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Modelling of contact-friction interactions in entangled fibrous materials

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Abstract A finite element model is proposed in this paper to simulate the non linear mechanical behaviour of assemblies of entangled fibres submitted to large transformations. The key point of the model lies in an automatic detection of contact-friction interactions between fibres. In all areas where contact is likely to occur, intermediate geometries are constructed. They provide adapted contact search direction and offer a geometrical support for the building up of contact elements. The fibres are taken into account by an enriched kinematical 3D beam model, formulated in large transformations. Periodic conditions are applied to randomly determined samples of fibrous media. Simulation results for the compression (up to 90 %) of five different random samples, show similar behaviours, and allow the identification of exponents of power laws.

Key words: entangled media, mechanical properties of fibrous assemblies, contact-friction interactions, finite element

INTRODUCTION

Various materials or structures are made of fibres, which may be differently arranged all together, depending on whether they are twisted, weaved, braided, or more generally, simply entangled. Such materials usually present a high deformability due to the possibility for constitutive fibres to move with respect to one another. When submitted to large deformations, those materials often show a complex and non linear behaviour which can be mainly attributed to contact-friction interactions occurring between fibres. The purpose of this paper is to present a model intended to simulate the mechanical behaviour of generally entangled fibrous media submitted to large displacements and deformations, in order to identify their constitutive laws. As application of this model, we shall consider the case of randomly determined samples of entangled media, that could represent materials such as glass wool [1] or other fibre networks [2,3]. Nevertheless, the methodology presented here can be as well applied to more classical types of arrangements such as twisting (metallic wire ropes and textile yarns) or weaving (textile structures and composites) [4]. The first section describes the original part of the model, that is the way contact-friction interactions within the collection of fibres are automatically detected and taken into account. To determine which material particles are predicted to enter into contact, instead of starting from one particle and using the normal to the surface at this particle to search for a corresponding particle on the opposite surface, we build up an intermediate geometry, defined as the average of the contacting geometries, and use the normal directions to this geometry to associate material particles that make up the contact elements. This way, contact-friction interactions are detected and discretized from this intermediate geometry, and not directly from the finite element discretization. The next section is devoted the enriched kinematical 3D beam model used to represent the fibres. The way samples of fibrous media are randomly generated, in such a way that periodic conditions could be easily prescribed

to them is then presented. Finally, we give and discuss simulation results for five different random samples submitted to a 90 % compression. Similar behaviours may be observed for these samples and different exponents for the power laws may be identified, depending on the density range.

AUTOMATIC DETECTION OF CONTACT INTERACTIONS IN A COLLECTION OF FIBRES

1. A priori discretization of the contact problem Contact between fibres should be theoretically considered as a continuous phenomenon. However, due to the discretization of the fibres by finite elements, the discretized surfaces of fibres, even if they are in contact, can not generally conform exactly to each other. For this reason, contact conditions can not be satisfied on continuous areas, but only at some discrete points distributed on the surfaces. From this principle, we choose for our model, to check contact only at some discrete locations, where we assume contact takes place between two material particles. Such a pair of two material particles entering into contact at a given place will be called a *contact element*.

The determination of places where the discrete contact elements have to be created is a part of the process of contact determination. These places do not depend directly on the finite element discretization, that is, they are determined regardless of the positions of nodes or integration points, but according to geometrical criteria. This is the reason why this discretization is described as a priori. Nevertheless, the distance between contact elements, which may be viewed as a discretization size, has to be linked with the mesh sizes of interacting beams.

The goal we assign to this a priori discretization of the contact problem is to determine a set of pairs of material particles, considered as contact elements, distributed all over the collection of fibres, at suitable locations, and with an adapted discretization step.

2. The problem of the contact search direction To form the pairs of material particles predicted to enter into contact at a given place, a direction, called the contact search direction, is used. The determination of this direction is a difficult issue of the global process. In classical contact strategies, the normal to one of the contacting surfaces is usually taken as contact search direction. For the problem we are interested in, such a choice may lead to bad results, especially if the fibres we consider may have high curvatures. Moreover, one argument that might be opposed to this choice of the normal to the surface as contact search direction, is that it leads to an unsymmetrical treatment of contact. If the normals to the surfaces at the two points associated for contact are different, starting from one point or the other will not actually give the same result. These deficiencies may be attributed to the fact that considering the normal to only one surface is not sufficient to predict the geometry of contact, which depends on both surfaces. To overcome this obstacle, we propose to introduce an intermediate geometry.

