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## Spin up/down in linearly stratified fluid

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### Abstract :

*Vertical transport of horizontal momentum due to baroclinic instabilities is investigated in a rotating stratified fluid. The experiments were carried out in the large rotating tank of the Coriolis-LEGI laboratory in Grenoble with flat bottom and linear density stratification. The mean flow was generated by increasing (spin up) or decreasing (spin down) the rotation rate of the platform. The velocity fields were measured by PIV (Particle Image Velocimetry) technique. The dimensionless parameters that are important in determining the observed response are the Burger number and the Rossby number.*

*Our experimental study reveals two very different behaviors. For  $O(1)$  Burger numbers, the bulk of the flow remains axisymmetric with a very slow decay. We have checked that the decay is well described by the usual vertical diffusion law, except in a region near the boundaries. The mismatch pertains to the presence of Ekman pumping/suction, but this mechanism is confined by stratification to a relatively shallow layer.*

*For Burger numbers  $\ll O(1)$ , baroclinic instabilities are observed. The increased vertical momentum transport leads to a much faster spin than the previous case. Neither the diffusion nor the Ekman pumping/suction can be considered responsible for this enhanced transport of momentum and we interpret it as the effect of baroclinic instabilities. Eady's theory is used to analyse the quasi-geostrophic baroclinic stability problem and a model is proposed to estimate the azimuthal velocity decay. The results are presented and discussed.*

**Key-words : spin up/down; rotating flow; stratified flow**

### 1 Introduction

The spin of a fluid in a container represents an important process as it describes an adjustment mechanism which arises frequently in both the atmosphere and oceans. The modern era of spin up studies was initiated by Greenspan & Howard (1963), who considered the effect on a flow of small changes in the rotation rate of its container. They showed that the spin up takes place in three time scales corresponding to three distinct physical mechanisms: (1) the Ekman boundary layers development as a result of the stresses on the rigid horizontal boundaries; (2) the spin up of the fluid caused by a secondary, meridional circulation due to an imbalance between viscous, centrifugal and pressure forces; (3) the viscous decay of any residual motion.

The literature has shown that the stratification adds a wealth of flow behaviors and instabilities. The continuous model restricts the role of viscosity to boundary layer regions adjacent only to solid surfaces and the interior core region can be regarded as essentially inviscid in character unless the effect of viscosity permeates the entire fluid, which is not plausible if the cinematic viscosity is  $O(10^{-6}) m^2 s^{-1}$ , as the fluid usually considered.

A theoretical analysis of the linear spin-up process for a stably stratified fluid was presented by Holton (1965), which proved to be qualitatively correct but had some inaccuracies in the treatment of the sidewall layer, and by Pedlosky (1967), but he incorrectly predicted that spin up would then be achieved by a diffusive mechanism. The correct description of the linear spin up process was later presented by Walin (1969), who predicted that the effect of a stable stratification was to restrict the recirculation of the fluid from the Ekman layers to a localized region near the horizontal boundaries.

Buzyna & Veronis (1971), Saunders & Beardsley (1975), and Lee (1975) performed a sequence of experiments in a cylinder filled with a linearly stratified fluid. These three studies clearly showed that the decay rate was faster than predicted by the linearized theory of Walin. Later experiments by Greenspan (1980) confirmed the intricate nature of stratified spin up.

Spin down received less attention in the past, but it has been long understood that the resulting flow may be subjected to sidewall instabilities due to an imbalance between centrifugal and pressure gradient forces. The development of this type of instabilities was discussed by Maxworthy (1971). Later, computations and experiments by Neitzel & Davis (1981) and Mathis & Neitzel (1985) verified that such centrifugal instabilities arise, leading to the formation of an array of Taylor-Görtler vortices along the sidewall. The most recent treatments of this problem are by Lopez (1996) and Lopez & Weidman (1996). It appears from their studies that spin down is a problem with various instability mechanisms and a complex dynamics.

