Graph theory and representation of distances : chronomaps, and other representations

Alain L'Hostis

PART 3

Towards Multilevel Graph Theory

Chapter 9

Graph Theory and Representation of Distances: ChronoMaps and Other Representations

9.1. Introduction

Born from the resolution of practical questions which cannot be easily solved by using graphs (such as the problem of crossing the bridges of Königsberg (Figure 9.1), which was solved by Euler, or the problem of the four colors, which was introduced for the first time by the cartographer Guthrie in 1852), graph theory constitutes a mathematical framework that makes it possible to tackle problems in a very vast field.

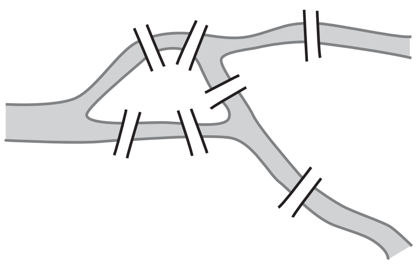

Figure 9.1. *Bridges of Königsberg*

Chapter written by Alain L'HOSTIS.

In the domains of geography, urban development and spatial planning, graph theory is used to tackle questions arising in the field of networks (whether they are transport and communication or even social networks). The general procedure is based on the translation of these problems into the mathematical language of graph theory so as to be able to treat them using, for example, the properties of connectivity or by exploring minimum paths. For this reason, in its applications to the social sciences, graph theory deals mainly with mathematical properties and much less with the graphic representation of nodes and arcs[1]. By refocusing on the graphical dimension, from which it was born, here we wish to develop the aspect of graph theory that relates to representation (plotting of graphs) in its applications to problems of spatial analysis.

Since graph theory is particularly adapted to the modeling of transport networks, the representation of the network graph may constitute an investigative direction that can answer the difficult question of the representation of distances.

A classical illustration of the difficult problem of the representation of distances is provided by Müller [MUL 79, p. 216]. In Figure 9.2 we laid out three cities noted by p, q and r. There is a highway connecting the cities q and r, and another between q and p, whereas r and p are connected by a small road. The shortest path by duration between the two latter cities passes by the city q and follows the highway. The question raised by Müller is as follows: "How is it possible to determine, on a map with a time scale, the position of points p, q and r when the shortest path from one point to another is no longer a straight line?" [MUL 79, p. 215].

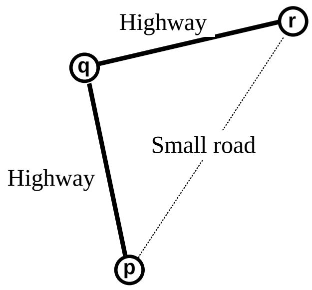

Figure 9.2. *Representation of the shortest paths*

1 When we study graph theory, one of the few properties that relates to their representation – their plotting – is the concept of a planar graph. A graph is known as planar if it can be drawn on a plane so that the nodes are distinct points, the edges are simple curves and no two edges meet apart from at their ends.

The straight line almost never corresponds to the general form of movement, i.e., to the general form of geographical distance. Consequently, as Angel and Hyman put it: “if we wish to study transport within a geometrical framework, we must develop non-Euclidean geographical theories” [ANG 72, p. 366]. In this spirit we will start by establishing a general form of distance on a graph on the basis of the basic definitions of graph theory. The goal is to give a mathematical form to a geographical measurement: the distance built in this fashion will make it possible to find support in mathematics, to better envisage the spatial dimension associated with transport and movement and, finally, to question the cartographic representations used in geography.

To deal with the domain of representations we will create a method for reading distances on a map. The comparison of these measurements which are read with those provided by the geographical data will make it possible to characterize the various representations according to their capacity to represent geographical distances.

9.2. A distance on the graph

Mathematical distances are measurements between objects that respect the properties of positivity, uniqueness, separation and triangular inequality. From the mathematical point of view as well as from the point of view of geographical interpretation, one of the most fundamental properties of distances is that of triangular inequality because it introduces the idea of the minimum [LHO 97, p. 102 and 113]. The distance between two objects is associated with the measurement of the smallest gap that can be found. To illustrate this principle, on Müller’s diagram the distance between points p and r can have two forms: if we measure it in kilometers, the distance is given by the small road, whereas, if we calculate it in duration of transport, the distance follows the highway route passing through point q.

In graph theory we have a particular application which associates a value to the shortest path: if a and b are two nodes of a graph G, the measure e (a, b) is given by the length of the shortest path between a and b.

If we consider the case of a valuated graph[2], the length of a path corresponds to the sum of the values associated with the arcs, of which it consists. With the measure e, we construct an application that associates a value in R to a pair of elements of the set of nodes S. It should be noted that in this definition there is no

2 In a valuation graph a numerical value is associated to each arc.

specification of the mathematical properties of the application. In particular, it is not specified, if the measure e is metric, i.e., if it respects the properties of distances.