3. Introduction of an intermediate geometry Because the consideration of only one of the contacting surfaces is too poor to approximate geometrically the actual contact surface, we suggest to build up, in all areas where contact is likely to occur, an intermediate geometry, defined as the average of the two parts of surface assumed to interact. This intermediate geometry provides, by the mean of its normals, contact search directions that are better adapted to the common geometry of contact. The use of the normals to the intermediate geometry enables a symmetrical treatment of contact. This intermediate geometry will also be employed as a geometrical support for the discretization of contact-friction interactions, that is to say, the discrete places where contact conditions are to be checked, will be defined with respect to this geometry rather than with respect to the beam finite elements.

In order to construct the intermediate geometry, a bijection between the two associated parts of surface has to be defined, which may reveal a difficult point. However, as far as we consider interactions between beams, the finding out of the positions of contacts between beams will be performed, in a first step, referring to the center lines of the beams rather than to their surfaces. This means that, in the case of contact between beams, the intermediate geometry will be represented by a part of line instead of a surface. This lineic intermediate geometry is defined, in that case, as the average between two parts of lines, using a simple bijection based on a relative curvilinear abscissa on each part.

4. Determination of proximity zones between beams The first step in the construction of intermediate geometries is to determine which parts of the lines of centroids are close together. To do this, for each pair of beams, and for points regularly distributed of one of the beams with a coarse discretization step, we compute the distance between these points and their corresponding closest point (usually the orthogonal projection) on the opposite beam. The parts comprised between couples of close points form what we call a proximity zone (*Fig. 1*).

Assuming each beam could be described, in a reference configuration by a cylinder, denoted $\Omega_{\xi}^{(i)}$, and defined by :

$$\Omega_{\xi}^{(i)} = \left\{ \boldsymbol{\xi} = (\xi_1, \xi_2, \xi_3) \in \mathbb{R}^3, \xi_1^2 + \xi_2^2 \leq r^{(i)2}, 0 \leq \xi_3 \leq l^{(i)} \right\} \quad (1)$$

where $r^{(i)}$ and $l^{(i)}$ are respectively the radius and the length of the beam (i), we define a proximity zone, denoted $PZ^{(k)}$ between the beams (l) and (m), by :

$$PZ^{(k)} = \{ ([a^{(l)}, b^{(l)}], [a^{(m)}, b^{(m)}]) \subset [0, l^{(l)}] \times [0, l^{(m)}] \} \quad (2)$$

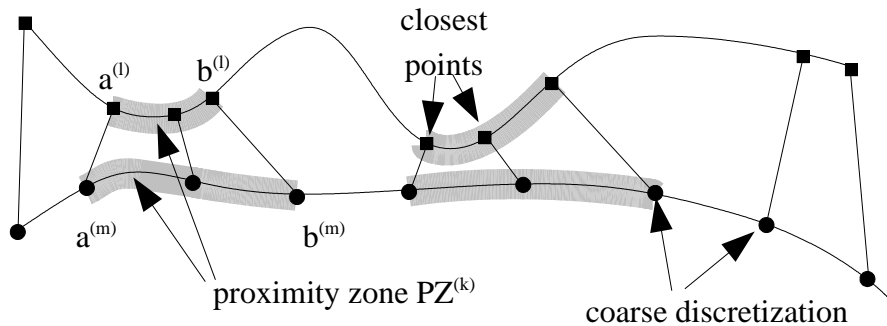


Fig. Number range Fig. Determination of proximity zones

5. Definition of intermediate geometries between beams All points of the intermediate geometry are described by their relative curvilinear abscissa varying from 0 to 1. Their positions are calculated in the following way :

$$\forall s \in [0, 1], \mathbf{x}_{PZ^{(k)}}(s) = \frac{1}{2} \left(\mathbf{x}_{0,t}^{(l)}((1-s)a^{(l)} + sb^{(l)}) + \mathbf{x}_{0,t}^{(m)}((1-s)a^{(m)} + sb^{(m)}) \right), \quad (3)$$

where $\mathbf{x}_{0,t}^{(l)}(s^{(l)})$ is the position of the centroid of the beam (l), at the abscissa $s^{(l)}$ and at time t .

