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## **HYBRID ACOUSTICAL METHOD FOR SOLVING EXTERIOR PROBLEMS**

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### **ABSTRACT**

Predicting noise is a step that cannot be ignored in the automotive industry during the vehicle design cycle.

This is classically achieved through Finite Element and Boundary Element methods. When dealing with exterior problems, Boundary Element Methods is quite efficient but may induce ill-conditioned equations. On the other hand, Finite Element Methods, if easier to handle are not initially adapted to unbounded media.

In this paper a hybrid method, which combines the advantages of both techniques is presented. This method, inspired from Substructure Deletion Method, which is well-known in Civil Engineering, consists in dividing a complex unbounded problem into two easier ones to solve finite and infinite problems. Instead of considering a geometrically complex structure, a prismatic bounding volume is first studied using BEM. Then a classical Finite Element computation is performed on the volume left between the box and the structure of interest. Advantage of this technique is that when testing and comparing several geometries contained in such a box, only one Boundary Element calculation is needed. Efficiency of this method is discussed in the present document.

## 1 INTRODUCTION

In the automotive industry, evaluating the emitted noise is an important step during units dimensioning or during the vehicle design cycle. Well-known methods exist to answer this issue in low frequency domain. Among these, Finite Element Methods ([1], [2], [3], [4]), adapted to closed domains, are quite easy to implement whereas Boundary Element Methods ([3], [5], [6]) are more adapted to infinite domains, but may induce singularity problems.

In this paper, a hybrid method inspired from the Substructure Deletion Method is described. The SDM was first presented by Dasgupta in the late 70's [7], and developed in Civil Engineering [8], [9], [10]. In order to investigate the dynamic response of a building embedded foundation under seismic loading, the non-excavated soil impedance matrix is first calculated. Then the embedment region composed of the same material as the soil is studied. At last the impedance matrices are combined according to continuity and equilibrium hypothesis to obtain the impedance matrix of the embedded foundation.

The first part of this article will introduce the hybrid method which will be then applied to an academic case. Finally some results will be presented.

## 2 PRESENTATION OF THE HYBRID METHOD

### 2.1 Global problem formulation

A complex vibrating structure, as shown in Fig. 1, is considered. The acoustical radiation problem is governed by the Helmholtz equation and conditions on the finite and infinite boundaries (cf Eq. 1).

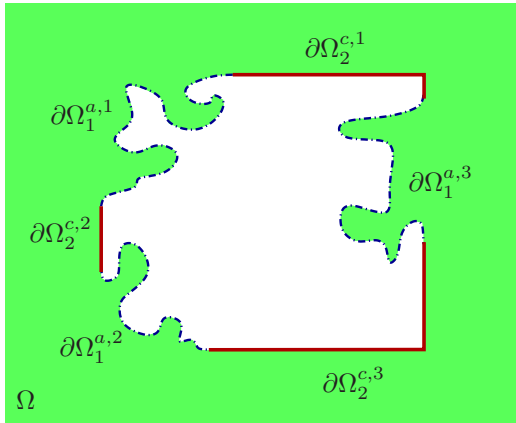


Fig. 1: Global problem domain definition

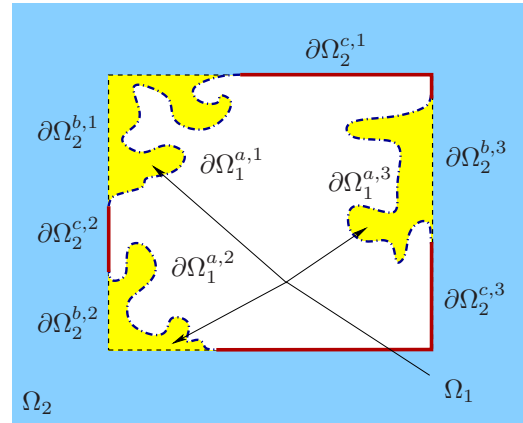


Fig. 2: Global problem division into two sub-domains

$$\begin{cases} \Delta p + k^2 p = 0 & \text{in } \Omega \\ \frac{\partial p}{\partial n} = q & \text{on } \partial\Omega \\ jkp - \frac{\partial p}{\partial r} = O(r) & \text{Sommerfeld condition at infinity} \end{cases} \quad (1)$$

In this article, only Neumann and Sommerfeld conditions will be considered. The weak formulation (or integral formulation) is obtained by applying the Green-Gauss theorem.