In the general case of a non-valuated graph Flament indicates the construction of a metric on the basis of the measure e [FLA 68, p. 39]. The only condition for the graph is that the value associated with a link between two nodes is identical in each direction. The graph must therefore be symmetrical. The measure between one point and another is considered zero. Defined on a symmetrical non-valuated graph, with conventions for infinite and zero paths, the margin is metric.

On a valuated graph in order for the measure e to be a distance in the mathematical sense, we must verify one by one the metric properties (symmetry, positivity, separation and triangular inequality) for any pair of nodes. That leads to two conditions, which make it possible to establish a graph distance [LHO 97, p. 137]: the graph must be symmetrical and for a valuated graph the values associated with the arcs must be strictly positive.

The distance criterion of minimum paths calculation can integrate the duration of transport, the cost, the length of the journeys, but also the various modes of transport (with a quality of connection that may or may not be homogenous) or even the nature of what is transported. If the graph considered is not symmetrical, we can construct only a non-symmetrical metric, a structure that belongs to the more general field of impoverished metrics[3]. Although non-symmetry is the general case for geographical distances (in time or cost) we will limit ourselves here to the analysis of symmetrical distances by considering a simplification of the collected data.

The distance defined in this fashion is purely mathematical: it is only the connection between pairs of places with a measurement that takes the form of a measure e. It is a mathematical object that characterizes geographical data, but *a priori* does not have immediate and univocal translation in the sphere of cartographic representations.

9.3. A distance on the map

Although a geographical map primarily provides information about the location of places, it also informs us about the relations between these locations. In particular, reading a roadmap makes it possible to work out routes and determine the distances associated with them.

---

3 For more on these structures and their use in spatial analysis see [HUR 90, p. 225].

To define the function of distance that we have in a map Müller [MUL 79, p. 224] introduced the concept of “graphic distance”. This distance is defined in an Euclidean form: we consider that the length of a path between two points on the map is directly proportional to the length of the rectilinear segment which connects them. However, movements carried out in the geographical space use transport infrastructures that have very little chance to follow a straight line between two places chosen at random. Thus, in order to measure a distance in kilometers on a roadmap the reader must transform the graphic distance mentally to obtain a useful measurement: the sinuosity of the road and the extent of detours along the way make it possible to correct, revising up, the Euclidean length of a route. In this context the Euclidean distance offers a first approximation of measurement. It is then adjusted by information about the infrastructure used. In fact, the concept of distance “as the crow flies”, on which Müller’s graphic distance is based, cannot be used to read a cartographic representation if we wish to understand the logic of movements.

For this reason it is necessary to introduce a different concept, that of “visual length” [LHO 97, p. 137], which is derived from Müller’s graphic distance. The visual length of a connection between two points is equal to the length of the corresponding path. If, for example, the path takes the form of several segments placed end-to-end, the visual length corresponds to the sum of the lengths of these segments. Visual length is, thus, not an Euclidean concept. It should be noted that, according to this definition, visual length does not exist between two points unless there is a path. Visual length makes it possible, in particular, to provide a measurement of the routes and paths on the basis of a graph plot, i.e., on the basis of the map of a transport network.

By abandoning graphic distance in favor of visual length we lose the reference to the metric. The motivation of this choice becomes apparent when we refer to Müller’s diagram (Figure 9.2). The visual length of the connection between p and q corresponds to the distance between p and q, if it is measured in kilometers. However, that is not true if the distance is calculated in journey duration. In this case the visual length read on the map along the small road is not a distance since it does not correspond to the minimum possible measurement in space-time.

By uncoupling the concepts of distance (geographical concept) and visual length (cartographic concept) we pave the way for a better understanding of the representations of distance.

Thusly defined, the concept of visual length calls into question the usual use of map scale. Indeed, on a traditional geographical map the scale has the function to provide measurements of distance in kilometers as the crow flies: it is clearly an Euclidean notion of distance. This use of scale implies the postulate of verifying the isotropy and the uniformity of the areas represented: two properties that cannot be regarded as given when geographical spaces are dealt with.

Visual length establishes a measurement which, converted by using the scale of representation, produces a geographical distance. It is a non-Euclidean use of the map scale. A cartographic representation of distances is considered to be satisfactory, if the distances read are coherent with the initial data, that is, with geographical distances.

For Bunge there are two manners of representing distances: we may show on a traditional map the paths that can be sinuous, or we may represent distances in a simplified form but on a deformed map [BUN 66]. The second approach refers to the anamorphoses introduced by Tobler [TOB 63, p. 59-78] and consists of moving the locations of places according to distances[4]. This second approach is the one that interests us because it directly involves the representation of the transport network.

