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# Acoustical method for radiation calculation of complex structures

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## Abstract

In the automotive industry, like in other transport industries, predicting noise during design cycle is a necessary step. Well-known methods exist to answer this issue in low frequency domain. Among these, Finite Element Methods, adapted to closed domains, are quite easy to implement whereas Boundary Element Methods are more adapted to infinite domains, but may induce singularity problems. In this article, a new method based on the well-known elastodynamics Substructure Deletion Method is presented. Analogies between acoustical and seismic problems are made to transpose the method. It consists in dividing a complex unbounded problem into two easier ones to solve finite and infinite problems. Instead of considering a geometrically complex structure, a prismatic bounding volume is first studied. Then a classical Finite Element computation is performed on the volume left between the box and the considered structure. The advantage of this technique is that when testing and comparing several geometries contained in such a box, only one boundary element calculation is needed. Efficiency of this method is discussed in the present document.

## 1 Introduction

In the automotive industry, evaluating the emitted noise is an important step during units dimensioning or during the vehicle design cycle as vehicles have to respect the European legislation concerning noise. Moreover, acoustics have become an important choice criteria when buying a car. To achieve this, two main methods exist that is Finite Element Methods ([1], [2], [3], [4]) and Boundary Element Methods ([3], [5], [6]). When dealing with radiation problems, Boundary Elements are usually employed. Since they may induce singularity problem, they are not really easy to use whereas Finite Elements are. They are well adapted to bounded media as this is impossible to discretize the infinity into finite cells.

The idea developed in this paper is to take advantage of both methods. The presented method is inspired from the Substructure Deletion Method is described. The SDM was first presented by Dasgupta in the late 70's [7], and developed in Civil Engineering [8], [9], [10]. In order to investigate the dynamic response of a building embedded foundation under seismic loading, the non-excavated soil impedance matrix is first calculated. Then the embedment region composed of the same material as the soil is studied. At last the impedance matrices are combined according to the continuity and equilibrium assumptions to obtain the impedance matrix of the embedded foundation.

## 2 Theoretical formulation of the method

### 2.1 General formulation of a radiation problem in acoustics

A complex vibrating structure, as shown in Fig. 1, is considered. The acoustical radiation problem is governed by the Helmholtz equation and conditions on the finite and infinite boundaries (cf Eq. 1).

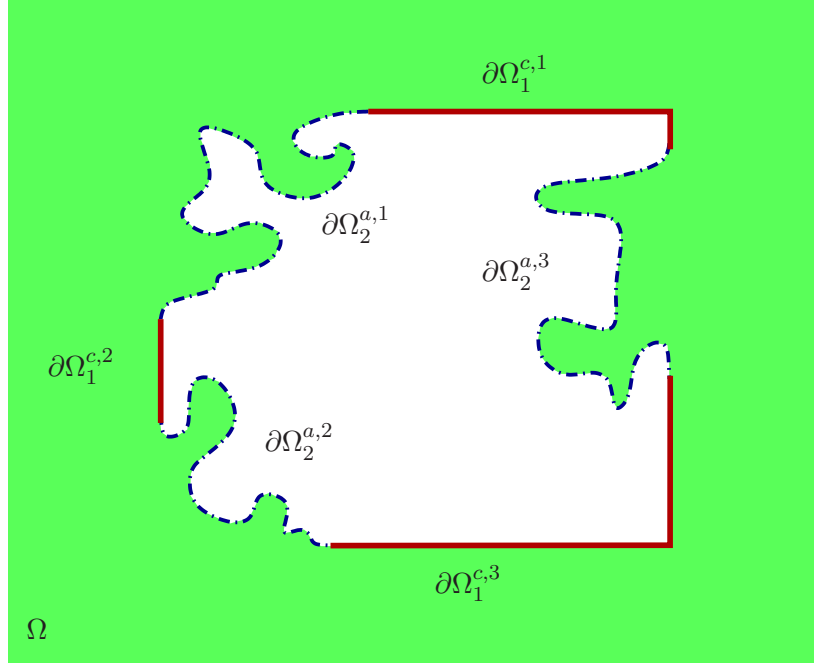


Figure 1: General radiation problem presentation

$$\begin{cases} \Delta p + k^2 p = 0 & \text{in } \Omega \\ \frac{\partial p}{\partial n} = q_v & \text{on } \partial\Omega|_v \\ p = q_p & \text{on } \partial\Omega|_p \\ jkp - \frac{\partial p}{\partial r} = O(r) & \text{Sommerfeld condition at infinity} \end{cases} \quad (1)$$