$$b(p, \delta p) = (q, \delta p)_{L^2(\Omega)}, \forall \delta p \text{ regular test function in } \Omega \quad (2)$$

Where:

$$\begin{cases} b(p, \delta p) = -k^2 \iiint_{\Omega} \delta p p \, dV + \iiint_{\Omega} \nabla \delta p \nabla p \, dV \\ (q, \delta p)_{L^2(\Omega)} = \iint_{\partial\Omega} \delta p \frac{\partial p}{\partial n} \, dS \end{cases}$$

Whatever Finite Element or Boundary Element methods are used for the discretization, the integration leads to the following discretized equation:

$$p = \mathbb{A} \frac{\partial p}{\partial n} \quad (3)$$

The present method aims at calculating the admittance matrix  $\mathbb{A}$ .

As in elastodynamics, this is achieved by dividing the  $\Omega_u$  domain into two domains as illustrated in Fig. 2, where  $\Omega_2$  is an unbounded domain and  $\Omega_1$  a bounded domain. The weak formulation (cf Eq. 2) can then be decomposed on these two domains and the integral equation (Eq. 2) of the complex problem (Fig. 1) becomes:

$$b(p_1, \delta p) + b(p_2, \delta p) = (q_1, \delta p)_{L^2(\partial\Omega_1)} + (q_2, \delta p)_{L^2(\Omega_2)}, \forall \delta p \in \mathbf{H}^1(\Omega_1) \cup \mathbf{H}^1(\Omega_2) \quad (4)$$

The most suitable discretization method is then chosen for each type of problem.

## 2.2 Exterior problem

A main advantage of Boundary Element methods is they are well adapted to infinite domain studies. Let's consider again the integral equation written on Eq. 2, substituting  $\Omega$  with  $\Omega_2$ . After applying a second time Green theorem, the fundamental solution  $g$ , called Green function, is introduced.  $g$  is the exact solution of the following equation :

$$\Delta g + k^2 g = \delta \text{ where } \delta \text{ is the dirac function,} \quad (5)$$

Hence, the boundary integral equation is obtained:

$$c_i p_i = \iint_{\partial\Omega_2} \frac{\partial p}{\partial n} \cdot g \, dS - \iint_{\partial\Omega_2} p \cdot \frac{\partial g}{\partial n} \, dS$$

$$\text{with: } c_i = \begin{cases} 1 & P \in \Omega_2 \setminus \partial\Omega_2 \\ \frac{1}{2} & P \text{ regular point on } \partial\Omega_2 \\ \frac{\text{solid angle}}{4\pi} & P \text{ singular point on } \partial\Omega_2 \end{cases} \quad (6)$$

The discretization of this equation on  $\partial\Omega_2$  leads to :

$$\mathbb{G} \frac{\partial p}{\partial n_2} = \mathbb{H} p_2 \Leftrightarrow -j\rho_f \omega \mathbb{G} v_{n_2} = \mathbb{H} p \Leftrightarrow p_2 = \mathbb{A}_2 v_{n_2} \quad (7)$$

### 2.3 Interior problem

In this case, the use of finite elements avoids the difficulties due to singularities encountered with boundary elements. Moreover this discretization method is more adapted to bounded domains.

As in structural dynamics mass-like and stiffness-like matrices can be inferred from the variational formulation described by Eq. 2 by introducing suitable shape functions. As a consequence the interior problem is governed by:

$$(\mathbb{K}^f - \omega^2 \mathbb{M}^f) p_1 = F, \quad \text{with:} \quad \begin{cases} \mathbf{K}_{ij}^f = \iiint_{\Omega_1} \frac{\partial N_i}{\partial n} \frac{\partial N_j}{\partial n} dV \\ \mathbf{M}_{ij}^f = \iiint_{\Omega_1} N_i N_j dV \\ F_i = -j\rho_f \omega \iint_{\partial\Omega_1} \bar{v}_n N_i dS \end{cases} \quad (8)$$

To be compatible with the boundary integral formulation in  $\Omega_2$ , these matrices have to be reduced on the domain boundary degrees of freedom. An exact condensation or a Guyan reduction is proceeded to obtain:

$$\mathbb{S}_{red} p_1 = \mathbb{F}_{red} v_{n1} \Leftrightarrow p_1 = \mathbb{A}_1 v_{n1} \quad (9)$$

