one hole is located one of the $t_{2 g}$ orbitals, yield the effective splittings between the $a_{1 g}$ and $e_{g}^{\prime}$ orbitals. The relative order of the $t_{2 g}$ orbitals is displayed on figure 3. One sees im-


FIG. 3: Schematic representation of the effective cobalt $t_{2 g}$ orbital energies in meV . Let us recall that the $\mathrm{Co}(1)$ cobalt is located directly under or on top a $\mathrm{Na}^{+}$ion.
mediately that the $a_{1 g}$ orbitals are on both sites about 250-300 meV higher than the $e_{g}^{\prime}$ ones. Similarly, in our dimer calculations we see that the $\mathrm{Co}^{4+}(1)$ ion is globally of lower energy than the $\mathrm{Co}^{4+}(2)$ ion. Indeed, our calculations yield a global charge order of about 0.17 electrons in favor of the $\mathrm{Co}(1)$ site. This result is in agreement with the common sense expectations that will favor a smaller valence for the cobalt with the largest number of $\mathrm{Na}^{+}$ cations neighbors. Indeed, computing the electrostatic potential difference between the two cobalt sites one finds the $\mathrm{Co}(1)$ site about 315 meV lower than the $\mathrm{Co}(2)$ site. This result is however in contradiction with the Bond Valence Sum (BVS) model, since the later yields acharge ordering of 0.12 electron in favor of the $\mathrm{Co}(2)$ site ${ }^{6}$, how ever it is in agreement with the LDA +U calculations 29 $(\sim 0.16 \bar{e}$ in favor of $\mathrm{Co}(1))$.

## B. Orbital hybridization

Let us now focus on the cobalt 3d orbital hybridization. These orbitals can hybridize in two ways : with the oxygen ligands 2 p orbitals, but also within the cobalt 3 d shell between the $t_{2 g}$ and $e_{g}$ sets of the regular octahedron. Indeed, it was shown (see ref. 27 for details) that not only this hybridization is symmetry allowed by the trigonal distortion of the octahedron, but also that it is responsible for the lowering of the $e_{g}^{\prime}$ orbitals compared to the $a_{1 g}$ one. The $t_{2 g}-e_{g}$ hybridization angle is reported in table 1 and found to be a little weaker in this system than in the superconducting compound, as expected from the lower trigonal distortion of the $\mathrm{CoO}_{2}$ layer in this system. Indeed, the hybridization angle is about 13 degrees in the latter ${ }^{27}$. Let us now focus on the cobalt ligand

$$
\begin{array}{c|cc}
\text { Atom } & e_{g 1}^{\prime} & e_{g 2}^{\prime} \\
\hline \mathrm{Co}(1) & -11.59 & -8.21 \\
\mathrm{Co}(2) & -11.54 & -10.18
\end{array}
$$

TABLE I: $t_{2 g}-e_{g}$ mixing angle (degrees) in the occupied $e_{g}^{\prime}$ cobalt orbitals.
hybridization. This hybridization seems quite insensitive to the local environment of the cobalt since we find similar values for both crystallographic sites. For the $e_{g}^{\prime}$ and $a_{1 g}$ orbitals the hybridization accounts for $\sim 10 \%$. For the $e_{g}$ orbitals the hybridization depends on the cobalt average charge with $\sim 45 \%$ for a formal $\mathrm{Co}^{4+}$ ion and $\sim 30 \%$ for a formal cobalt charge of 3.5.

## C. Interatomic interactions

Let us now focus on the interactions between two first neighbor cobalt atoms. Many authors assume that the low energy physics of this compound is determined by the $a_{1 g}$ orbitals only. We thus computed the effective integrals both for a $t-J$ model and a one-band Hubbard model.