The spin problem is clearly central to an understanding of the dissipative mechanisms for large scale motions, where rotational effects are dominant. Moreover, the stratification causes vertical shear in the azimuthal flow: the baroclinic instability may taps the available potential energy of the sloping isopycnal surfaces, thereby indirectly the kinetic energy of the mean flow. The formation of large scale eddies through the development of the instabilities thus provides an additional mechanism for the transport of angular momentum from solid boundaries to the bulk of the fluid.

The onset of the baroclinic instabilities during spin up has been studied by Smirnov et al. (2005), but they did not address its effect on the azimuthal velocity decay. The main goal of our work is thus to deepen the role of the baroclinic instability as source of momentum transport and to model the resulting dissipation on the relative flow.

## 2 Experimental Procedure

A series of spin-up and spin-down experiments, described in fig 1, were performed in the large cylindrical rotating tank, 13 m diameter, of the Coriolis-LEGI laboratory.

EXP	$T_f$	$\Omega_f$	$\Delta\Omega$	$R_o$	$B_u$
UP 1	50	0.126	$+\Omega_f/12$	0.08	0.06
DW 1	50	0.126	$-\Omega_f/12$	0.08	0.06
UP 2	50	0.126	$+\Omega_f/6$	0.17	0.06
DW 2	50	0.126	$-\Omega_f/6$	0.17	0.06
UP 3	100	0.063	$+\Omega_f/6$	0.17	0.26
DW 3	100	0.063	$-\Omega_f/6$	0.17	0.26
UP 4	200	0.031	$+\Omega_f/3$	0.32	1.16
DW 4	200	0.031	$-\Omega_f/3$	0.32	1.16

Figure 1: List of performed experimental runs.  $T_f$  is the final (reference) rotation period of the tank [s],  $\Omega_f$  is the final rotation rate of the tank [ $rad\ s^{-1}$ ],  $\Delta\Omega$  is the change in rotation rate of the tank [ $rad\ s^{-1}$ ],  $R_o$  and  $B_u$  are the Rossby and Burger number [-].

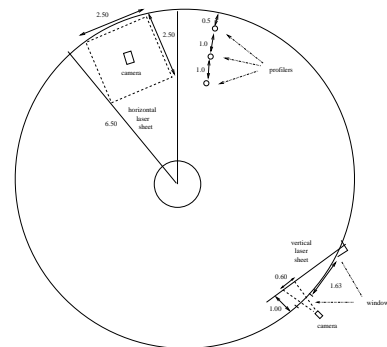


Figure 2: Experimental set-up, top view. The change in the rotation rate of the tank was performed during a time interval of approximately five second and may be considered as impulsive.

The tank was filled while rotating with linearly stratified fluid. The stratification was characterized by the buoyancy frequency  $N$ , defined by  $N^2 \equiv (g/D)(\Delta\rho/\rho_0) \simeq 0.49\ s^{-2}$ , with  $g \equiv |\vec{g}|$  the gravity acceleration,  $D = 60\ cm$  the total depth of the fluid layer,  $\rho$  and  $\rho_0$  the background and reference densities. The buoyancy force on a small fluid element displaced from equilibrium is proportional to  $N^2$ . When  $N$  is large, the buoyancy force is strong and

tends to inhibit motion parallel to  $\vec{g}$ . The Coriolis force, by contrast, tends to make motions transverse to  $\vec{\Omega}$  vertically uniform. When  $\vec{\Omega}$  and  $\vec{g}$  are parallel, as in our case, the two forces compete and their relative importance is measured by the Burger number.

