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## A fast algorithm for the computation of 2-D forward and inverse MDCT

J.S. Wu<sup>a, d</sup>, H.Z. Shu<sup>a, d</sup>, L. Senhadji<sup>b, c, d</sup>, L.M. Luo<sup>a, d</sup>

<sup>a</sup>Laboratory of Image Science and Technology, School of Computer Science and Engineering, Southeast University, 210096, Nanjing, China

<sup>b</sup>INSERM, U642, Rennes, F-35000, France

<sup>c</sup>Université de Rennes 1, LTSI, Rennes, F-35000, France

<sup>d</sup>Centre de Recherche en Information Biomédicale Sino-Français (CRIBs)

Information about the corresponding author:

Huazhong Shu, Ph.D

Laboratory of Image Science and Technology  
School of Computer Science and Engineering  
Southeast University, 210096, Nanjing, China

Tel: 00-86-25-83 79 42 49

Fax: 00-86-25-83 79 26 98

Email: shu.list@seu.edu.cn

**Abstract**—A fast algorithm for computing the two-dimensional (2-D) forward and inverse modified discrete cosine transform (MDCT and IMDCT) is proposed. The algorithm converts the 2-D MDCT and IMDCT with block size  $M \times N$  into four 2-D discrete cosine transforms (DCTs) with block size  $(M/4) \times (N/4)$ . It is based on an algorithm recently presented by Cho [7] for the efficient calculation of one-dimensional MDCT and IMDCT. Comparison of the computational complexity with the traditional row-column method shows that the proposed algorithm reduces significantly the number of arithmetic operations.

**Keywords**—2-D MDCT, 2-D DCT, fast algorithm, image coding

## I. Introduction

The forward and inverse modified discrete cosine transform (MDCT and IMDCT) are extensively used to realize the analysis/synthesis filter banks of time domain aliasing cancellation scheme for subband coding [1]. Such a filter bank is equivalent to the modulated lapped transform (MLT) introduced by Malvar [2]. Many fast algorithms have been reported in the literature for computing the one-dimensional (1-D) MDCT/IMDCT (or MLT/MLT<sup>-1</sup>). For example, Britanak and Rao [3] proposed an efficient approach for implementing the  $M$ -point MDCT and IMDCT based on the  $M/4$ -point DCT/DST and corresponding  $M/4$ -point IDCT/IDST, respectively. Lee [4] then suggested an improvement in the computational speed of this algorithm. By using a matrix representation, Cheng and Hsu [5] presented various approaches for efficient implementation of the MDCT and IMDCT. Recently, Truong *et al.* [6] developed a fast algorithm for computing the  $M$ -point MDCT and IMDCT through  $M/2$ -point DCT. Among these approaches, the algorithms reported in [4], [5] and [6] are probably the most efficient for computing the MDCT in terms of the arithmetic complexity. However, the algorithm presented by Cho *et al* in [7], which does not contain recursive structure, seems to achieve a good balance between the arithmetic complexity and computational structure. A comprehensive list of references on this subject is available in [8] and [9]. The two-dimensional (2-D) MDCT/IMDCT (or MLT/MLT<sup>-1</sup>), belonging to the lapped transforms, have a better performance compared to the non-lapped transforms (like the 2-D DCT/IDCT), not only because they have higher coding gains, but also they lead to a strong reduction in “blocking effects” in image coding [10]. Therefore, the 2-D MDCT/IMDCT have found their applications in image coding [11, 12], spectral image analysis [13] and digital image

watermarking [14].

During the past decades, many fast algorithms for computing the 1-D and 2-D DCT have been proposed [15-43]. A comprehensive survey of DCT algorithms can be found in [44] and comments on various fast algorithms for 2-D DCT was given in [45]. For the 1-D case, Lee's algorithm [15] and Hou's algorithm [16] are probably the most attractive radix-2 algorithms for computing the  $2^m$ -point DCT. Loeffler *et al.* [17] presented a fast algorithm for computing the 8- and 16-point DCT with minimum computational complexity. Chan and Siu [18] presented a mixed radix-3/6 algorithm to realize the DCT of length- $M = 2^m 3^n$ ,  $m, n \geq 1$ . Kok [19] then suggested a generalized radix-2 algorithm that can be used to compute the even-length DCT. Recently, Bi and Yu [20] derived an efficient mixed-radix algorithm for computing the DCT of composite sequence length  $M = p \cdot 2^m$  where  $p$  is an odd integer.