Analyzing the low temperature crystallographic structure of $\mathrm{Na}_{0.5} \mathrm{CoO}_{2}$ (10 K data of reference 6), one sees that there are four types of independant $\mathrm{Co}-\mathrm{Co}$ bonds (see figure (1), the $\mathrm{Co}(1)-\mathrm{Co}(1)$ bond, the $\mathrm{Co}(2)-\mathrm{Co}(2)$ bond and two kinds of $\mathrm{Co}(1)-\mathrm{Co}(2)$ bonds, namely those where the cobalt-cobalt interactions are mediated by one $\mathrm{O}(1)$ and one $\mathrm{O}(3)$ oxygen ligands and those where the cobalt-cobalt interactions are mediated by one $\mathrm{O}(2)$ and one $\mathrm{O}(3)$ oxygen ligands. The effective magnetic ex-


FIG. 4: (color online) Schematic representation of the $\mathrm{CoO}_{2}$ layers in the $\mathrm{Na}_{0.5} \mathrm{CoO}_{2}$ compound. There are two different cobalt sites represented in light and darker turkoise color blue, three different oxygen ligands and the four different CoCo bonds represented in orange $[\mathrm{Co}(1)-\mathrm{Co}(1)]$, red $[\mathrm{Co}(2)-$ $\mathrm{Co}(2)]$, dark blue $[\mathrm{Co}(1)-\mathrm{Co}(2)$ bridged by $\mathrm{O}(2)$ and $\mathrm{O}(3)$ oxygens] and black $[\mathrm{Co}(1)-\mathrm{Co}(2)$ bridged by $\mathrm{O}(1)$ and $\mathrm{O}(3)$ oxygens].
change integrals are

$$
\begin{aligned}
J(11) & =-11 \mathrm{meV} \quad J(22) & =-27 \mathrm{meV} \\
J\left(12^{[13]}\right) & =-19 \mathrm{meV} \quad J\left(12^{[23]}\right) & =-19 \mathrm{meV}
\end{aligned}
$$

where $J(i j)$ refers to the $\mathrm{Co}(\mathrm{i})-\mathrm{Co}(\mathrm{j})$ bond and the $[a b]$ superscript refers to the bridging oxygens. In our notations, the negative integrals correspond to antiferromagnetic coupling. We find all exchange integrals in the $\mathrm{Na}_{0.5} \mathrm{CoO}_{2}$ compound antiferromagnetic in agreement with the neutron scattering data 30 . One notice that the exchange integrals are strongly modulated according to the local environment. Indeed, they differ by a factor larger than two. It results a strongly anisotropic $t-J$ model.
Let us now focus on the hopping integrals. One can get them either from the charge fluctuation in the singlet state of the $\mathrm{Co}^{4+}-\mathrm{Co}^{4+}$ fragments, or from the spectroscopy of the $\mathrm{Co}^{3+}-\mathrm{Co}^{4+}$ fragments. A simple analysis shows that the two hopping integrals are somewhat different. In the first case, there is only one spectator electron on the bond, while in the second case, there are

$$
\frac{\downarrow}{d_{l}} \frac{\perp}{d_{r}} \stackrel{\mathbf{t}(1 \overline{\mathbf{e}})}{\longleftrightarrow} \frac{\downarrow \downarrow}{d_{l}} \frac{\downarrow}{d_{r}} \quad \frac{\downarrow}{d_{l}} \frac{\perp}{d_{r}} \stackrel{\mathbf{t}(2 \overline{\mathbf{e}})}{\longleftrightarrow} \frac{\downarrow}{d_{l}} \frac{\downarrow \downarrow}{d_{r}}
$$