In order to avoid spurious perturbations caused by shear stress exerted by the air on the fluid surface during the filling, the tank was covered with a floating foam canopy, removed just before the experiments. The fluid was seeded with light-scattering particles as to use PIV (Particle Image Velocimetry) system and the flow evolution was monitored for about 100 rotation periods. There were two laser sheets, a horizontal one,  $2.5\text{ m} \times 2.5\text{ m}$ , and a vertical one,  $0.6\text{ m} \times 0.6\text{ m}$ , as sketched in fig. 2. The horizontal laser sheet could scan seven different vertical positions in the bulk of the fluid. Images were taken by a digital CCD camera with spatial resolution  $1024\text{ pixels} \times 1024\text{ pixels}$ . Each velocity field was obtained from a burst of four images, allowing a choice of an optimum time interval for the image cross-correlation. Such bursts were repeated every  $5\text{ s}$  at each horizontal level providing a measurement periodicity at a given height of  $35\text{ s}$ . A similar procedure was used for the fixed vertical laser sheet.

### 3 Experimental Results

Following Smirnov et al., the nature of the flow considered depends on six dimensional parameters, i.e., the rotation rate,  $\Omega$ ; the change in the rotation rate,  $\Delta\Omega$ ; the buoyancy frequency,  $N$ ; the horizontal length scale,  $L$ ; the depth of the fluid layer,  $D$ ; the kinematic viscosity,  $\nu$ . Using dimensional analysis, the problem depends on four dimensionless parameters which we take as the Rossby number,  $R_o \equiv \Delta\Omega/\Omega$ ; the Burger number,  $B_u \equiv (ND/fL)^2$ , with  $f \equiv 2\Omega$  the Coriolis parameter; the (vertical) Ekman number,  $E_v \equiv \nu/(\Omega D^2)$ ; the aspect ratio  $\delta \equiv D/L$ . Recognizing that the fluid depth was kept constant and the Ekman number was very small,  $E_v = O(10^{-4})$ , the conditions for the flow development were primarily determined by the parameters  $R_o$  and  $B_u$ .

We chose to investigate the experiments UP-DW 1 and UP-DW 4, characterized respectively by  $B_u = 0.06$  and  $B_u = 1.16$ . The temporal evolution of the azimuthal velocity<sup>1</sup> in the bulk of the fluid is shown in Figs 3-4. In the exp. UP 4 the velocity vector field is characterized by circular streamlines quite perfectly oriented along the azimuthal direction for all the duration of the acquisition. The effect of this behavior is clear in Fig. 3, black line: the stability of

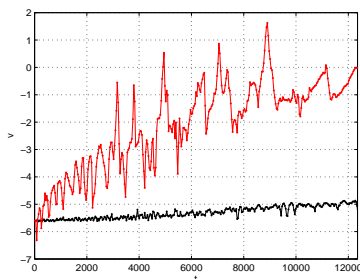


Figure 3: (left) Spin up - azimuthal velocity [ $\text{cm s}^{-1}$ ] temporal evolution [s]: red = UP 1 black = UP 4 - averaged in a small box at  $r = 535\text{ cm}$   $h = 40\text{ cm}$ .

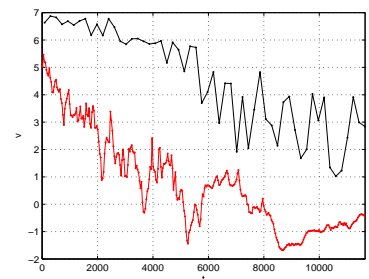


Figure 4: (right) Spin down - azimuthal velocity [ $\text{cm s}^{-1}$ ] temporal evolution [s]: red = UP 1 black = UP 4 - averaged in a small box at  $r = 535\text{ cm}$   $h = 40\text{ cm}$ .

the flow implies a slow velocity decay with fluctuations of the order of  $10^{-1}\text{ cm s}^{-1}$ . In the exp. UP 1 the velocity vector field shows two different regimes: at the outset of the exp. just a relative small discrepancy is evident by respect to the previous case, and the streamlines preserve a circular shape. In the temporal evolution - Fig. 3, red line - it corresponds to a series of growing oscillations, quite regular in period, superimposed to a faster decay trend than the exp. UP 4. During the second stage, after about 55 reference rotation periods ( $t \simeq 2750\text{ s}$ ), a drastic change occurs in the flow development, characterized by nonaxisymmetric disturbances which

can even lead to local reversal flow. The result on the azimuthal velocity temporal evolution consists in wider and lesser regular oscillations, superimposed to a slower decay trend.