In this article, only Neumann and Sommerfeld conditions will be considered as shown on the equation (1), that is to say the normal velocities resulting from a frequency response of the structure are assumed to be known.

$$\begin{cases} \Delta p + k^2 p = 0 & \text{in } \Omega \\ \frac{\partial p}{\partial n} = q & \text{on } \partial\Omega \\ jkp - \frac{\partial p}{\partial r} = O(r) & \text{Sommerfeld condition at infinity} \end{cases} \quad (2)$$

After having multiplied the Helmholtz equation by a test function  $\delta p$ , the weak formulation (or integral formulation) can be derived by applying the Green-Gauss theorem.

$$b(p, \delta p) = (q, \delta p)_{L^2(\Omega)}, \quad \forall \delta p \text{ regular test function in } \Omega \quad (3)$$

Where:

$$\begin{cases} b(p, \delta p) = -k^2 \iiint_{\Omega} \delta p p dV + \iiint_{\Omega} \nabla \delta p \nabla p dV \\ (q, \delta p)_{L^2(\Omega)} = \iint_{\partial\Omega} \delta p \frac{\partial p}{\partial n} dS \end{cases}$$

The function  $b$  is a sesquilinear form whereas  $(\cdot, \cdot)$  is a linear operator.

As it will be demonstrated in the next paragraphs, whatever Finite Element or Boundary Element methods are used for the discretization, the integration leads to the following discretized equation:

$$p = \mathbb{A} \frac{\partial p}{\partial n} \quad (4)$$

The present method aims at calculating the admittance matrix  $\mathbb{A}$ .

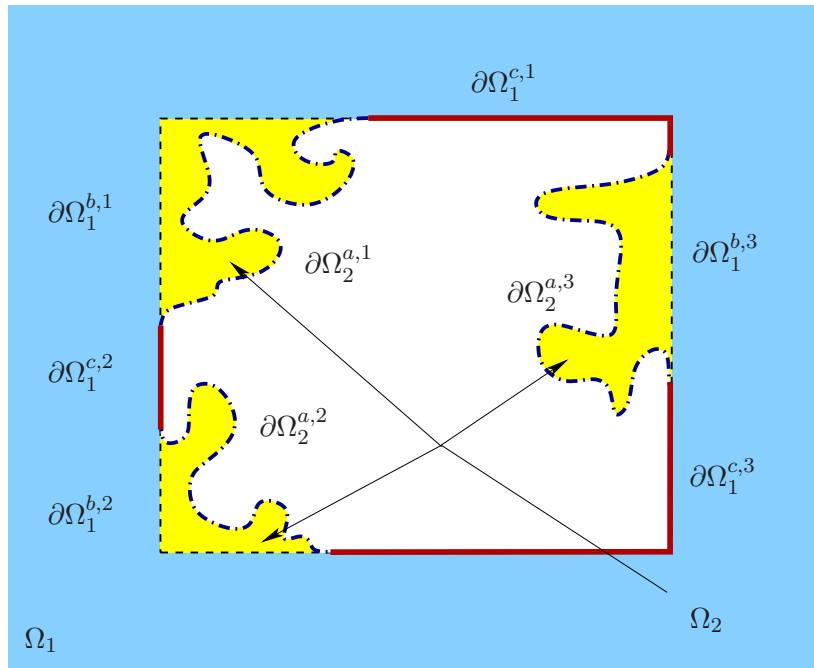


Figure 2: Global problem division into two subdomains

As in elastodynamics, this is achieved by dividing the  $\Omega_u$  domain into two domains as illustrated in Fig. 2, where  $\Omega_2$  is an unbounded domain and  $\Omega_1$  a bounded domain. The weak formulation (cf Eq. 3) can then be decomposed on these two domains and the integral equation (Eq. 3) of the complex problem (Fig. 1) becomes:

$$b(p_1, \delta p) + b(p_2, \delta p) = (q_1, \delta p)_{L^2(\partial\Omega_1)} + (q_2, \delta p)_{L^2(\Omega_2)}, \quad \forall \delta p \in \mathbf{H}^1(\Omega_1) \cup \mathbf{H}^1(\Omega_2) \quad (5)$$

The most suitable discretization method is then chosen for each type of problem.