FIG. 5: Schematic representation of the effective hopping integrals as a function of the spectator electrons on the bond. $l$ and $r$ labels refers to the arbitrary "left" and "right" atoms in the fragment.
two of them (see figure 5). It results in a different evaluation of the hopping integrals according to the number of spectator electrons.
$t(1 \bar{e})=\left\langle d_{l}\right| h\left|d_{r}\right\rangle+\left\langle d_{l} \bar{d}_{l}\right| 1 / r_{12}\left|d_{r} \bar{d}_{l}\right\rangle$
$t(2 \bar{e})=\left\langle d_{l}\right| h\left|d_{r}\right\rangle+\left\langle d_{l} \bar{d}_{l}\right| 1 / r_{12}\left|d_{r} \bar{d}_{l}\right\rangle+\left\langle d_{l} \bar{d}_{r}\right| 1 / r_{12}\left|d_{r} \bar{d}_{r}\right\rangle$
where the $l$ and $r$ labels refers to the arbitrary "left" and "right" atoms in the fragment, and $h$ is the single electron Hamiltonian part.
$t(1 \bar{e})$ can be extracted from the $\mathrm{Co}^{4+}-\mathrm{Co}^{4+}$ fragment. Indeed, mapping a single band Hubbard model on the computed low-energy singlet and triplet states wave functions and energies, one gets after a little algebra

$$
\begin{align*}
t(1 \bar{e}) & =J \frac{1}{2 \cos \varphi \tan \alpha}  \tag{1}\\
\bar{U}-V & =J\left(1-\frac{1}{\cos 2 \varphi \tan ^{2} \alpha}\right) \tag{2}
\end{align*}
$$

assuming the following form for the computed singlet and triplet wave functions

$$
\begin{aligned}
\psi_{S g} & =\cos \alpha \frac{\left|d_{l} \bar{d}_{r}\right\rangle+\left|d_{r} \bar{d}_{l}\right\rangle}{\sqrt{2}} \\
& +\sin \alpha\left(\cos \varphi \frac{\left|d_{l} \bar{d}_{l}\right\rangle+\left|d_{r} \bar{d}_{r}\right\rangle}{\sqrt{2}}+\sin \varphi \frac{\left|d_{l} \bar{d}_{l}\right\rangle-\left|d_{r} \bar{d}_{r}\right\rangle}{\sqrt{2}}\right) \\
& + \text { small terms }
\end{aligned}
$$

$$
\psi_{T p}=\frac{\left|d_{l} \bar{d}_{r}\right\rangle-\left|d_{r} \bar{d}_{l}\right\rangle}{\sqrt{2}}+\text { small terms }
$$

$J=E_{S g}-E_{T p} . \bar{U}-V$ is the effective on-site repulsion of a simple Hubbard model. Let us remind that it accounts for both the average on-site repulsion between the two sites, $\bar{U}=\left(U_{l}+U_{r}\right) / 2$, and the nearest neighbor effective repulsion between the cobalt atoms, $V=V_{l r}$. The (electron-)hopping integrals between the $a_{1 g}$ orbitals thus come as

$$
\begin{gathered}
t_{00}^{1 \bar{e}}(11)=84 \mathrm{meV} \quad t_{00}^{1 \bar{e}}(22)=139 \mathrm{meV} \\
t_{00}^{1 \bar{e}}\left(12^{[13]}\right)=115 \mathrm{meV} \quad t_{00}^{1 e}\left(12^{[23]}\right)=111 \mathrm{meV}
\end{gathered}
$$

Once again, one sees that the effective transfer integrals are strongly dependant on the local environment of the cobalts and that the $\mathrm{Na}_{0.5} \mathrm{CoO}_{2}$ system is more anisotropic than first thought.

The transfer integrals with two spectator electrons on the bond, $t^{2 \bar{e}}$, can be extracted from the low-energy states
of the $\mathrm{Co}^{3+}-\mathrm{Co}^{4+}$ fragments. There are six low energy states according to the localization of the hole on the $a_{1 g}$ and $e_{g}^{\prime}$ orbitals. The associated effective Hamiltonian can be written as