5. Construction of contact elements on intermediate geometries Contact elements are dedicated to point out the two particles of interacting beams that come into contact at a given point of the intermediate geometry. Let \mathbf{x}_c be a point of the intermediate geometry, the contact element at this point should theoretically be defined as

$$E_c(\mathbf{x}_c, t) = (\xi^{(l)}, \xi^{(m)}) \in \Omega_\xi^{(l)} \times \Omega_\xi^{(m)}, \text{ such that : } \mathbf{x}_t^{(l)}(\xi^{(l)}) = \mathbf{x}_t^{(m)}(\xi^{(m)}) = \mathbf{x}_c \quad (4)$$

where $\mathbf{x}_t^{(l)}(\xi^{(l)})$ is the position of the particle $\xi^{(l)}$ at time t . However, the problem of determining which particles come into contact can not be directly solved, and couples of contacting particles can only be determined by successive predictions.

Contact elements are created for a set of discrete points regularly distributed on the intermediate geometry. This set of discrete points may be viewed as a mesh for the contact problem, denoted $\mathbf{M}_c(PZ^k)$, and defined by

$$\mathbf{M}_c(PZ^k) = \left\{ \mathbf{x}_{PZ^k}(s_i); s_i = ih, i \in \mathbb{N}, 0 \leq i \leq \frac{1}{h} \right\}. \quad (5)$$

The discretization step h , used in the above definition, should be related to the mesh size of the beams. If quadratic elements are used for the beams, it seems to be convenient to take as discretization step for the contact, the half of the smallest element size of the two interacting beams.

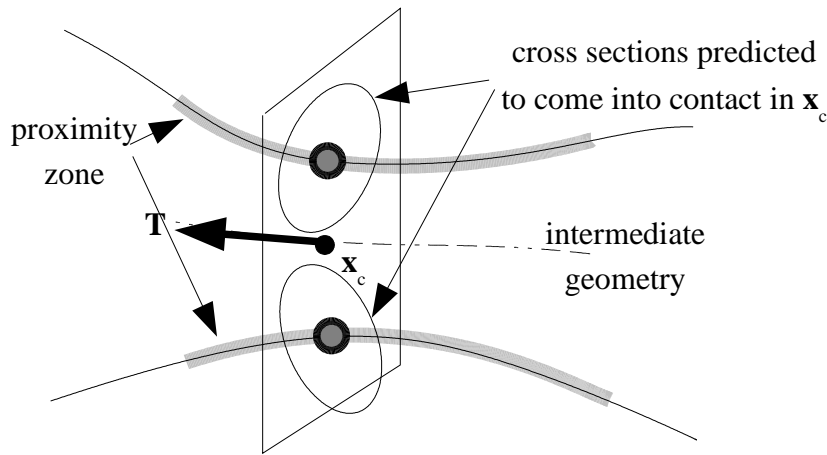


Fig. Number range Fig. Determination of cross sections predicted to come into contact

The determination of the couples of particles predicted to come into contact follows two steps. First, using the normal directions to the intermediate geometry, we select beam cross-sections which are likely to contact each other. These cross-sections are positioned at the intersections between normal planes to the intermediate geometry and the lines of centroids of the beams (see Fig. 2). Then, particles candidate to contact are chosen on the outline of these cross-sections, using the direction between the two centroids (Fig. 3).

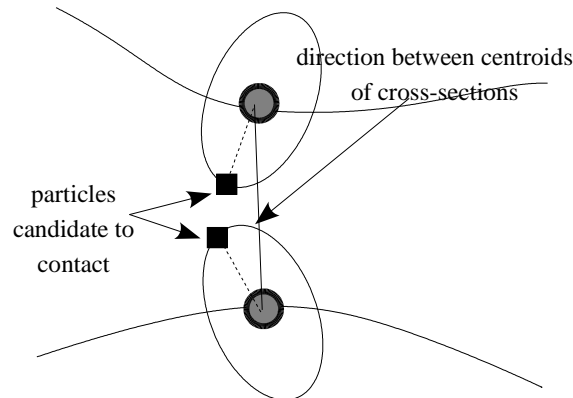


Fig. Number range Fig. Determination of particles candidate to contact on the outline of cross sections

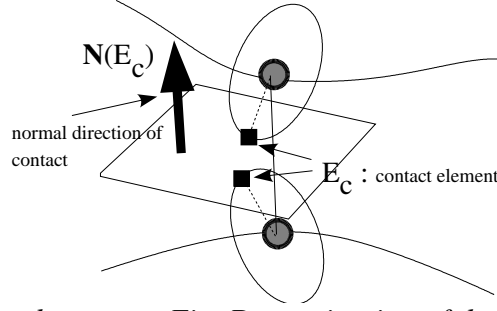


Fig. Number range Fig. Determination of the normal direction of contact

6. Determination of the normal direction of contact A normal direction of contact has to be attached to each element in order to set linearized kinematical contact conditions. These conditions prevent the particles of the contact element from going through a plane defined by its normal vector $\mathbf{N}(E_c)$, and may be expressed, for a contact element E_c , as follows :

$$gap(E_c) = (\mathbf{x}_t^{(l)}(\xi^{(l)}) - \mathbf{x}_t^{(m)}(\xi^{(m)}), \mathbf{N}(E_c)) \geq 0 \quad (6)$$