## 2.2 Exterior problem

A main advantage of Boundary Element methods is they are well adapted to infinite domain studies. Let's consider again the integral equation written on Eq. 3, substituting  $\Omega$  with  $\Omega_1$ . After applying a second time Green theorem, the fundamental solution  $g$ , called Green function, is introduced.  $g$  is the exact solution of the following equation :

$$\Delta g + k^2 g = \delta \text{ where } \delta \text{ is the dirac function,} \quad (6)$$

Hence, the boundary integral equation is obtained:

$$c_i p_i = \iint_{\partial\Omega_1} \frac{\partial p}{\partial n} \cdot g \, dS - \iint_{\partial\Omega_1} p \cdot \frac{\partial g}{\partial n} \, dS \quad \text{with: } c_i = \begin{cases} 1 & P \in \Omega_1 \setminus \partial\Omega_1 \\ \frac{1}{2} & P \text{ regular point on } \partial\Omega_1 \\ \frac{\text{solid angle}}{4\pi} & P \text{ singular point on } \partial\Omega_1 \end{cases} \quad (7)$$

The discretization of this equation on  $\partial\Omega_2$  leads to :

$$\mathbb{G} \frac{\partial p}{\partial n_1} = \mathbb{H} p_1 \Leftrightarrow -j\rho_f\omega \mathbb{G} v_{n_1} = \mathbb{H} p$$

$$\text{where the elementary matrices are such as: } \begin{cases} G_{ij}^\alpha &= \iint_{\Sigma_j} \varphi_\alpha \cdot g_i \, dS \\ H_{ij}^\alpha &= \bar{H}_{ij}^\alpha + c_i \delta_{ij} \\ \bar{H}_{ij}^\alpha &= \iint_{\Sigma_j} \varphi_\alpha \cdot \frac{\partial g_i}{\partial n} \, dS \end{cases} \quad (8)$$

$\delta_{ij}$  is the kronecker symbol.

Finally, the following expression is derived:

$$p_1 = \mathbb{A}_1 v_{n_1} \quad (9)$$

### 2.3 Interior problem

In this case, the use of finite elements avoids the difficulties due to singularities encountered with boundary elements. Moreover this discretization method is more adapted to bounded domains.

As in structural dynamics mass-like and stiffness-like matrices can be inferred from the variational formulation described by Eq. 3 by introducing suitable shape functions. As a consequence the interior problem is governed by:

$$\left( \mathbb{K}^f - \omega^2 \mathbb{M}^f \right) p_2 = \mathbb{F} v_{n_2}, \quad \text{with: } \begin{cases} \mathbb{K}_e^f &= \iiint_{\Omega_e} \left[ \frac{\partial N}{\partial n} \right]^t \left[ \frac{\partial N}{\partial n} \right] dV \\ \mathbb{M}_e^f &= \frac{1}{c^2} \iiint_{\Omega_e} N N^t dV \\ \mathbb{F}_e &= -j\rho_f\omega \left[ \iint_{\partial\Omega_e} N n^t N^t dS \right] n^{-t} \end{cases} \quad (10)$$

To be compatible with the boundary integral formulation in  $\Omega_2$ , these matrices have to be reduced on the domain boundary degrees of freedom. An exact condensation or a Guyan reduction is proceeded to obtain the transformation matrix  $\mathbb{T}$  such as:

$$\begin{aligned} \mathbb{M}_{red} &= \mathbb{T}^t \mathbb{M}^f \mathbb{T} \\ \mathbb{K}_{red} &= \mathbb{T}^t \mathbb{K}^f \mathbb{T} \end{aligned}$$

Hence a similar expression as equation (9) can be derived :

$$\begin{aligned} [\mathbb{K}_{red} - \omega^2 \mathbb{M}_{red}] p_2 &= \mathbb{F} v_{n_2} \\ \Leftrightarrow p_2 &= \mathbb{A}_2 v_{n_1} \end{aligned} \quad (11)$$