9.4. Spring maps

To illustrate the impact of the modes of transport on spatial relations Tobler considers the representation of distances in a mountainous zone [TOB 97]. The initial data set consists of measurements of real distances in kilometers that often follow sinuous routes. The degree of circuity of the associated graph (which is defined as the measurement of the difference between the distances in kilometers and the distances as the crow flies [KAN 63, p. 93-121]) is thus very high[5]. The spring map (Figure 9.3) is constructed on the basis of a set of places which are positioned at their real location [CAU 84, p. 40], i.e., their position on the plane of the map is not modified by anamorphoses.

The principle of spring representation is to depict the link existing between two points in space in the shape of a spring whose length is proportional to the length in kilometers. By reference to the Euclidean distance, connections which are longer than those "as the crow flies" are represented in the shape of a generally compressed spring. This graphic principle makes it possible to represent paths which are longer than the straight line without modifying the location of the ends.

To read such a map, it is necessary to get rid of our Euclidean reflexes because the distance between places is indicated by a path which is not a straight line. This distance can be understood only by using visual length because the graphic distance defined by Müller results in Euclidean measurements. The map is read by evaluating

4 On this subject see [CAU 96] (www.cybergeo.presse.fr).
5 For developments and analyses of the degree of circuity see [CHA 97, p. 330].

the intensity of the compression of the springs: the more compressed the spring is, the more difficult the connection (in other words it contains more turns and slopes due to the mountainous terrain, which lowers the average speed) and the more it moves away from the straight line. The representation is built around a direct reference to the Euclidean distance, but a reference that is implicit. Here we find the status of the Euclidean space as proposed by Cauvin, that is, as a reference space [CAU 84, p. 72].

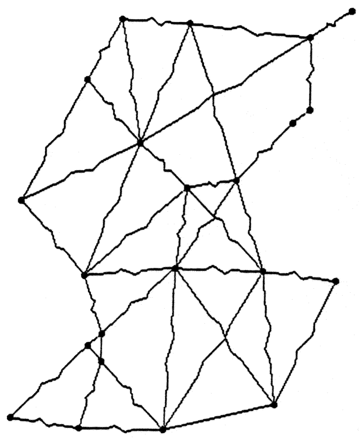

Figure 9.3. *Spring graph of the road network of Western Colorado*

The spring graph may be seen as the plotting of a graph. The graph is that of the transport network considered, which is valuated by the measurements taken for the network, and its plotting undergoes three constraints:

– the nodes of the graph are placed at the geographical position of the nodes, which they model;

– the length of the arcs of the graph is proportional to the associated value;

– the arcs of the graph are drawn in the shape of springs which ensure the proportionality of the lengths.

The distances in kilometers are considered here to be symmetrical and they define a graph distance. The construction principle of the spring map makes it possible to establish a representation which is coherent with the associated mathematical structure. Therein lies the main merit of this type of map, which is to offer a representation of distances that remains coherent with real measurements, although they elude the Euclidean domain.

On the basis of the measurements represented on the spring map, Tobler then proposes three-dimensional anamorphoses. The adopted principle is to slide the location of the places on a vertical axis to stretch the distances "as the crow flies" in three dimensions. A regression using least squares makes it possible to find a configuration which minimizes the cumulated error for all the links (Figure 9.4.).

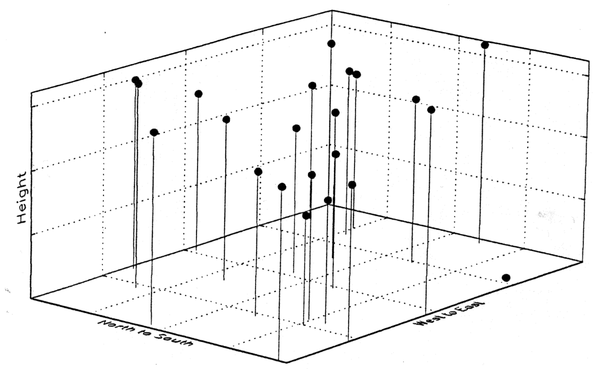

Figure 9.4. *Road distances approximated by displacing the nodes in the third dimension*

Then, a transport surface can be calculated by interpolation, by building a continuum based on the previously positioned sites (Figure 9.5). It is possible to find the distances between places by using a measurement along a two-dimensional plane.

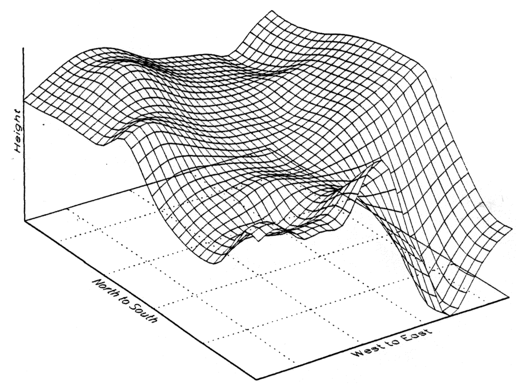

Figure 9.5. *Transport surface constructed by interpolation*

Let us imagine tracing the original transport network on this surface. This plotting of the graph would make it possible to find a coherence between the real distances and the represented distances, similarly to what is proposed by the spring map. If we consider that the initial data makes it possible to construct a graph distance (where positivity and symmetry are verified), the suggested map is coherent with this data, as well as with the mathematical structure that they imply and therefore also with the geographical reality.