For the 2-D case, fast DCT algorithms can be classified into three categories: indirect algorithms, direct algorithms and optimal algorithms based on complexity theory or tensor approach. The indirect algorithms calculate the 2-D DCT through other transforms such as 2-D fast Fourier transform (FFT) [21, 22], 4-D FFT [23], or polynomial transform [24-26]. Among them, by using a polynomial transform (PT), Duhamel and Guillemot [24] developed the most efficient 2-D DCT algorithm for the block sizes  $2^m \times 2^m$ ,  $m \geq 3$ . Zeng *et al.* [26] also presented a PT-based multidimensional DCT algorithm, which can be used to compute the 2-D DCT for the rectangular block sizes  $2^m \times 2^n$ ,  $m, n \geq 2$ . Tatsaki *et al.* [23] derived a prime-factor DCT algorithm for computing the 2-D DCT of the block size  $N \times N$  with  $N \neq 2^m$ . The direct algorithms include the calculation of 2-D DCT through  $N$  sets of  $N$ -point 1-D DCTs plus a post-addition stage [27-31], matrix factorization or recursive computation [32-37],

constant geometry algorithm [38,39], and Chebyshev polynomial [40]. Among them, Britanak and Rao [36] developed an efficient recursive 2-D DCT algorithm for a rectangular  $2^m \times 2^n$  block sizes. Bi *et al.* [37] suggested an algorithm that supports transform sizes  $p \cdot 2^m \times q \cdot 2^n$ , where  $p$  and  $q$  are odd integers. Note that the algorithms reported in [28]-[30] and [35] require the same number of multiplications and similar number of additions as that of the algorithm presented in [24] for computing the  $2^m \times 2^m$ -point DCT, but they have more regular computational structures compared to [24]. The optimal algorithms based on complexity theory or tensor approach [41-43] are mainly proposed to reach the minimum multiplicative complexity. For example, by using the 1-D DCT-based tensor approach, Feig and Winograd [41] obtained the lower bound of the multiplicative complexity which is  $2^m(2^{m+1}-m-2)$  for the  $2^m \times 2^m$  block sizes DCT. That is to say, the lower bounds of the multiplicative complexity for  $8 \times 8$ - and  $16 \times 16$ -point DCT are 88 and 416, respectively. As noted in [45], by combining Loeffler's 1-D DCT algorithm [17] with Cho's 2-D DCT algorithm [28], the multiplications needed for the  $8 \times 8$ - and  $16 \times 16$ -point DCT are 88 and 496, respectively. Recently, by using the shifted Fourier transform-based tensor approach, Grigoryan and Agaian [43] proposed an approach in which 84 and 460 multiplications are required for computing the  $8 \times 8$ - and  $16 \times 16$ -point DCTs, respectively. By utilizing the distributed arithmetic (DA) structure of the 2-D DCT, Pan [46] reported that only 64 multiplications are required for the computation of  $8 \times 8$ -point DCT. Although many algorithms have been reported to reduce the arithmetic complexity of 2-D DCT, to the authors' knowledge, little attention has been paid on the fast computation of 2-D MDCT/IMDCT. Frantzeskakis and Karathanasis [47] developed a time-recursive approach for real-time computation of the 2-D MLT. In most cases, the 2-D

MDCT/IMDCT are calculated with the row-column method [11-14], which requires evaluating  $M$  sets of  $N$ -point MDCTs/IMDCTs and  $N$  sets of  $M$ -point MDCTs/IMDCTs for an  $M \times N$ -point 2-D MDCT/IMDCT. As noted in [24], the true 2-D techniques are more efficient than the row-column approach. Therefore, proper 2-D algorithms need to be developed.

In this paper, the 1-D MDCT/IMDCT algorithm presented in [7] is extended to two dimensions to obtain a new 2-D MDCT/IMDCT algorithm. In section II, a simple variation of the algorithm in [7] is described. The algorithm is then generalized to 2-D in Section III. The computational complexity of the method is analyzed and compared to the row-column method in Section IV. Section V concludes the work.

## II. 1-D MDCT/IMDCT algorithm

Let  $\{x(m)\}, m \in [0, M-1]$  denote a windowed input data sequence. The unnormalized 1-D forward and inverse MDCT are respectively defined as [1]

$$X(k) = \sum_{m=0}^{M-1} x(m) \cos \left[ \frac{\pi}{2M} \left( 2m+1 + \frac{M}{2} \right) (2k+1) \right], k \in [0, M/2-1], \quad (1)$$

$$\hat{x}(m) = \sum_{k=0}^{M/2-1} X(k) \cos \left[ \frac{\pi}{2M} \left( 2m+1 + \frac{M}{2} \right) (2k+1) \right], m \in [0, M-1], \quad (2)$$

where  $M$  is assumed to be divisible by 4, i.e.,  $M = 4p$ .