This normal direction needs to be carefully determined, particularly near crossing between fibres. Without some particular precautions, if the fictitious plane placed between the particles to prevent penetrations is not well oriented, there is a risk that fibres go through each other. To avoid this problem in the neighbourhood of crossings, it is suitable to take for this direction, the vector product between the two tangents to the lines of centroids of the beams.

MODELS FOR CONTACT-FRICTION INTERACTIONS

1. Regularized penalty method for pure contact The mechanical counterpart of the kinematical measure of the gap between particles, is the normal reaction that is generated whenever a penetration is detected. To stabilize the algorithm, a quadratic regularization is considered for small penetrations. It helps for contact elements whose status oscillates, by ensuring a continuity of the derivative of the normal reaction at the origin. A depth of regularization, denoted δ_{reg} , is defined, and the normal reactions exerted between the particles of the contact element is calculated as follows :

$$\text{if } gap(E_c) \leq 0; R_N(E_c) = \mathbf{0}$$

$$\text{if } 0 \leq gap(E_c) \leq \delta_{reg}; R_N(E_c) = \frac{k_c}{2\delta_{reg}} gap(E_c)^2 \quad (7)$$

$$\text{if } gap(E_c) > \delta_{reg}; R_N(E_c) = k_c \left(gap(E_c) - \frac{\delta_{reg}}{2} \right)$$

Adaptation of the penalty coefficient The penetration allowed by the penalty method is linked to the normal force through the penalty coefficient k_c . During the simulation, the normal forces developed in the different contact zones may vary largely, and therefore very disparate penetrations might appear. Some of them could even be greater than the radii of the fibres, which would mean the fibres go through one another at this place. To avoid these problems, we have chosen to control the maximum penetration for each contact zone. To this aim, a different penalty coefficient is attached to each proximity zone, and regularly updated so that the maximum penetration for this zone do not exceed a given value.

2. Regularized Coulomb's law for friction A regularized Coulomb's model, allowing a small reversible

relative displacement before gross sliding occurs is used. One difficulty with this model is that it considers history Dependant parameters associated with contact elements, whereas these elements do not have any continuity in time. This requires the informations related to these parameters could be attached to the material configuration and locally recovered any time contact elements are regenerated.

3D BEAM MODEL WITH DEFORMABLE SECTIONS

1. Introduction A large amount of beam models are available in the literature. Among them, models considering rigid sections, with six degrees of freedom per section, are commonly used. The handling of large rotations required by these models raise nevertheless some difficulties in their development. The search of model that may be formulated more easily lead us to develop and use an enriched kinematical beam model taking into account some deformations of the sections.

2. Kinematical model To set the kinematical model, we express the placement of each particle of the beam as a first order Taylor expansion with respect to its section coordinates (ξ_1, ξ_2) , as follows :

$$\mathbf{x}_t(\xi_1, \xi_2, \xi_3) = \mathbf{x}_t(\mathbf{0}, \mathbf{0}, \xi_3) + \xi_1 \partial_1 \mathbf{x}_t(\mathbf{0}, \mathbf{0}, \xi_3) + \xi_2 \partial_2 \mathbf{x}_t(\mathbf{0}, \mathbf{0}, \xi_3) + o(\xi_1, \xi_2) \quad (8)$$

According to this first order expansion, the placement of any particle may be expressed by the mean of three fields defined on the line of centroids of the beam :

$$\mathbf{x}_t(\xi_1, \xi_2, \xi_3) = \mathbf{x}_{0,t}(\xi_3) + \xi_1 \mathbf{g}_{1,t}(\xi_3) + \xi_2 \mathbf{g}_{2,t}(\xi_3) \quad (9)$$

where $\mathbf{x}_{0,t}$ is the placement of the cross-section center, and $\mathbf{g}_{1,t}$ and $\mathbf{g}_{2,t}$ are two vectors determining the shape and the orientation of the cross-section. The displacement field at time t, denoted \mathbf{u}_t , may be expressed in a similar way as :

$$\mathbf{u}_t(\xi_1, \xi_2, \xi_3) = \mathbf{u}_{0,t}(\xi_3) + \xi_1 \mathbf{h}_{1,t}(\xi_3) + \xi_2 \mathbf{h}_{2,t}(\xi_3) \quad (10)$$

where $\mathbf{u}_{0,t}$ is the displacement of the cross-section center, and $\mathbf{h}_{1,t}$ and $\mathbf{h}_{2,t}$ represent the variations of the section vectors. The main advantage of this model compared to classical model with rigid sections, is that no constraint needs to be put on the kinematical fields, so that these displacement fields and their variations are of the same kind, which simplifies greatly the formulation of the principle of virtual work in large transformations compared with the one derived from rigid sections model.