The construction principle of the interpolated transport surface comprises, however, two important differences with respect to the spring map representation mode. Firstly, regression and interpolation introduce an error, which, even if minimized, makes the representation enter the domain of approximation, since the measurements taken have to integrate an associated error. Secondly, the use of the third dimension implies a projection system onto a two-dimensional surface of a paper sheet that carries the image. And there too we introduce an approximation, though of a different nature, due to the projection angles. This approximation can be largely reduced, if we help the reader to reconstruct the third dimension mentally. In Figure 9.7 the deformation of the grid, the hidden surfaces (in the south of the surface) and the three-dimensional coordinates system invite the reader to enter into the third dimension of the map and help him to reconstruct the surface topology.

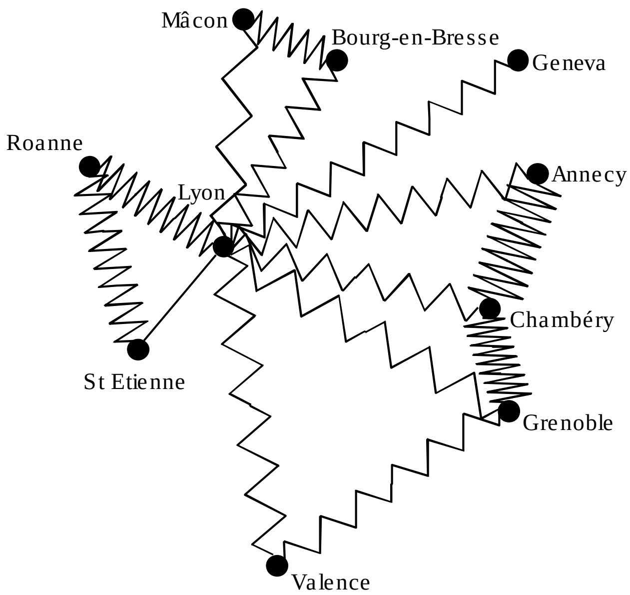

Figure 9.6. *"Spring" graph of the Lyon region*

The second application that we propose [PLA 87, p. 12] (Figure 9.6) shows the quality of service of the rail network in the Lyon region. The value of the arcs in the graph indicates the quality of service, which in turn is constructed on the basis of the quality of service of every line.

Visually, on spring maps, it is difficult to directly perceive the exact length of the arcs. We first see the generally “compressed” aspect of the spring which provides an indication of its length. The notion of visual length, which is given by the overall length of the spring, does not result here from a direct visual perception. In the case of the spring graph we are closer to the idea of a mental transformation carried out by the map reader, than of a direct and proportional visual perception. However, does this mental transformation not proceed from the learning inherent to any non-traditional system of representation? As Tobler proposes, these representations may seem strange at first, but this is largely due to our more favorable attitude towards more traditional and also more familiar maps, which means that we tend to consider only conventional maps as realistic and correct [TOB 61, p. 164].

To answer the problem of the representation of distances, the plotting of a three-dimensional graph offers other latitudes and allows other modes of representation, which are to some extent connected with Tobler’s explorations. We will now present the space-time relief maps, from the viewpoint of coherence with the initial data, but also by way of comparison with the systems presented up until now.

9.5. ChronoMaps: space-time relief maps

Space-time relief maps, or chronomaps, aim at representing a space of transport deformed by the coexistence of modes of transport with different performances [LHO 96, p. 37-43]. In the domain of passenger transport speed and cost per kilometer vary very strongly according to the transport modes, but also within a modes depending on the quality and type of the infrastructure used. An inhabitant of Aurillac who wishes to go by car to the prefecture of the area, Clermont-Ferrand, starts by driving on a trunk road with an average speed of 60 km/h until he reaches a highway entry, from where he drives at 110 km/h on average. In this example, the highway portion of the journey is covered almost twice as quickly as the portion on the trunk road. The possibility of swapping between the sub-networks of the transport system are very strongly conditioned by the nature of the networks and, therefore, the cartography of such a transport space must take into account the form of the networks and the heterogeneous nature of the system. This difficult problem, which in anamorphoses is dealt with by dilating the space in a non-homogenous manner according to the quality of the connections, is solved on the relief map by employing the third dimension. To represent these distances, we do not change the location of the places, but the way of drawing the connections.