In this section, we briefly describe the algorithm proposed in [7]. Using the following permutations introduced in [4] and [7]

$$w(m) = \begin{cases} -x(3M/4-1-m) - x(3M/4+m), & m \in [0, M/4-1] \\ x(m-M/4) - x(3M/4-1-m), & m \in [M/4, M/2-1] \end{cases} \quad (3)$$

and

$$y(m) = \begin{cases} -\hat{x}(3M/4 + m), & m \in [0, M/4 - 1] \\ \hat{x}(m - M/4), & m \in [M/4, M - 1] \end{cases} \quad (4)$$

Equations (1) and (2) can be rewritten as

$$X(k) = \sum_{m=0}^{M/2-1} w(m) \cos\left[\frac{\pi}{2M}(2m+1)(2k+1)\right], k \in [0, M/2 - 1], \quad (5)$$

$$y(m) = \sum_{k=0}^{M/2-1} X(k) \cos\left[\frac{\pi}{2M}(2m+1)(2k+1)\right], m \in [0, M/2 - 1]. \quad (6)$$

The above equations show that the forward and inverse MDCT can be realized by the same DCT-IV algorithm.

Equation (5) can further be computed as follows.

$$A(k) = X(2k) + X(2k - 1) = 2 \sum_{m=0}^{M/4-1} u(m) \cos\left[\frac{2\pi}{M}(2m+1)k\right], k \in [0, M/4 - 1], \quad (7)$$

$$B(k) = X(2k) - X(2k - 1) = 2 \sum_{m=0}^{M/4-1} v(m) \sin\left[\frac{2\pi}{M}(2m+1)k\right], k \in [1, M/4], \quad (8)$$

where

$$u(m) = w(m) \cos\left[\frac{\pi}{2M}(2m+1)\right] + w(M/2 - 1 - m) \sin\left[\frac{\pi}{2M}(2m+1)\right], m \in [0, M/4 - 1], \quad (9)$$

$$v(m) = -w(m) \sin\left[\frac{\pi}{2M}(2m+1)\right] + w(M/2 - 1 - m) \cos\left[\frac{\pi}{2M}(2m+1)\right], m \in [0, M/4 - 1], \quad (10)$$

with the initial conditions  $X(0) = A(0)$  and  $X(M/2 - 1) = -B(M/4)$ .

### III. 2-D MDCT/IMDCT algorithm

The corresponding 2-D MDCT and IMDCT are respectively defined by

$$X(k, l) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} x(m, n) \cos\left[\frac{\pi}{2M}\left(2m+1+\frac{M}{2}\right)(2k+1)\right] \cos\left[\frac{\pi}{2N}\left(2n+1+\frac{N}{2}\right)(2l+1)\right], \quad (11)$$

$$k \in [0, M/2 - 1], l \in [0, N/2 - 1],$$

$$\hat{x}(m, n) = \sum_{k=0}^{M/2-1} \sum_{l=0}^{N/2-1} X(k, l) \cos\left[\frac{\pi}{2M}\left(2m+1+\frac{M}{2}\right)(2k+1)\right] \cos\left[\frac{\pi}{2N}\left(2n+1+\frac{N}{2}\right)(2l+1)\right],$$

$$m \in [0, M-1], n \in [0, N-1],$$
(12)

where  $M$  and  $N$  are both assumed to be divisible by 4.

**Step 1: Mapping  $M \times N$ -point forward and inverse MDCT to  $(M/2) \times (N/2)$ -point DCT-IV.**

Letting

$w(m, n)$

$$= \begin{cases} \left[ x\left(\frac{3M}{4}-1-m, \frac{3N}{4}-1-n\right) + x\left(\frac{3M}{4}-1-m, \frac{3N}{4}+n\right) \right] + \left[ x\left(\frac{3M}{4}+m, \frac{3N}{4}-1-n\right) \right. \\ \left. + x\left(\frac{3M}{4}+m, \frac{3N}{4}+n\right) \right], & m \in \left[0, \frac{M}{4}-1\right], n \in \left[0, \frac{N}{4}-1\right] \\ \left[ x\left(\frac{3M}{4}-1-m, \frac{3N}{4}-1-n\right) - x\left(\frac{3M}{4}-1-m, n-\frac{N}{4}\right) \right] + \left[ x\left(\frac{3M}{4}+m, \frac{3N}{4}-1-n\right) \right. \\ \left. - x\left(\frac{3M}{4}+m, n-\frac{N}{4}\right) \right], & m \in \left[0, \frac{M}{4}-1\right], n \in \left[\frac{N}{4}, \frac{N}{2}-1\right] \\ \left[ x\left(\frac{3M}{4}-1-m, \frac{3N}{4}-1-n\right) + x\left(\frac{3M}{4}-1-m, \frac{3N}{4}+n\right) \right] - \left[ x\left(m-\frac{M}{4}, \frac{3N}{4}-1-n\right) \right. \\ \left. + x\left(m-\frac{M}{4}, \frac{3N}{4}+n\right) \right], & m \in \left[\frac{M}{4}, \frac{M}{2}-1\right], n \in \left[0, \frac{N}{4}-1\right] \\ \left[ x\left(\frac{3M}{4}-1-m, \frac{3N}{4}-1-n\right) - x\left(\frac{3M}{4}-1-m, n-\frac{N}{4}\right) \right] - \left[ x\left(m-\frac{M}{4}, \frac{3N}{4}-1-n\right) \right. \\ \left. - x\left(m-\frac{M}{4}, n-\frac{N}{4}\right) \right], & m \in \left[\frac{M}{4}, \frac{M}{2}-1\right], n \in \left[\frac{N}{4}, \frac{N}{2}-1\right] \end{cases} \quad (13)$$