## 2.4 The Substructure Deletion Method

Since the FE and BE matrices have been calculated, only boundary conditions (given below Eq. 12) are lacking to solve the problem. These conditions guarantee the compatibility and equilibrium between the two problems described before.

$$\begin{cases} p_1^b = \mathbb{A}_1^{b,b} v_{n1}^b + \mathbb{A}_1^{b,c} v_{n1}^c \\ p_1^c = \mathbb{A}_1^{c,b} v_{n1}^b + \mathbb{A}_1^{c,c} v_{n1}^c \\ p_2^a = \mathbb{A}_2^{a,a} v_{n2}^a + \mathbb{A}_2^{a,b} v_{n2}^b \\ p_2^b = \mathbb{A}_2^{b,a} v_{n2}^a + \mathbb{A}_2^{b,b} v_{n2}^b \end{cases} \quad \text{and} \quad \begin{cases} p_1^b = p_2^b \\ v_{n1}^b + v_{n2}^b = 0 \\ p_2^a = p_3^a \\ v_{n2}^a = v_{n3}^a \\ p_1^c = p_3^c \\ v_{n1}^c = v_{n3}^c \end{cases} \quad (12)$$

This leads to the following expression of  $\mathbb{A} = \mathbb{A}_3$ :

$$p_3 = \begin{bmatrix} \mathbb{A}_3^{a,a} & \mathbb{A}_3^{a,c} \\ \mathbb{A}_3^{c,a} & \mathbb{A}_3^{c,c} \end{bmatrix} v_{n3} \quad (13)$$

$$\text{with} \quad \begin{cases} \mathbb{A}_3^{a,a} = \mathbb{A}_2^{a,a} - \mathbb{A}_2^{a,b} \left[ \mathbb{A}_1^{b,b} + \mathbb{A}_2^{b,b} \right]^{-1} \mathbb{A}_2^{b,a} \\ \mathbb{A}_3^{a,c} = \mathbb{A}_2^{a,b} \left[ \mathbb{A}_1^{b,b} + \mathbb{A}_2^{b,b} \right]^{-1} \mathbb{A}_1^{b,c} \\ \mathbb{A}_3^{c,a} = \mathbb{A}_1^{c,b} \left[ \mathbb{A}_1^{b,b} + \mathbb{A}_2^{b,b} \right]^{-1} \mathbb{A}_2^{b,a} \\ \mathbb{A}_3^{c,c} = \mathbb{A}_1^{c,c} - \mathbb{A}_1^{c,b} \left[ \mathbb{A}_1^{b,b} + \mathbb{A}_2^{b,b} \right]^{-1} \mathbb{A}_1^{b,c} \end{cases} \quad (14)$$

## 3 Application and results

### 3.1 Presentation of the studied models

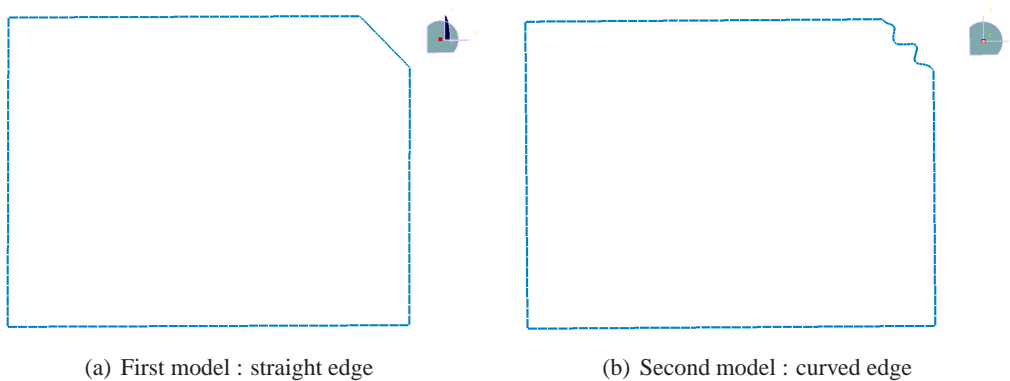


Figure 3: Unbounded complex problems meshes

The Substructure Deletion Method is applied here on two-dimensional examples. The different models are presented on figure 3. These problems are divided into two subproblems respectively illustrated on figures 4 and 5. The same mesh is used to solve the two unbounded problems.