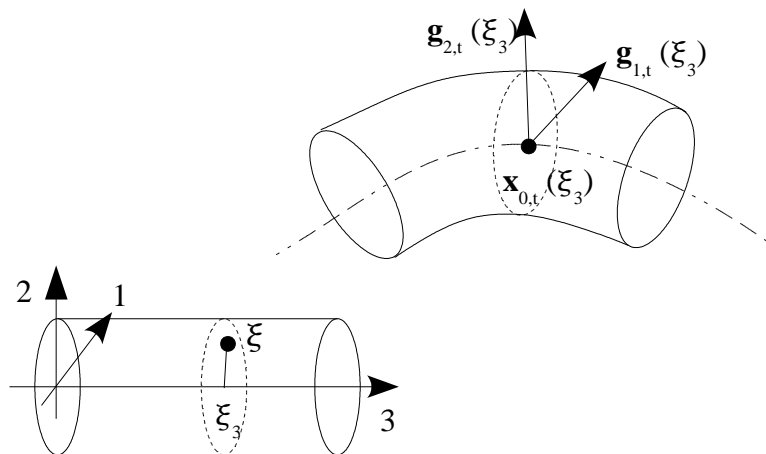


Fig. Number range Fig. Enriched kinematical beam model

The proposed beam model comprises nine degrees of freedom for each section. According to this model,

sections remain plane, but may have planar deformations. In particular, this means this model is able to account for the reduction of section due to the elongation of the beam (through Poisson's coefficient).

2. Adapted rheological model If we decompose the Green-Lagrange strain tensor with respect to the section coordinates, we obtain a part which remains constant with respect to these coordinates, and higher order terms accounting for linear and quadratic variations of the strain tensor with respect to these coordinates. Only the constant part of the tensor is really 3-dimensional, with a priori non-zero components. For higher order terms of the strain tensor expansion, some components are automatically zero, which implies a real 3D constitutive law can not be used with these terms. For this reason, two rheological models are used. For terms constant in the section, we use a classical 3-dimensional Hooke's law. For higher order terms, to avoid locking effects linked to the fact they can not represent all kind of deformations, we take a zero Poisson's coefficient for the Hooke's law.

GLOBAL SOLUTION ALGORITHM

Several non linearities are embedded in the global problem, which require adapted algorithms. The process of detection and taking into account of contact-friction interactions is a predictive process, which depends on the solution. This makes necessary to iterate on this process at two levels. A first level loop is dedicated to iterations on the determination of contact elements, while an inner loop iterates on the normal contact directions used to measure gaps. These two quantities being fixed, a Newton algorithm is employed to solve other nonlinearities.

GENERATION OF RANDOM SAMPLES OF ENTANGLED MEDIA

The generation of random samples for the simulation of entangled media is a delicate issue. The samples have to be compatible with the prescription of periodic conditions, in order to reduce the size of the problem. To this aim, to generate a sample, we first build up a long random line, with a given mean curvature. Then, we define the dimensions of the box that will contain the sample, and place the random line inside this box, starting from one of its faces. Whenever the line goes through another face, we cut it, and consider the part inside the box as a new fibre. To create the next fibre, we place the rest of the line, on the opposite face, at the same relative position with respect to the face. This way, each extremity of a fibre corresponds to the extremity of another fibre, at the same position on the opposite face, and with the same orientation with respect to it. Therefore, periodic conditions may be easily prescribed to the fibre extremities. Such a construction corresponds to the assumption that the fibres have an infinite length.