Eq. (11) becomes

$$X(k, l) = \sum_{m=0}^{M/2-1} \sum_{n=0}^{N/2-1} w(m, n) \cos\left[\frac{\pi}{2M}(2m+1)(2k+1)\right] \cos\left[\frac{\pi}{2N}(2n+1)(2l+1)\right],$$

$$k \in [0, M/2-1], l \in [0, N/2-1]$$
(14)

Using a mapping analogous to (4)

$$y(m, n) = \begin{cases} \hat{x}(3M/4+m, 3N/4+n), & m \in [0, M/4-1], n \in [0, N/4-1] \\ -\hat{x}(3M/4+m, n-N/4), & m \in [0, M/4-1], n \in [N/4, N-1] \\ -\hat{x}(m-M/4, 3N/4+n), & m \in [M/4, M-1], n \in [0, N/4-1] \\ \hat{x}(m-M/4, n-N/4), & m \in [M/4, M-1], n \in [N/4, N-1] \end{cases} \quad (15)$$



Equation (12) can be written as

$$y(m, n) = \sum_{k=0}^{M/2-1} \sum_{l=0}^{N/2-1} X(k, l) \cos\left[\frac{\pi}{2M}(2m+1)(2k+1)\right] \cos\left[\frac{\pi}{2N}(2n+1)(2l+1)\right], \quad (16)$$

$$m \in [0, M-1], n \in [0, N-1].$$

Note that

$$y(m, n) = -y(M-1-m, n) = -y(m, N-1-n) = y(M-1-m, N-1-n), \quad (17)$$

$$m \in [0, M/2-1], n \in [0, N/2-1].$$

Therefore, only  $y(m, n), m \in [0, M/2-1], n \in [0, N/2-1]$  needs to calculate. Equations (14) and (16) show that the 2-D forward and inverse MDCT can be realized by the same 2-D DCT-IV algorithm.

**Step 2: Decomposing  $(M/2) \times (N/2)$ -point DCT-IV into four  $(M/4) \times (N/4)$ -point DCTs.**

Instead of computing (14) directly, we propose in this subsection an algorithm suitable for fast computation.

Letting

$$C(k, l) = C_1(k, l) + C_2(k, l), \quad (18)$$

$$D(k, l) = C_1(k, l) - C_2(k, l), \quad (19)$$

$$C'(k, l) = C'_1(k, l) + C'_2(k, l), \quad (20)$$

$$D'(k, l) = C'_1(k, l) - C'_2(k, l), \quad (21)$$

where

$$C_1(k, l) = \frac{1}{4}[X(2k, 2l) + X(2k, 2l-1)], \quad k \in [0, M/2-1], l \in [0, N/2], \quad (22)$$

$$C_2(k, l) = \frac{1}{4}[X(2k-1, 2l) + X(2k-1, 2l-1)], \quad k \in [1, M/2], l \in [0, N/2], \quad (23)$$

$$C'_1(k, l) = \frac{1}{4}[X(2k, 2l) - X(2k, 2l-1)], \quad k \in [0, M/2-1], l \in [0, N/2], \quad (24)$$

$$C'_2(k, l) = \frac{1}{4}[X(2k-1, 2l) - X(2k-1, 2l-1)], \quad k \in [1, M/2], l \in [0, N/2]. \quad (25)$$

**A. Computation of  $C(k, l)$  and  $D(k, l)$ .**

1) Computation of  $C_1(k, l)$  and  $C_2(k, l)$ .

From (22), we have

$$C_1(k, l) = \frac{1}{4} \sum_{m=0}^{M/2-1} \cos \left[ \frac{\pi}{2M} (2m+1)(4k+1) \right] \sum_{n=0}^{N/2-1} w(m, n) \left\{ \cos \left[ \frac{\pi}{2N} (2n+1)(4l+1) \right] + \cos \left[ \frac{\pi}{2N} (2n+1)(4l-1) \right] \right\}. \quad (26)$$