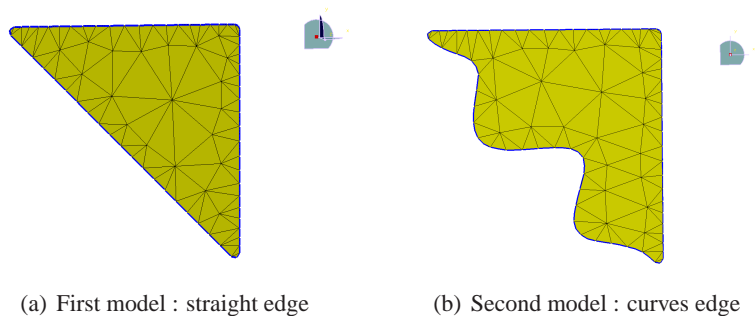


Figure 4: Bounded problems meshes

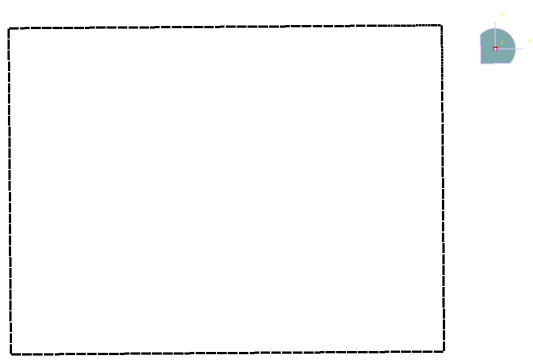


Figure 5: Common unbounded problem mesh

**3.2 Results**

Figures 7, 8 and 9 show the pressure level at different points on the wetted surface. Those points can be located on figure 6. The results of only three representative nodes are presented here.

SDM results are compared with BEM results. These were computed with the Matlab open source OpenBEM routines. For all nodes, the Substructure Deletion Method shows a quite good agreement with BEM.

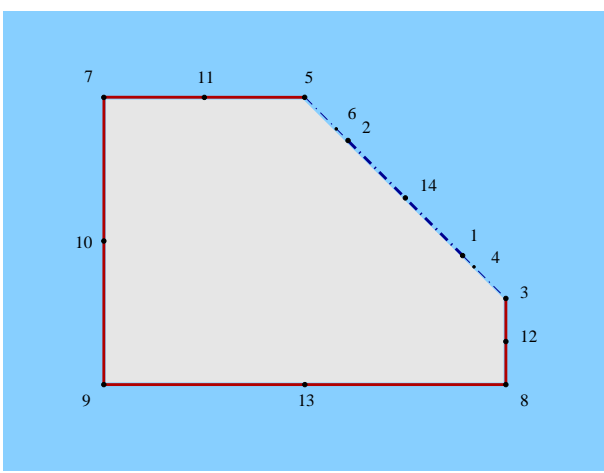
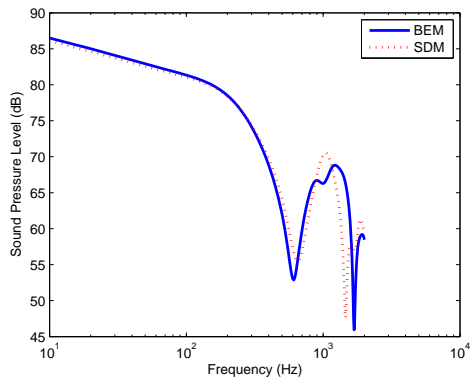
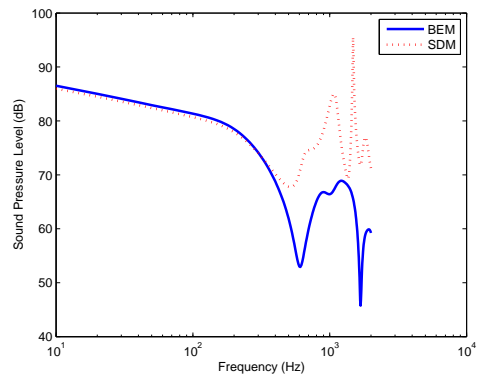