Chronomaps are constructed on the basis of a graph (Figure 9.7 of the central book, stage b). The graph representing the transport network must be planar and saturated in order to enable the construction of a three-dimensional surface (Figure 9.7, stage g). To respect the heterogeneity of the network the graph adopts the shape of a p-graph[6], in which each sub-network appears as a partial graph[7]. Thus, between Nantes and Angers, the highway and the road are superimposed in the p-graph and each infrastructural network is considered as a partial graph of the p-graph. The nodes of the graph are again assigned to the main cities of the area considered, but also to the singular places, whose consideration is necessary for a thorough understanding of the transport network, such as the Essarts highway interchange located between La Roche-sur-Yon and Cholet. The choice of the density of nodes in the space considered is one of the determining elements of the form of the relief and, thus, of the produced image and the message that it conveys. Low density can make it possible to show the effect of networks with a broad grid (TGV and highway) over the entire area, whereas greater density makes it possible to express the complexity of local distortions.

Representing distances in a homogenous network with connections that are not far from the straight line is easy: in the plot in the plan of the highway graph (stage c) the length of the arcs plotted is directly proportional to the time-distance and the associated errors are small. On the other hand, in a heterogeneous network, where the differential journey speeds (or costs) are considerable, the proportionality of the lengths of the arcs cannot be respected if they are drawn in the form of segments in the plan. In space-time relief maps the arcs with poorer performances are traced under the plane of the nodes in the shape of two segments, in such a way that their length is proportional to the journey durations. The worse a connection performs, the more an arc becomes concave and moves away from the straight line on the plane (stages d, e and f). During the last stage the coloring of the graph surfaces in different shades of gray makes it possible to reconstruct the transport surface and facilitates reading the relief.

The chronomap is constructed as the plotting of a graph representing a multimodal transport network. The principle of plotting follows three rules that can be stated as follows:

– the position of the nodes in the graph corresponds to their real position;

– the length of the arcs in the graph is proportional to their effective length (be it in kilometers, hours, etc);

– to enable the proportionality of the lengths, the arcs are drawn in the third dimension.

---

6 In a p-graph there can be no more than p distinct arcs connecting the same pair of nodes.

7 A partial graph G' of a graph G contains all of the latter's nodes, but only a part of its arcs.

From the point of view of graph plotting, the relationship of this representation principle with that of Tobler's spring maps is very strong. The location of the nodes is the same and the principle of proportionality for the length of the arcs is common. The only difference lies in the manner of tracing the arcs.

Another interesting point of comparison of the two models is their way of representing the continuum. According to Tobler, the passage from graph plotting to the transport surface is achieved by using a regression along a vertical axis followed by interpolation. In the space-time relief map the passage from graph plotting is established by tracing the facets of the graph on the basis of the arcs. Each triangular facet (the graph must be planar and saturated) is in turn divided into four triangles which are traced from the middle of the facets. The surface constructed in this manner then undergoes the deformation of graph arcs. The surface is associated with the road graph and not to the highway graph because access to normal space is only possible by using the road. This idea of universality of the road network [DUP 94, p. 146], defined by Dupuy, is expressed by Tobler by defining the border of the network: the border of the road network is constituted by the edge of the carriageway, whereas the border of the highway network is constituted by the access points [TOB 61, p. 89]. On the relief map the slope of the arcs directly translates the associated relative speed: the more marked the slope, the lower the connection speed is with respect to the possible maximum speed appearing in the plan of the nodes occupied by the highway network. By geometrical construction the slope of the facets in the graph is sharper than that of the arcs upon which they are based. This means that the performance of the interstitial sub-network that services the area of the facet is poorer than that of the connections bordering it. This principle is coherent with that of network hierarchy: in transport networks the performance drops with the size of the grid [PLA 91, p. 22]. The representation of the continuum, which is defined as the dual of the network, does not contradict the principle of relief representation. However, it is necessary to draw attention to the fact that the distances shown by the relief map are, first of all, those produced by the transport network. Here, the first function of the continuum is graphic: the shades of gray of the facets help the reader to imagine the third dimension, just as the deformed grid used by Tobler did.

Chronomaps do not show space in the form of plane layers, but of superimposed complex surfaces. The image in Figure 9.8 of the central book shows two sub-networks of the road transport network which are arranged in superimposed layers in the third dimension and coming into contact only at the sites of highway interchanges. This is a direct illustration of the tunnel effect of fast infrastructures.

Each space is registered according to its own relief: the highway is in the plane of the cities where the straight line draws the fastest path but where space is reduced to a set of points (network space), whereas the other networks follow a relief which becomes more marked as accessibility diminishes (banal space). The space of the road is a continuous but irregular surface, less accessible, but where the concept of proximity still operates. The map shows a space dualized by speed.