Using (7), we can easily get

$$C_1(k, l) = \frac{1}{2} \sum_{n=0}^{N/4-1} \cos \left[ \frac{2\pi}{N} (2n+1)l \right] \sum_{m=0}^{M/2-1} u(m, n) \cos \left[ \frac{\pi}{2M} (2m+1)(4k+1) \right], \quad (27)$$

where

$$u(m, n) = w(m, n) \cos \left[ \frac{\pi}{2N} (2n+1) \right] + w(m, N/2-1-n) \sin \left[ \frac{\pi}{2N} (2n+1) \right]. \quad (28)$$

Similarly, we have

$$C_2(k, l) = \frac{1}{2} \sum_{n=0}^{N/4-1} \cos \left[ \frac{2\pi}{N} (2n+1)l \right] \sum_{m=0}^{M/2-1} u(m, n) \cos \left[ \frac{\pi}{2M} (2m+1)(4k-1) \right]. \quad (29)$$

2) Computation of  $C(k, l)$  and  $D(k, l)$ .

Substituting (27) and (29) into (18), we obtain

$$C(k, l) = \sum_{m=0}^{M/4-1} \sum_{n=0}^{N/4-1} u_u(m, n) \cos \left[ \frac{2\pi}{M} (2m+1)k \right] \cos \left[ \frac{2\pi}{N} (2n+1)l \right], \quad (30)$$

$$k \in [0, M/4-1], l \in [0, N/4-1]$$

where

$$u_u(m, n) = u(m, n) \cos \left[ \frac{\pi}{2M} (2m+1) \right] + u(M/2-1-m, n) \sin \left[ \frac{\pi}{2M} (2m+1) \right]. \quad (31)$$

For the computation of  $D(k, l)$ , we have

$$D(k, l) = \frac{1}{2} \sum_{n=0}^{N/4-1} \cos \left[ \frac{2\pi}{N} (2n+1)l \right] \sum_{m=0}^{M/2-1} u(m, n) \left\{ \cos \left[ \frac{\pi}{2M} (2m+1)(4k+1) \right] - \cos \left[ \frac{\pi}{2M} (2m+1)(4k-1) \right] \right\}. \quad (32)$$

By using (8), we have

$$D(k, l) = \sum_{m=0}^{M/4-1} \sum_{n=0}^{N/4-1} v_u(m, n) \sin\left[\frac{2\pi}{M}(2m+1)k\right] \cos\left[\frac{2\pi}{N}(2n+1)l\right], \quad (33)$$

where

$$v_u(m, n) = -u(m, n) \sin\left[\frac{\pi}{2M}(2m+1)\right] + u(M/2-1-m, n) \cos\left[\frac{\pi}{2M}(2m+1)\right]. \quad (34)$$

Equation (33) can be rewritten as

$$D\left(\frac{M}{4}-k, l\right) = \sum_{m=0}^{M/4-1} \sum_{n=0}^{N/4-1} (-1)^m v_u(m, n) \cos\left[\frac{2\pi}{M}(2m+1)k\right] \cos\left[\frac{2\pi}{N}(2n+1)l\right], \quad (35)$$

$$k \in [0, M/4-1], l \in [0, N/4-1]$$

## B. Computation of $C'(k, l)$ and $D'(k, l)$ .

1) Computation of  $C'_1(k, l)$  and  $C'_2(k, l)$ .

By proceeding in a similar way as for  $C_1(k, l)$  and  $C_2(k, l)$ , we obtain

$$C'_1(k, l) = \frac{1}{2} \sum_{n=0}^{N/4-1} \sin\left[\frac{2\pi}{N}(2n+1)l\right] \sum_{m=0}^{M/2-1} v(m, n) \cos\left[\frac{\pi}{2M}(2m+1)(4k+1)\right], \quad (36)$$

$$C'_2(k, l) = \frac{1}{2} \sum_{n=0}^{N/4-1} \sin\left[\frac{2\pi}{N}(2n+1)l\right] \sum_{m=0}^{M/2-1} v(m, n) \cos\left[\frac{\pi}{2M}(2m+1)(4k-1)\right], \quad (37)$$

where

$$v(m, n) = -w(m, n) \sin\left[\frac{\pi}{2N}(2n+1)\right] + w(m, N/2-1-n) \cos\left[\frac{\pi}{2N}(2n+1)\right]. \quad (38)$$

2) Computation of  $C'(k, l)$  and  $D'(k, l)$ .