Figure 6: Visualization points

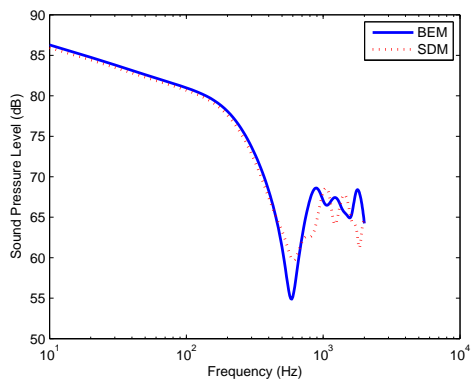


(a) First model : straight edge

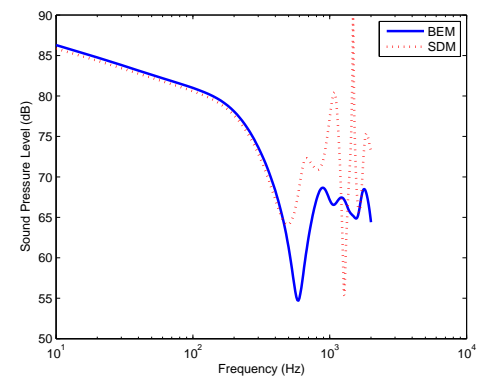


(b) Second model : curved edge

Figure 7: Sound Pressure Level at node 2



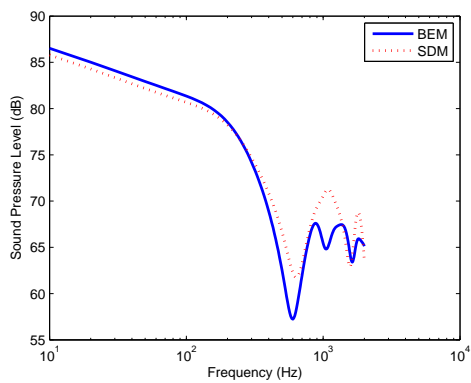
(a) First model : straight edge



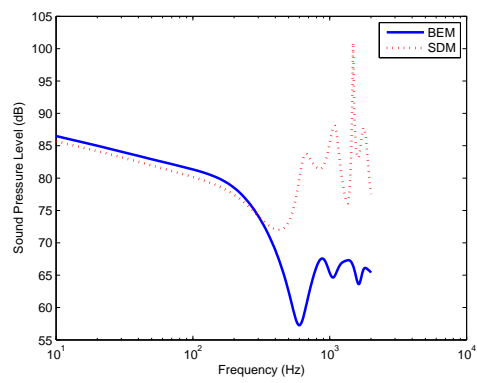
(b) Second model : curved edge

Figure 8: Sound Pressure Level at node 9

Even if some inaccuracies remains, qualitative behaviour is generally respected, and quantitative behaviour is satisfying for a use during preliminary architectural studies



(a) First model : straight edge



(b) Second model : curved edge

Figure 9: Sound Pressure Level at node 14



Number of iterations	SDM (Matlab routines)	BEM (Matlab routines)
	CPU Time	CPU Time
1	1 BEM computation : 2h30 1 FEM and SDM computation : 3 min Total : 2h33	1 BEM computation : 2h30 Total : 2h30
10	1 BEM computation : 2h30 10 FEM and SDM computations : 30 min Total : 3h	10 BEM computations : 102h30 Total : 25h

Table 1: CPU Time depending on the number of iterations

## 4 Conclusion

In this article, a new method has been described. It consists in dividing a complex radiation problem into two subproblems : a bounded one, solved thanks to Finite Elements, and a unbounded one studied here with Boundary Elements. The application on an academical two-dimensional example shows quite good agreement with reference BEM results. Since this method is to be used on simplified geometries during prospecting phase, the results may be accurate enough to compare several architectural concepts. More, as it is illustrated by the table 1, this method is adapted to optimization : the more numerous geometries to be tested, the more CPU time may be gained by using the Substructure Deletion Method.

At last, if the Substructure Deletion Method shows good results on an academical case, its accuracy on an industrial case remains to be demonstrated. If it is, this method should allow to conceive better and earlier.

## Acknowledgements

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