The large highway axes which serve the territory appear clearly inscribed in the plane of the cities. However, in the interstices of this network the space-time relief reflects the lack of accessibility due to the inferior quality of the infrastructure. The network space of the Riviera, from Avignon to Menton, which is well connected thanks to an extensive highway grid, strongly contrasts with the poorly accessible space-time of Alpes-de-Haute-Provence and Hautes-Alpes.

The ChronoMap of the road and the highway renders readable the minimum duration transport routes by showing the degradation of the relative conditions of circulation outside of the highway type network. Thus, the Grenoble-Nice connection is ensured via the Rhone valley along a highway route that traces a vast loop whose visual length remains lower than that of the route closer to the straight line through Gap and Digne which passes through an irregular space-time relief.

In formalizing the construction principle of relief maps, the first constraint relates to the length of the arcs without taking their visual length into consideration. In this respect we may note a difference with the construction principle of "spring maps". In spring maps visual length is directly proportional to the effective length of the arcs, even though the visual length of springs is more inferred than directly read.

Relief maps are particular views of a general three-dimensional model. The length of the arcs of the construction principle is a constraint on this general model. The images, whether projected onto a screen or onto a sheet of paper, are two-dimensional. The concept of visual length may be used only with respect to the images. An image is created by the transformation of a three-dimensional structure – the model – into a two-dimensional form – the map. Just as on the interpolated transport surface of Tobler, the complex play of three-dimensional structure projections onto the plane of the map causes disturbances in the lengths of the arcs. These disturbances justify the fact that we only have an "approximate scale" of space-time in relief maps. The visual length which is available for images is not exactly proportional to the effective length present in the data. Mathematically, this results in the fact that a three-dimensional surface and the final image are not isometric: there is a distortion due to the projection on the plane. For arcs belonging to the third dimension, the distortion between visual lengths and effective lengths is not homogenous. It is therefore difficult to evaluate visually: it is necessary to find the position of the segments in three dimensions. It is the role of gray shaded facets, of viewing angle indication, to help to mentally reconstruct the relief in order to make the distortion understandable. Let us note that the general principle of relief maps construction places no constraints on images, meaning that the adjustment of the image generation parameters arises from the choices made by the cartographer. An infinite number of possible images correspond to the single model, according to the choice of the viewing angle, or also of the point of view[8].

Although the approximation due to the projection of the three-dimensional structure is common to that of the interpolated transport surfaces of Tobler, quite unlike the latter, from the point of view of the construction principle, space-time relief maps address the exact domain. This means that the length of arcs of the three-dimensional model is strictly equal to that coming from initial data.

9.6. Conclusion

Distance belongs to concepts applied in disciplines that are very far from each other. In Tobler (for whom any map results from a mathematical transformation whose parameters it suffices to modify), in Müller [MUL 79, p. 215-227], or in Huriot and Perreur [HUR 90, p. 225] the issue of the representation of distances, in geography as well as in other disciplines that concern space, insistently refers to mathematics.

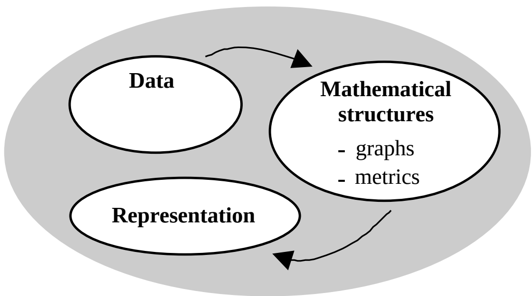

Figure 9.9. *The role of mathematical structures in the representation of distances*

In this spirit, the representation procedures exposed here are all based on graphs. The very object of this contribution (the issue of the representation of distances) is a call for a positioning in the field of metrics. Graph distance has been defined in this manner. The representations all are constructed on the basis of a graph plotting. Reading the map requires measuring instruments (which are the scale of representation and the graphic length), which make it possible to reconstruct metric space and understand its distance.

---

8 For relief maps of the Atlantic coast according to an unusual point of view see [MAT 96, p. 97-111].

Whether implicit or explicit, the reference to mathematical structures is necessary for the procedure of representing distances (Figure 9.9). The use of mathematical forms enables the clarification of principles and ensures the freedom of contradiction [BUN 66, p. 2].

The principles of representation clarified here show the essential contributions of graphs use in the cartography of distances and the understanding of spatial relations. All the avenues are far from having been explored and the models still hold important development prospects. Indeed, if the relief maps presented show the connections in the form of two broken segments, nothing prevents us from imagining other configurations using curves, for example.

The cartographies of distance can hardly be understood without instructions (a scale of correspondence and the way to use it), which sometimes disturb the established order of cartographic conventions.

9.7. Bibliography

[ANG 72] ANGEL S., HYMAN G.M., "Urban spatial interaction", *Environment and planning*, GB, vol. 4, p. 350-367, 1975.

[BUN 66] BUNGE W., *Theoretical geography*, (revised 2nd e.), Gleerup (Lund studies in Geography), Lund, Sweden, 1966 (1962).