By proceeding in a similar way as for  $C(k, l)$  and  $D(k, l)$ , we have

$$C'(k, l) = \sum_{m=0}^{M/4-1} \sum_{n=0}^{N/4-1} u_v(m, n) \cos\left[\frac{2\pi}{M}(2m+1)k\right] \sin\left[\frac{2\pi}{N}(2n+1)l\right], \quad (39)$$

$$D'(k, l) = \sum_{m=0}^{M/4-1} \sum_{n=0}^{N/4-1} v_v(m, n) \sin\left[\frac{2\pi}{M}(2m+1)k\right] \sin\left[\frac{2\pi}{N}(2n+1)l\right], \quad (40)$$

where

$$u_v(m, n) = v(m, n) \cos\left[\frac{\pi}{2M}(2m+1)\right] + v(M/2-1-m, n) \sin\left[\frac{\pi}{2M}(2m+1)\right], \quad (41)$$

$$v_v(m, n) = -v(m, n) \sin\left[\frac{\pi}{2M}(2m+1)\right] + v(M/2-1-m, n) \cos\left[\frac{\pi}{2M}(2m+1)\right]. \quad (42)$$

Equations (39) and (40) can also be rewritten as

$$C'\left(k, \frac{N}{4}-l\right) = \sum_{m=0}^{M/4-1} \sum_{n=0}^{N/4-1} (-1)^n u_v(m, n) \cos\left[\frac{2\pi}{M}(2m+1)k\right] \cos\left[\frac{2\pi}{N}(2n+1)l\right], \quad (43)$$

$$k \in [0, M/4-1], l \in [0, N/4-1]$$

$$D'\left(\frac{M}{4}-k, \frac{N}{4}-l\right) = \sum_{m=0}^{M/4-1} \sum_{n=0}^{N/4-1} (-1)^{m+n} v_v(m, n) \cos\left[\frac{2\pi}{M}(2m+1)k\right] \cos\left[\frac{2\pi}{N}(2n+1)l\right], \quad (44)$$

$$k \in [0, M/4-1], l \in [0, N/4-1]$$

The final outputs of (11) can be obtained by

$$\begin{aligned} X(2k, 2l) &= C(k, l) + D(k, l) + C'(k, l) + D'(k, l), \quad k \in [0, M/4-1], l \in [0, N/4-1] \\ X(2k, 2l-1) &= C(k, l) + D(k, l) - C'(k, l) - D'(k, l), \quad k \in [0, M/4-1], l \in [1, N/4] \\ X(2k-1, 2l) &= C(k, l) - D(k, l) + C'(k, l) - D'(k, l), \quad k \in [1, M/4], l \in [0, N/4-1] \\ X(2k-1, 2l-1) &= C(k, l) - D(k, l) - C'(k, l) + D'(k, l), \quad k \in [1, M/4], l \in [1, N/4]. \end{aligned} \quad (45)$$

For some special values of  $k$  and  $l$ , equation (45) can be further simplified as

$$\begin{aligned} X(2k, 0) &= C(k, 0) + D(k, 0), \\ X(2k-1, 0) &= C(k, 0) - D(k, 0), \\ X(2k, N/2-1) &= -[C'(k, N/4) + D'(k, N/4)], \\ X(2k-1, N/2-1) &= -[C'(k, N/4) - D'(k, N/4)], \end{aligned} \quad k \in [0, M/4] \quad (46)$$

$$\begin{aligned} X(0, 2l) &= C(0, l) + C'(0, l), \\ X(0, 2l-1) &= C(0, l) - C'(0, l), \\ X(M/2-1, 2l) &= -[D(M/4, l) + D'(M/4, l)], \\ X(M/2-1, 2l-1) &= -[D(M/4, l) - D'(M/4, l)], \end{aligned} \quad l \in [0, N/4] \quad (47)$$

and

$$\begin{aligned} X(0, 0) &= C(0, 0), \\ X(0, N/2-1) &= -C'(0, N/4), \\ X(M/2-1, 0) &= -D(M/4, 0), \\ X(M/2-1, N/2-1) &= D'(M/4, N/4). \end{aligned} \quad (48)$$

#### IV. Computational complexity and comparison analysis

In this section, we analyze the computational complexity of our proposed 2-D MDCT/IMDCT algorithm and compare it with the traditional row-column method. Assuming that a butterfly computation is implemented with 3 multiplications and 3 additions, then the decomposition costs are given as follows

- 1)  $3MN/4$  additions for  $w(m, n)$  in (13).
- 2)  $3MN/8$  multiplications and  $3MN/8$  additions for (28) and (38).
- 3)  $3MN/8$  multiplications and  $3MN/8$  additions for (31), (34), (41), and (42).
- 4)  $MN/2 - M - N$  additions for (45)-(48).

In summary, the computational complexity of the proposed 2-D MDCT algorithm is given by

$$M_{M \times N}^{\text{MDCT}} = 4M_{(M/4) \times (N/4)}^{\text{DCT}} + 3MN/4, \quad (49)$$

$$A_{M \times N}^{\text{MDCT}} = 4A_{(M/4) \times (N/4)}^{\text{DCT}} + 2MN - M - N. \quad (50)$$

For the computation of 2-D IMDCT, the manipulation in (15) is just a process of data shift,  $3MN/4$  additions can thus be saved. Fig. 1 shows the block diagram of the proposed 2-D MDCT algorithm for the case of  $M = N = 8$ .