$$
H=\left(\begin{array}{cccccc}
a_{1 g}^{l} & e_{g 1}^{\prime l} & e_{g 2}^{\prime l} & a_{1 g}^{r} & e_{g 1}^{\prime r} & e_{g 2}^{\prime r}  \tag{3}\\
\varepsilon_{0}^{l} & t p_{10}^{l} & t p_{20}^{l} & t_{00}^{2 \bar{e}} & t_{10}^{2 \bar{e}} & t_{20}^{2 \bar{e}} \\
t p_{10}^{l} & \varepsilon_{1}^{l} & t p_{12}^{l} & t_{10}^{2 \bar{e}} & t_{11}^{2 \bar{e}} & t_{12}^{2 \bar{e}} \\
t p_{20}^{l} & t p_{12}^{l} & \varepsilon_{2}^{l} & t_{20}^{2 \bar{e}} & t_{12}^{2 \bar{e}} & t_{22}^{2 \bar{e}} \\
t_{00}^{2 \bar{e}} & t_{10}^{2 \bar{e}} & t_{20}^{2 \bar{e}} & \varepsilon_{0}^{r} & t p_{10}^{r} & t p_{20}^{r} \\
t_{10}^{2 \bar{u}} & t_{11}^{2 \bar{r}} & t_{12}^{2 \bar{e}} & t p_{10}^{r} & \varepsilon_{1}^{r} & t p_{12}^{r} \\
t_{20}^{2 \bar{e}} & t_{12}^{2 \bar{e}} & t_{22}^{2 \bar{e}} & t p_{20}^{r} & t p_{12}^{r} & \varepsilon_{2}^{r}
\end{array}\right)
$$

where $\varepsilon_{i}$ are the atomic effective orbital energies (see figure 3), $t_{i j}^{2 \bar{e}}$ are the effective transfer integrals (direct plus mediated by the oxygen ligands) between the $3 d_{i}$ orbital of one cobalt and $3 d_{j}$ of the other. $t p_{i j}$ are effective intraatomic transfer integrals between the $d_{i}$ and $d_{j}$ orbitals of the same atom. One may be surprised to find such terms, since the direct integrals are zero by $Y_{l}^{m}$ symmetry, however the coupling with the bridging oxygen $2 p$ orbitals yield in second order perturbation theory an effective intra-atomic transfer term of

$$
t p_{i j}=-\sum_{p} \frac{t_{i p} t_{j p}}{\Delta_{p}}
$$

where $i$ and $j$ refers to the $3 d$ orbitals of the same cobalt atom, the sum over $p$ runs over all the ligand bridging orbitals, $t_{i p}$ is the cobalt $3 d_{i}$-ligand hopping integrals and $\Delta_{p}$ is the excitation energy toward the ligand-tometal charge transfer configuration. For a more detailed description of the underlying mechanism, one can refer to reference 25. Summing the $t p_{i j}$ contributions coming from the six oxygens around a cobalt atom, one can show that they exactly cancel out in a symmetric system. In the present system, where the atomic $S_{6}$ symmetry is not exactly respected, these terms will certainly not exactly cancel out, however, one can expect that their sum will remain very weak.

Table III displays the different effective integrals involved in matrix 3 . One sees immediately that the dom-

TABLE II: Effective (electron-)hopping integrals between $a_{1 g}$ and $e_{g}^{\prime}$ orbitals of two neighboring cobalt atoms (in meV).
inant transfer integrals are the inter-atomic hopping between two $a_{1 g}$ orbitals and two $e_{g 1}^{\prime}$ orbitals, in agreement with what we found on the superconducting compound. All other integrals are relatively weak $(<30 \mathrm{meV})$ and can probably be omitted in a simple model. Comparing these values with what we found on the superconductor
system, one sees that they are of the same order of magnitude, except for the inter-atomic hopping between the $a_{1 g}$ orbital of one cobalt and the $e_{g 2}^{\prime}$ orbital of the other. Indeed, the larger value found in the superconducting systems $(-53 \mathrm{meV})$ is now spread over the three $t_{10}^{2 \bar{e}}, t_{20}^{2 \bar{e}}$ and $t_{12}^{2 \bar{e}}$ hopping terms, due to local symmetry breaking.

Let us notice that as in the hydrated system, the above hopping integrals are of the same order of magnitude as the atomic orbital energy splitting and thus it remains quite difficult to conclude on the pertinence of the one or three bands models. In any case, it does not seem to us that the present results differ enough from those found for the hydrated compound to support the hypothesis of a change in behavior (one-band versus three-bands ${ }^{11}$ ) at $x=0.5$. Let us notice that this hypothesis comes from $\mathrm{LDA}+\mathrm{U}$ calculations and is strongly dependent of the on-site repulsion $U$ parameter.