[CAU 84] CAUVIN C., Espaces cognitifs and transformations cartographiques, Strasbourg, Thesis, 1984.

[CAU 96] CAUVIN C., "Au sujet des transformations cartographiques de position", *Revue électronique Cybergéo* (www.cybergeo.presse.fr), 1996.

[CHA 97] CHAPELON L., Offre de transport et aménagement du territoire: évaluation spatio-temporelle des projets de modification de l'offre par modélisation multiéchelles des systèmes de transport, Thesis, Tours, 1997.

[DUP 94] DUPUY G., "Réseaux", *Encyclopédie d'économie spatiale: concepts - comportements – organisations*, J.-P. Auray, Antoine Bailly, P.-H. Derycke, J.-M. Huriotb (eds.), Economica, p. 145-151, Paris, 1994.

[FLA 68] FLAMENT C., *Théorie des graphes et structures sociales*, Paris: Gauthier-Villars/Mouton (Mathématiques et sciences de l'homme), 1968.

[HUR 90] HURIOT J.M., PERREUR J., "Distances, espaces et représentations", *RERU*, France, no. 2, p. 197-237, 1990.

[KAN 63] KANSKY K., "Structure of transport networks", Chicago: Department of Geography, *Research Paper no. 84*, 1963.

[LHO 96] L'HOSTIS A., "Transports et Aménagement du territoire: cartographie par images de synThesis d'une métrique réseau", *Mappemonde*, no. 3, p. 37-43, 1996.

[LHO 97] L'HOSTIS A., Image de synThesis pour l'aménagement du territoire: la déformation de l'espace par les réseaux de transport rapide, Thesis, Tours, 1997.

[MAT 96] MATHIS P., "La Stratégie des réseaux de transport dans le grand Ouest", *L'Entreprise Atlantique*, Yves Morvan (ed.), l'Aube, Paris, 1996.

[MUL 79] MÜLLER J.C., "La cartographie d'une métrique non euclidienne: les distances-temps", *L'Espace géographique*, France, vol. 8, no. 3, p. 215-227, 1979.

[PLA 87] PLASSARD H., ROUTHIER J.L., *Sémiologie graphique et évaluation*, ARTUR, Lyon, 1987.

[PLA 91] PLASSARD F., "La Révolution TGV", *TGV et aménagement du territoire*, Paris: Syros/alternatives (TEN), Paris, 1991.

[TOB 61] TOBLER W., *Map transformation of geographic space*, Washington, USA, 1961.

[TOB 63] TOBLER W.R., "Geographic area and map projections", *Geographical review*, USA, vol. 53, p. 59-78, 1963.

[TOB 97] TOBLER W.R., "Visualizing the impact of transport on spatial relations", *Paper for the Western Regional Science Association meeting*, Hawaii, 1997.