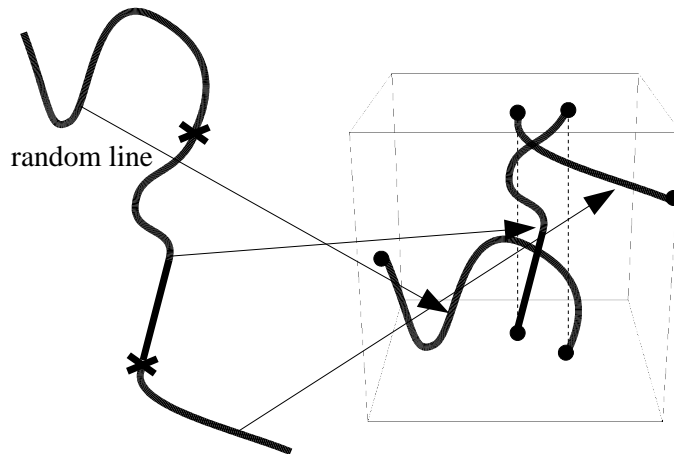


Fig. Number range Fig. Generation of random periodic samples

BOUNDARY CONDITIONS

1. Periodic boundary conditions The way samples are generated allows to apply easily periodic conditions to the extremities of fibres, since each extremity of a fibre corresponds to the extremity of another fibre, on the opposite face, at the same position and with the same orientation with respect to the face. Displacements in the plane of the face, and rotations (variations of the section vectors) at the end of a fibre are prescribed to remain equal to those at the beginning of the next fibre.

2. Conditions with the faces of the sample Two kinds of conditions are considered with the faces of the sample. First, extremities of fibres are prescribed to remain on the face they belong to, by freezing their displacements in the normal direction to the face. Displacements of the extremities in the plane of the faces are kept free. We add moreover contact conditions between fibres and faces in order they can not go outside the sample.

3. Deformation of the sample The loading is applied by prescribing the positions of the faces of the sample at any time.

NUMERICAL RESULTS

1. Presentation of the studied cases To test our global model, five different randomly determined samples have been used. All the samples have the same nominal characteristics, with fibre distributions derived from five different initial random lines. The samples have initially a side length equal to 1, and fibres have a radius of 0.166, and a mean radius of curvature equal to 0.27. The initial volume fraction of fibres in all samples is 2 %, which gives a number of fibres varying from 57 to 71, depending on the sample. The global meshes contain about 1800 nodes. At the end of the simulation, the number of contact elements is about 2000. A friction coefficient of 0.2 has been considered in the simulations.

2. Results analysis The samples have been compressed until the computations could not converge any more. Depending on the cases, a vertical compression from 90 % to 93 % could be reached, corresponding to final volume fractions of fibres exceeding 0.28. The views of the deformed meshes (*Fig. 9*) show the high densification of the assemblies at the end of the computation.

Interesting informations are given by the load/density curves (*Fig. 7*). Three different stages are observed, especially on the curves in logarithmic scales. The beginning of the loading, for a density between 2% and 3%, seems to correspond to a setting up of the sample, accompanied by a rather fast increasing of the number of contacts (*Fig. 8*). After that, a stable behaviour takes place, for volumic

fractions from 5 % to 15 %. This behaviour is characterized by a loading force proportional to the square of the density, while the number of contacts varies as the square root of the density. The results are not in accordance with the predictions of the van Wyk's theory [8], which gives an exponent equal to 3. For higher densities, the exponent of the power law followed by the load increases and reaches values near from 4. The evolution of the number of contacts, except at the beginning, remains stable and follows the square root of the density. Here again, the van Wyk theory would have predicted a different evolution, function of the square of the density.