It's therefore clear that the system has experienced a more rapid transport of momentum from the boundaries to the bulk of the fluid in the exp. UP 1 than in UP 4.

During spin down the fluid motion was perturbed by sidewall instabilities, while it was always centrifugally stable in the case of spin up. Fig. 4, black line, shows that, even if the fluid was baroclinically stable, the system has experienced a faster decay. In fact, the effect of the centrifugal instabilities was to decelerate the fluid more rapidly by extracting energy from the mean flow. We decided to not linger over it too long since this topic is beyond our scope.

The exp.s UP 1 and UP 4 are characterized by a different change in the rotation rate of the tank, which means different  $R_o$  numbers, and a different reference rotation period, which implies different  $B_u$  numbers. In both the exp.s  $R_o$  is relatively small and, since the effects of a weak nonlinearity were considered modest<sup>2</sup>, we supposed to be responsible for this discrepancy in the flow development the Burger number.

First, we decided to investigate the mechanism of momentum transport in absence of baroclinic instabilities. Since the baroclinic instability requires  $B_u < O(1)$ , we considered the exp.s UP 4 ( $B_u = 1.16$ ). We used the finite differences method of solving the vertical diffusion equation for the azimuthal velocity of the flow, with no-slip condition on the bottom and no-flux condition on the free surface. Fig. 5 shows a comparison between experimental and numerical

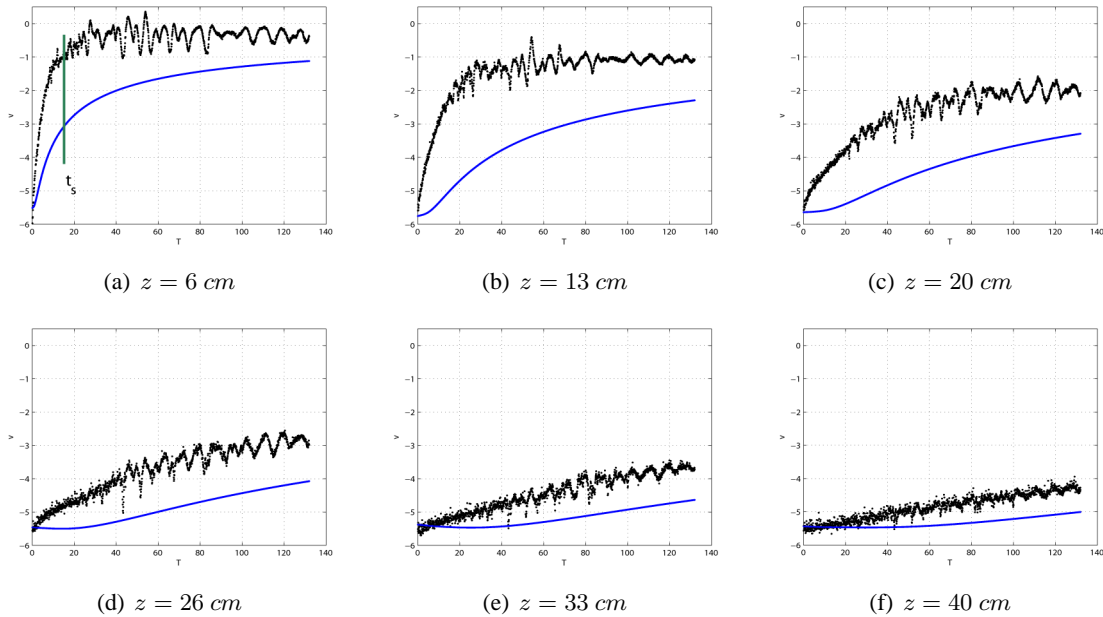


Figure 5: Spin up:  $B_u = 1.16$   $R_o = 0.32$  - azimuthal velocity [ $cm s^{-1}$ ]: black = exp. blue = num. - temporal evolution ( $T$  is the reference rotation period) at different vertical locations.