The time-recursive algorithm presented in [47] belonging to the recursive algorithm, is not efficient in terms of arithmetic complexity, but its regressive structure provides an efficient scheme for the parallel VLSI implementation [9]. For this reason, we compare only our algorithms with the traditional row-column method whose computational complexity is given by

$$M_{M \times N}^{\text{MDCT}} = MM_N^{\text{MDCT}} + NM_M^{\text{MDCT}}, \quad (51)$$

$$A_{M \times N}^{\text{MDCT}} = MA_N^{\text{MDCT}} + NA_M^{\text{MDCT}}. \quad (52)$$

Note that the above equations are also valid for the 2-D IMDCT.

In the following, we only give the comparison results of 2-D MDCT algorithm, since the results of 2-D IMDCT algorithm are only different in additions. We first consider the case where the data sequence length satisfies  $M = N = 2^m$ ,  $m \geq 4$ . For the proposed algorithm, we convert  $M \times N$  MDCT into four  $(M/4) \times (N/4)$  DCTs, which are then computed by the fast 2-D DCT algorithms presented in [28]-[30] or [35]. For the row-column method, we combine Lee's algorithm [4] or Cheng and Hsu's algorithm [5] with Kok's algorithm [19] for the fast computation of 1-D MDCT. The comparison result is listed in table I. Note that for  $8 \times 8$ -point MDCT, we used the initial values  $M_{2 \times 2}^{\text{DCT}} = 2$  and  $A_{2 \times 2}^{\text{DCT}} = 8$  [32, 34] in table I. Then, for the case  $M = N$ , both  $M$  and  $N$  being multiple of 4, but not power of two, we combine our 2-D MDCT algorithm with Tatsaki's algorithm [23], and compare it with row-column method, which uses the algorithm presented in [4] or [5] and Bi's algorithm [20] for the efficient computation of 1-D MDCT. The comparison result is shown in table II. For some image compression applications (e.g., [48]), it may require adaptive block sizes in different dimensions, so, we also consider the case where  $M \neq N$ . For the case  $M = 2^m$ ,  $N = 2^n$ ,  $m \geq 3$ ,  $n \geq 4$ , we combine our 2-D MDCT algorithm with Zeng *et al's* algorithm [26]. For the case  $M = p \cdot 2^m$ ,  $N = q \cdot 2^n$ ,  $m \geq 5$ ,  $n \geq 3$ , we combine our 2-D MDCT algorithm with Bi *et al's* algorithm [37]. For the row-column method, we still use the algorithm presented in [4] or [5] and Bi's algorithm [20] for the efficient computation of 1-D MDCT. The comparison results are shown in table III and table IV, respectively. It can be observed from these tables that the proposed 2-D MDCT algorithm significantly reduces the number of arithmetic operations in both multiplications and additions compared to the row-column method. Since the block sizes  $8 \times 8$ - and  $16 \times 16$ -point DCT are commonly used in image compression, we also consider the

computational complexity of the  $8 \times 8$ - and  $16 \times 16$ -point MDCT. It can be easily seen from table I that our algorithm needs only 56 (or 256) multiplications and 144 (or 776) additions for  $8 \times 8$  (or  $16 \times 16$ )-point DCT. However, if we use the row-column method, 128 (or 640) multiplications and 256 (or 1408) additions are required.

## V. Conclusions

A fast algorithm for the computation of 2-D MDCT/IMDCT is presented. It is an extended version of an 1-D MDCT/IMDCT algorithm recently introduced by Cho *et al.* The algorithm reduces significantly the number of arithmetic operations compared to the row-column method. Therefore, it could find its application in multi-signal and image processing tasks.