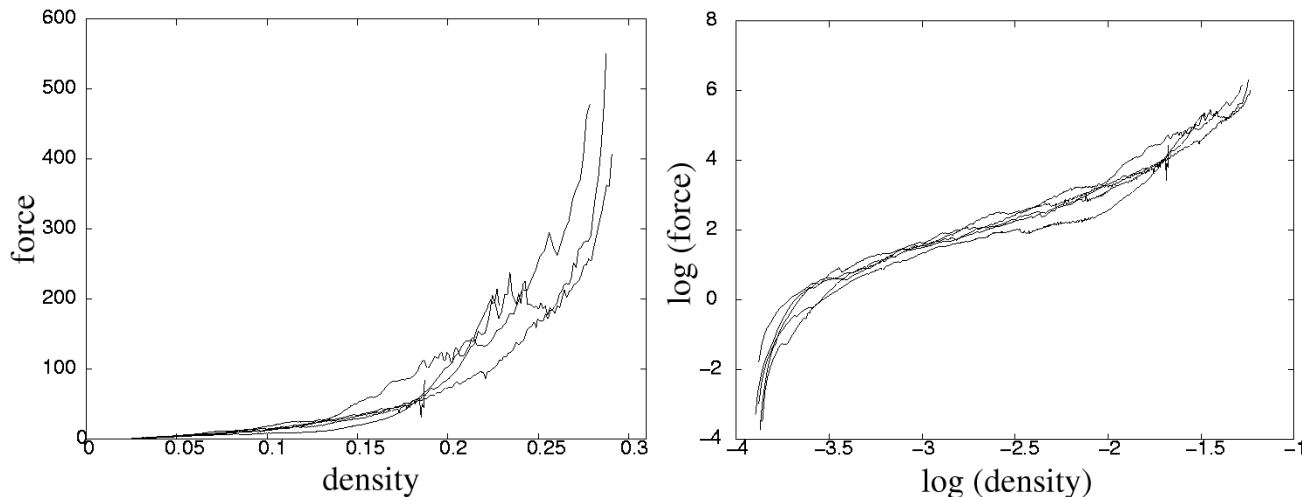


Fig. Number range Fig. Vertical load versus density for the five samples in natural scale (left) and logarithmic (right)

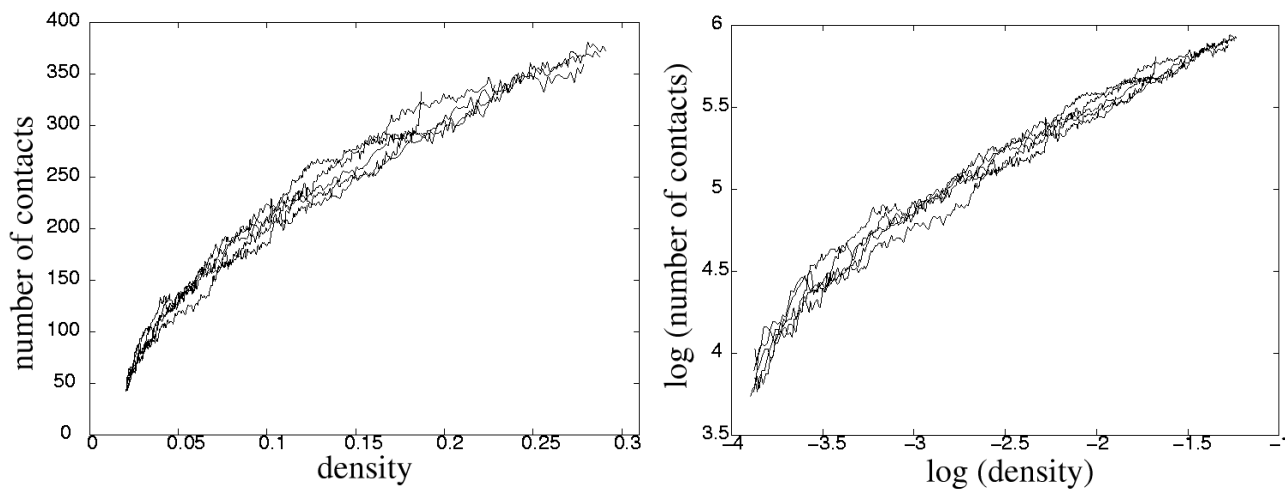


Fig. Number range Fig. Number of contacts versus density for the five samples in natural scale (left) and logarithmic (right)