results for the azimuthal velocity at different vertical locations. At lower levels - fig.s 5(a), 5(b) - the exp. velocity decays initially at a faster rate than the model and as the distance from the bottom increases the decay rate weakens progressively - fig.s 5(c), 5(d). This result was interpreted as showing the effect of the confinement of the meridional circulation by stratification, in accordance with the spatial structure of the streamfunction corresponding to the secondary circulation velocity field in Buzyna & Veronis. Later in time, no further Ekman pumping occurs because there is no difference in the local angular velocity between Ekman layer and adjacent

<sup>2</sup>A review on this topic is available in Duck & Foster (2001)

fluid: the decay rate of the residual motion is then well expressed by viscous diffusion since the two curves show the same slope - fig.s 5(a), 5(b), 5(c), 5(d). Our analysis is in agreement with the work of Walin, which recognized that the effect of the meridional circulation, in stratified fluid, is to bring only a partial spin up of the interior, while the total adjustment to the final state of solid body rotation is accomplished by viscous diffusion. The time  $t_s$  required for the partial spin up is  $t_s \simeq r/(\Omega_f \nu S^2)^{1/2}$ , where  $r$  is the radius of the tank and  $S \equiv N/f$ : using the parameters of UP 4 we obtain  $t_s \simeq 16,5 T_f$  - fig. 5(a). In the bulk of the fluid - fig.s 5(e), 5(f) -, where the secondary circulation doesn't penetrate, the exp. velocity decays faster than the model, according with the work of Hyun et al. (1981): the non-uniform spin up in stratified fluid produces azimuthal velocity gradients in the interior, which introduce viscous diffusion sooner than anticipated by Walin. A qualitative comparison between our exp. case ( $SD/L \simeq 1.08$ ) and the num. results of Hyun et al. ( $SD/L \simeq 1.03$ ) shows a good agreement.

Neither the diffusion nor the secondary circulation can be considered responsible for the enhanced transport of momentum experienced in the case of low Burger number, therefore the effect of the instabilities was to decelerate the relative flow more rapidly.

Next, we considered the exp. UP 1 ( $B_u = 0.06$ ). We focussed on the growing oscillations, due to the onset of the instabilities, developed just after the change in the rotation rate of the tank. In accordance with the normal mode analysis, the perturbations were interpreted as wave-like. In order to estimate the wavelength of the instability, the azimuthal velocity

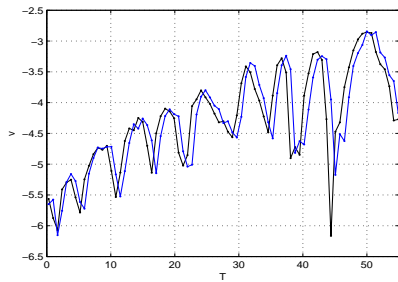
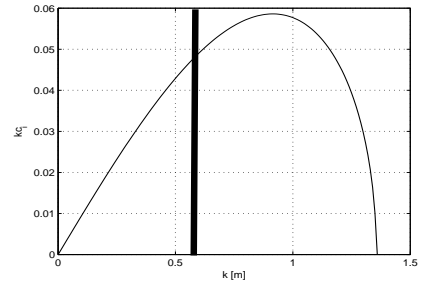


Figure 6: (left) Spin up - temporal evolution of azimuthal velocity [ $cm s^{-1}$ ] averaged in two boxes along the same streamline just after the change in the rotation rate of the tank.