## D. On-site repulsion

Let us now examine what we find for on-site the repulsion $U$ from our calculations. Indeed, as detailed in equation 2, the value of $\bar{U}-V$ can be extracted from the singlet and triplet energies and wave functions on the $\mathrm{Co}^{4+}-\mathrm{Co}^{4+}$ fragments. It comes for the two cobalt sites and for a single band model

$$
\begin{aligned}
U(1)-V(11) & =2.614 \mathrm{eV} \\
U(2)-V(22) & =2.859 \mathrm{eV} \\
\bar{U}-V\left(12^{[13]}\right) & =2.758 \mathrm{eV} \\
\bar{U}-V\left(12^{[23]}\right) & =2.562 \mathrm{eV}
\end{aligned}
$$

where $\bar{U}=\frac{U(1)+U(2)}{2}$
While these values are similar, they are quite weaker than what we found for the superconducting system. This fact may be explained by the larger distances between the cobalt and its first coordination shell. Indeed, the $3 d$ magnetic orbitals have a larger volume to spread over and thus the repulsion integral is weaker.

## IV. CONCLUSION

We determined the effective on-site and nearestneighbor coupling parameters for Hubbard and $t-J$ models of the $\mathrm{Na}_{0.5} \mathrm{CoO}_{2}$ compound. The effective integrals and orbital energies were computed using ab-initio quantum chemical methods treating exactly the strong corre-
lation effects within the cobalt $3 d$ shell and the screening effects up to the double-excitations.

We determined the ligand field splitting for the two crystallographically independent cobalts as well as their relative energetic ordering. As for the superconducting compound, we find the $a_{1 g}$ orbitals above the $e_{g}^{\prime}$ ones. The $t_{2 g}$ orbital splitting is however smaller in this system for the $\mathrm{Co}(1)$ atom $(\sim 260 \mathrm{meV})$ due to the weaker trigonal splitting. Indeed, while in the superconducting system the average distance between the oxygen and cobalt planes is $0.88 \AA$, in the $x=0.5$ compound it ranges between 0.95 and $0.98 \AA$. As described in reference 27, the relative order between the $a_{1 g}$ and $e_{g}^{\prime}$ orbitals is due to a small hybridization between the $t_{2 g}$ and $e_{g}$ orbitals of the regular octahedron. This hybridization is found to range between 8 and 11 degrees in this system while it was 13 degrees in the superconductor, in agreement with a larger $a_{1 g}-e_{g}^{\prime}$ splitting. The $a_{1 g}$ orbital of the $\mathrm{Co}(1)$ cobalt atom - on-top of a sodium cation - is found of slightly lower energy than for the $\mathrm{Co}(2)$ atom as expected from simple electrostatic considerations. We found that it results in a very weak charge ordering of $\pm 0.08 \bar{e}$ in favor of $\mathrm{Co}(1)$. This weak charge ordering is in agreement BVS estimations, however in reversed order as expected from electrostatic.

As far as the hopping integrals in the $t_{2 g}$ set are concerned, none of them are zero due to the local symmetry reduction of the $\mathrm{CoO}_{6}$ octahedra. Anyway, the dominant ones are as in the superconductor the $a_{1 g}-a_{1 g}$ and the $e_{g 1}^{\prime}-e_{g 1}^{\prime}$.

Finally, one should remember the strong variation of the effective exchange integrals as a function of the local distortions. This fact can of importance in the understanding of the changes in magnetic behavior of the $\mathrm{Na}_{\mathrm{x}} \mathrm{CoO}_{2}$ compounds as a function of $x$. It would thus be of interest to see if the local distortions observed for larger $x$ value will be able to reverse the sign of the magnetic exchange as supposed from neutron diffraction experiments. This point will be the object of further works.

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