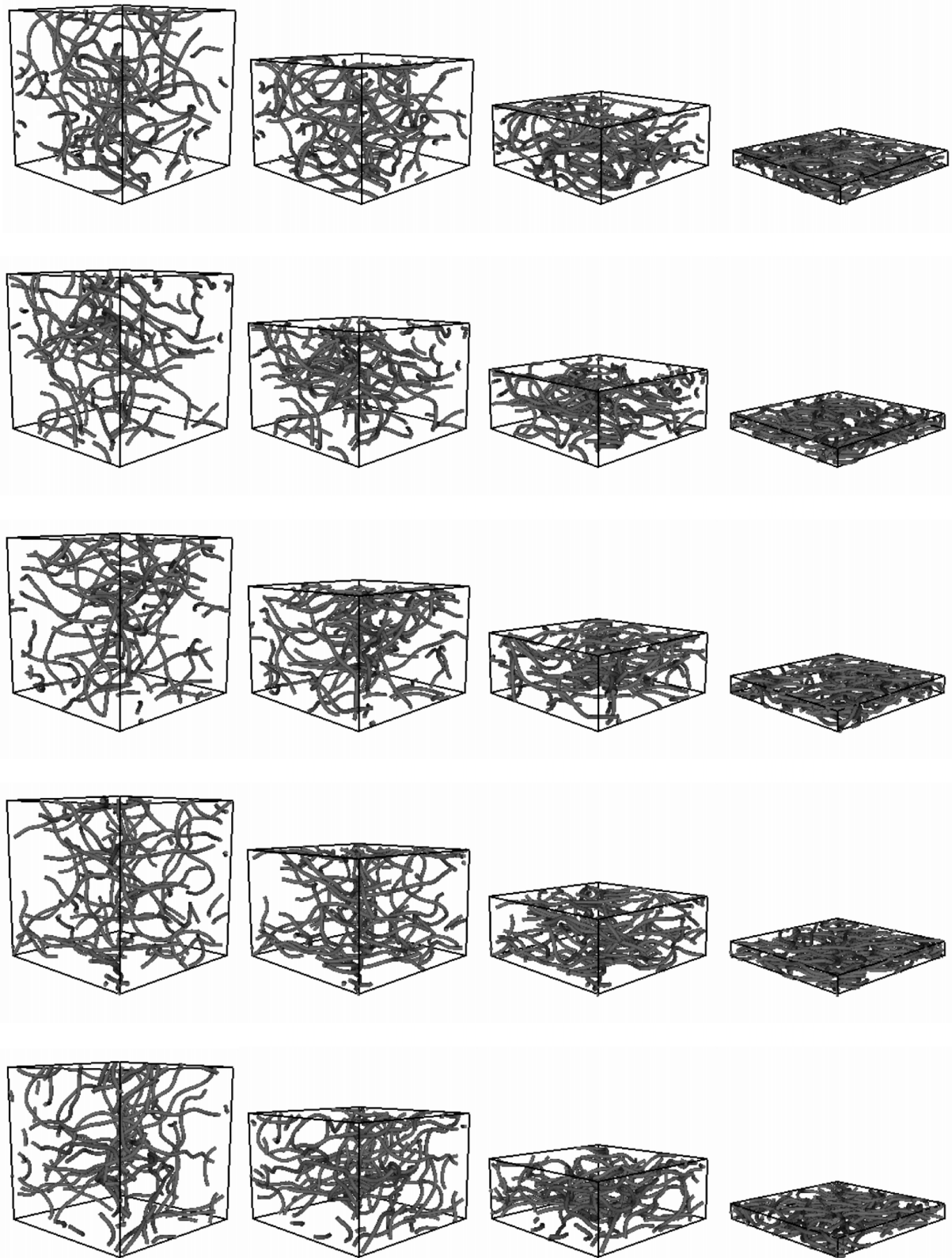


Fig. Number range Fig. Meshes of the five randomly determined samples at respectively 0 %, 30 %, 60 % and 90 % compression

CONCLUSION

Numerical results show the capabilities of the global model to handle complex configurations of fibrous media under large deformations. The automatic detection of the numerous contacts appearing in the fibre assemblies proves to be both reliable and efficient. Investigations are still needed to understand the discrepancies with the van Wyk's theory. The potentialities of the model may be used in a wide range of applications involving fibrous materials submitted to different loadings, in order to understand and identify the mechanisms responsible for their global nonlinear behaviour.

REFERENCES

- [1] M. Baudequin, G. Ryschenkow and S. Roux, *Non-linear elastic behaviour of light fibrous materials*, The European Physical Journal B, 12, (1999), 157-162.
- [2] S. Heyden, *Network modelling for the evaluation of mechanical properties of cellulose fibre fluff*, Doctoral Thesis,, Lund University, Sweden, (2000).
- [3] S. Toll and J.-A. E. Manson, *Elastic compression of a fiber network*, Journal of Applied Mechanics, 62, (1995), 223-226.
- [4] D. Durville, *Numerical simulation of the mechanical behaviour of textile structures at a mesoscopic scale*, Structural Membranes 2003, in *Textile composites and inflatable structures*, E. Oñate and B. Kröplin (Eds.), CIMNE, Barcelona, Spain, (2003), 310-315.
- [5] T.W. McDevitt and T. A. Laursen, *A mortar-finite element formulation for frictional contact problems*, Int. J. Numer. Methods Engng., 48, (2000) 1525-1547.
- [6] P. Wriggers and G. Zavarise, *On contact between three-dimensional beams undergoing large deflection*, Communications in Numerical Methods in Engineering, 13, (1997), 429-438.
- [7] G. Zavarise and P. Wriggers, *Contact with friction between beams in 3-D space*, Int. J. Numer. Methods Engng., 49, (2000), 977-1006
- [8] C.M. van Wyk, *Note on the compressibility of wool*, J. Textile Institute, 37, (1946), 285-292