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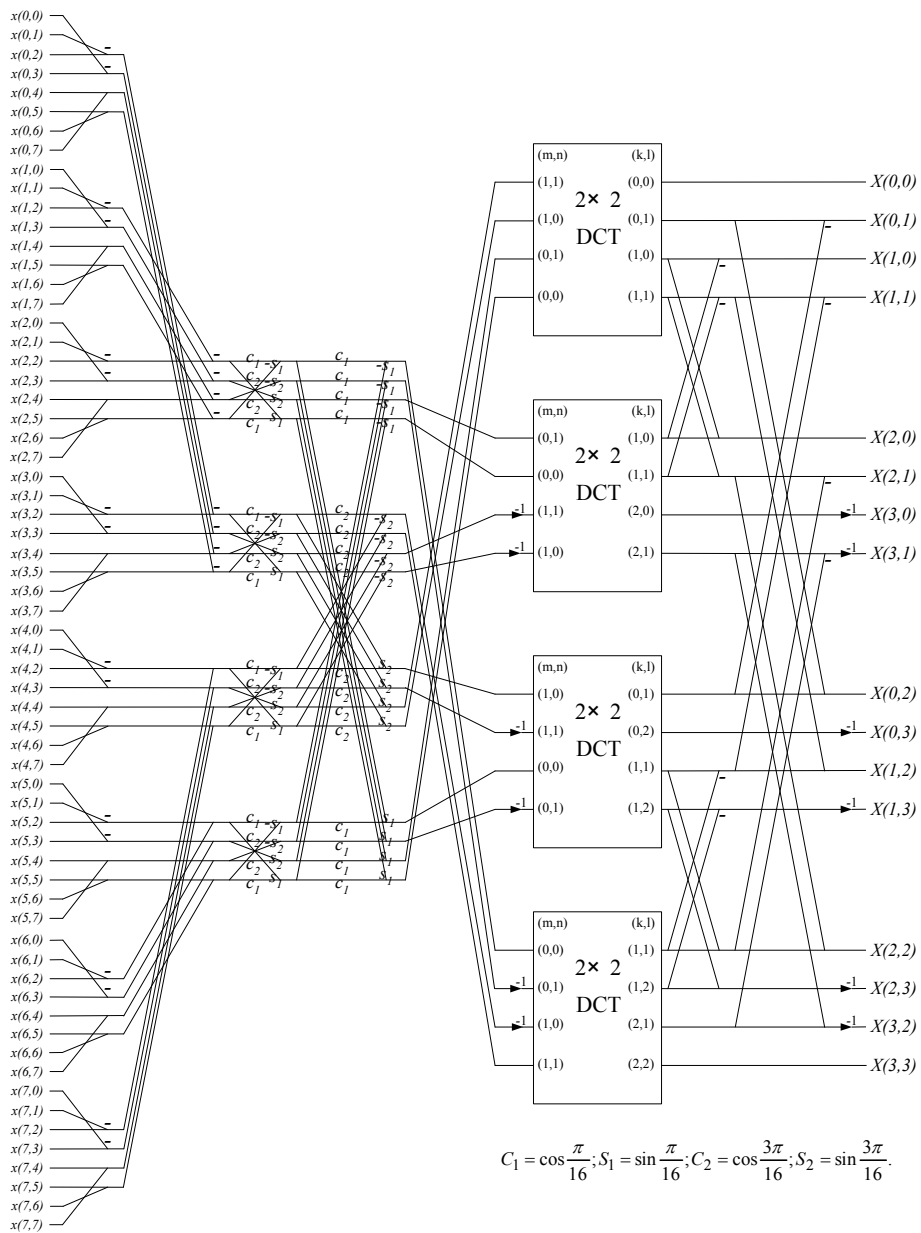


Fig. 1. The block diagram of the 8×8-point MDCT.

Table I Required number of arithmetic operations for 2-D MDCT with block size  $M \times N$ , where  $M = N = 2^m, m \geq 3$ .

Block Size	Proposed algorithm			Row-column method ([4] or [5])			save
	Mul	Add	Total	Mul	Add	Total	
8×8	56	144	200	128	256	384	48%
16×16	256	776	1032	640	1408	2048	50%
32×32	1152	3848	5000	3072	7168	10240	51%
64×64	5120	18184	23304	14336	34816	49152	53%

Table II Required number of arithmetic operations for 2-D MDCT with block size  $M \times N$ , where  $M = N \neq 2^m$ .

Block Size	Proposed algorithm			Row-column method ([4] or [5])			save
	Mul	Add	Total	Mul	Add	Total	
24×24	568	1848	2416	1248	3456	4704	49%
40×40	1784	5752	7536	4480	11840	16320	54%
48×48	2560	9000	11560	6144	17280	23424	51%
56×56	3768	12728	16496	14336	25984	40320	59%

Table III Required number of arithmetic operations for 2-D MDCT with block size  $M \times N$ , where  $M = 2^m, N = 2^n, m \geq 3, n \geq 4$ .

Block Size	Proposed algorithm			Row-column method ([4] or [5])			save
	Mul	Add	Total	Mul	Add	Total	
8×16	128	348	476	288	608	896	47%
16×32	576	1776	2352	1408	3200	4608	49%
32×64	2560	8536	11096	6656	15872	22528	51%
64×128	11264	39592	50856	30720	75776	106496	52%

Table IV Required number of arithmetic operations for 2-D MDCT with block size  $M \times N$ , where  $M = p \cdot 2^m, N = q \cdot 2^n, p=1, q=3, m \geq 5, n \geq 3$ .

Block Size	Proposed algorithm			Row-column method ([4] or [5])			save
	Mul	Add	Total	Mul	Add	Total	
32×24	984	2616	3600	1984	4992	6976	48%
64×48	4704	12320	17024	9472	24576	34048	50%

128×96	21648	57616	79264	44032	116736	160768	51%
256×192	99600	262000	361600	200704	540672	741376	51%