Figure 7: (right) The growth rate,  $kc_i$ , for the most unstable Eady mode. The solid vertical line is in correspondence of the experimental value  $k_{exp} \simeq 0.55 m^{-1}$ .



temporal profile was averaged in small boxes along the same circular streamline - fig. 6. In agreement with the relative velocity of the flow and its radial profile, we interpreted the minima (less negative points) as the velocity of the mean flow, while the maxima as the effect of the superimposed velocity of the baroclinic waves. Knowing the frequency of the signal as well as the time spent by the perturbation to move between the two boxes, we estimated the wavelength of the instability. A comparison with Eady's model shows a fine agreement - fig. 7.

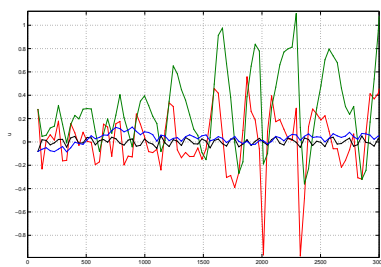
We supposed that the faster velocity decay experienced in the case of low Burger number could be connected to the adjustment of the isopycnal surfaces due to a conversion of available potential energy in kinetic energy of the perturbations: an important role should be played by the radial velocity associated to this process.

Using Boussinesq approximation in Navier-Stokes eq. for an inviscid, incompressible, stratified fluid, we proposed a simple model for the rate of change of the azimuthal velocity of the flow, averaged along the dimensions of the system:

$$\frac{\partial}{\partial t}(\overline{\rho u_\theta}) \simeq 2\Omega_f \overline{\rho v_r} \quad (1)$$

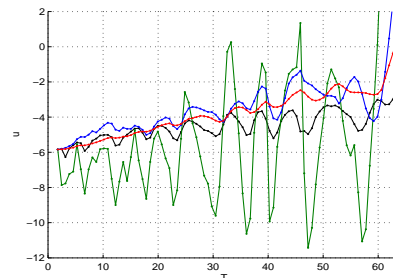
where  $u_\theta$  and  $u_r$  are the azimuthal and radial velocities of the flow. The transport term related to the vertical Reynolds stress was found negligible according to the quasi-geostrophic theory.

Fig. 8 shows the radial velocity and its cumulative sum for the exp.s UP 1 and UP 4. In the



**Figure 8:** (left) Temporal evolution of radial velocity [ $cm\ s^{-1}$ ] - black = UP 4, red = UP 1 - averaged in a rectangular patch and its cumulative sum - blu = UP 4, green = UP 1.

**Figure 9:** (right) UP 1: temporal evolution of azimuthal velocity [ $cm\ s^{-1}$ ] - black = exp. averaged in a rectangular patch, green = num. Num. filtered data using a windows size: blu = 10, red = 30 elements.



case of  $B_u = O(1)$ , baroclinically stable flow, the radial velocity - black line - remains almost zero for all the duration of the acquisition. The cumulative sum - blue line -, which provides information on its net effect, confirms this result. In the case of  $B_u \ll O(1)$ , the radial velocity was affected by baroclinic instabilities: its profile - red line - is characterized by an oscillation trend in the range  $0.1 - 0.4\ cm\ s^{-1}$  and the cumulative sum - green line - shows that the net effect is a positive radial velocity. This result is in accordance with the expectation since the isopycnal surfaces were sloped towards the lateral wall of the tank and the measurements were taken in the upper part of the fluid, where the lighter fluid spreaded over the heavier one.

Fig. 9 shows a comparison between experimental and numerical results for the azimuthal velocity during the regime corresponding to the growing oscillations: the model - green line - shows oscillations with the same period of the measurements - black line - but too wide. Better results were achieved filtering the data using different windows size to obtain a running average which could smooth the oscillations.

This work confirms that in the case of spin up with  $B_u \ll O(1)$ , the transport of momentum is governed by baroclinic instabilities. Although the stratification produces a confinement of the meridional circulation, the adjustment of the isopycnal surfaces allows a radial velocity field which enhances the transport of momentum with respect to the diffusion process.

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