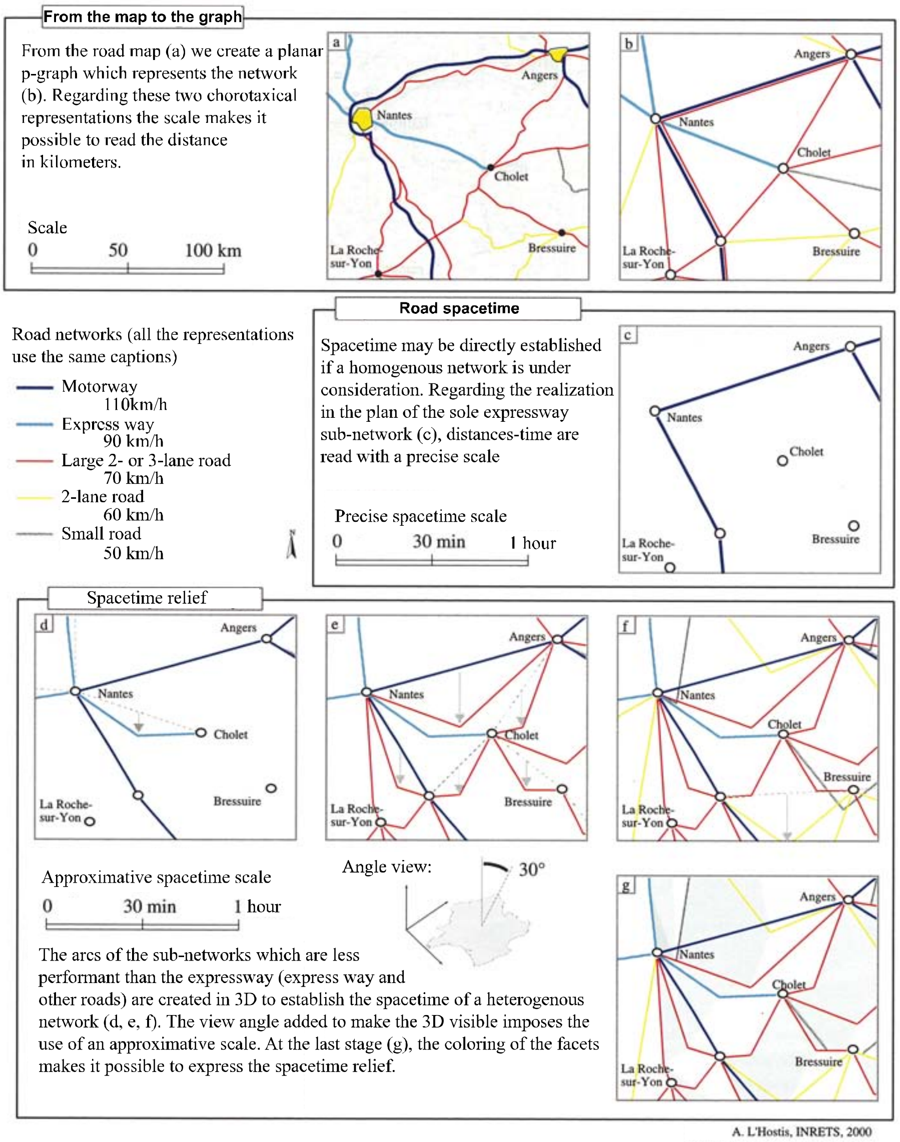

Figure 9.7. *Stages of the construction of a map in spacetime relief*

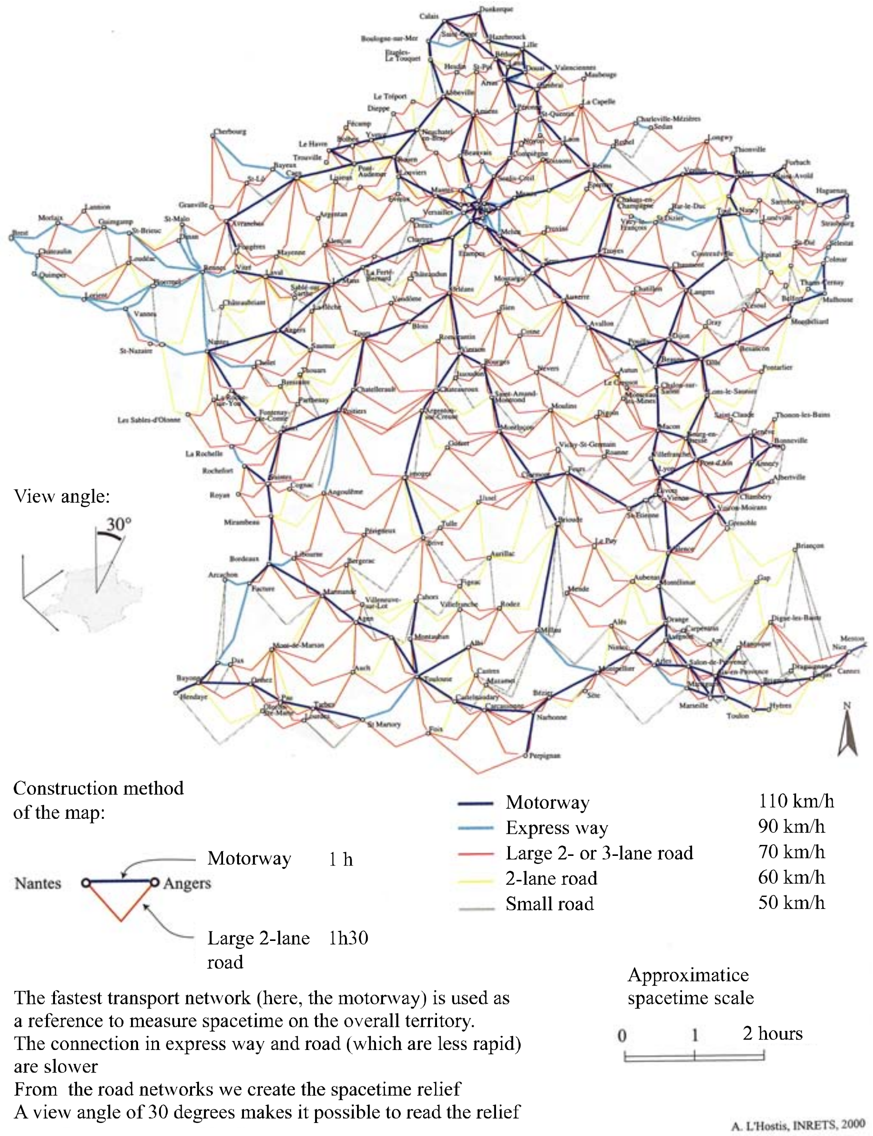

Figure 9.8. *Spacetime relief of the road and motorway*