### 2.4 Application of the Substructure Deletion Method

Since the FE and BE matrices have been calculated, only boundary conditions (given below Eq. 10) are lacking to solve the problem. These conditions guarantee the compatibility and equilibrium between the two problems described before.

$$\begin{cases} p_1^a = \mathbb{A}_1^{a,a} v_{n1}^a + \mathbb{A}_1^{a,b} v_{n1}^b \\ p_1^b = \mathbb{A}_1^{b,a} v_{n1}^a + \mathbb{A}_1^{b,b} v_{n1}^b \\ p_2^b = \mathbb{A}_2^{b,b} v_{n2}^b + \mathbb{A}_2^{b,c} v_{n2}^c \\ p_2^c = \mathbb{A}_2^{c,b} v_{n2}^b + \mathbb{A}_2^{c,c} v_{n2}^c \end{cases} \quad \text{and} \quad \begin{cases} p_1^b = p_2^b \\ v_{n1}^b + v_{n2}^b = 0 \\ p_1^a = p_3^a \\ v_{n1}^a = v_{n3}^a \\ p_2^c = p_3^c \\ v_{n2}^c = v_{n3}^c \end{cases} \quad (10)$$

This leads to the following expression of  $\mathbb{A} = \mathbb{A}_3$ :

$$p_3 = \begin{bmatrix} \mathbb{A}_3^{a,a} & \mathbb{A}_3^{a,c} \\ \mathbb{A}_3^{c,a} & \mathbb{A}_3^{c,c} \end{bmatrix} v_{n3} \quad \text{with} \quad \begin{cases} \mathbb{A}_3^{a,a} = \mathbb{A}_1^{a,a} - \mathbb{A}_1^{a,b} \left[ \mathbb{A}_2^{b,b} + \mathbb{A}_1^{b,b} \right]^{-1} \mathbb{A}_1^{b,a} \\ \mathbb{A}_3^{a,c} = \mathbb{A}_1^{a,b} \left[ \mathbb{A}_2^{b,b} + \mathbb{A}_1^{b,b} \right]^{-1} \mathbb{A}_2^{b,c} \\ \mathbb{A}_3^{c,a} = \mathbb{A}_2^{c,b} \left[ \mathbb{A}_2^{b,b} + \mathbb{A}_1^{b,b} \right]^{-1} \mathbb{A}_1^{b,a} \\ \mathbb{A}_3^{c,c} = \mathbb{A}_2^{c,c} - \mathbb{A}_2^{c,b} \left[ \mathbb{A}_2^{b,b} + \mathbb{A}_1^{b,b} \right]^{-1} \mathbb{A}_2^{b,c} \end{cases} \quad (11)$$

### 3 RESULTS

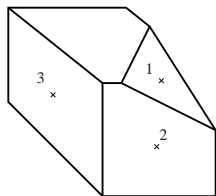


Fig. 3: Points position

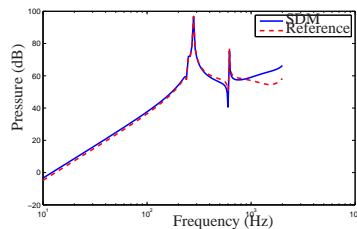


Fig. 4: Pressure at point 1

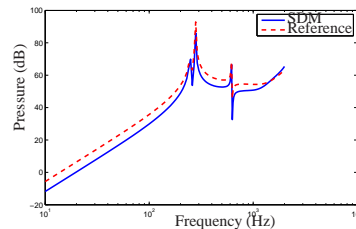


Fig. 5: Pressure at point 2

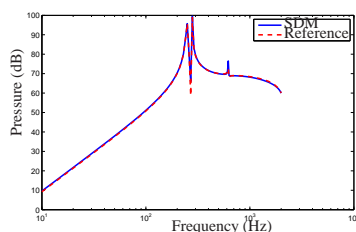


Fig. 6: Pressure at point 3

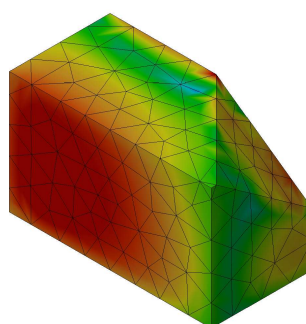


Fig. 7: SDM results (pressure) at 500 Hz

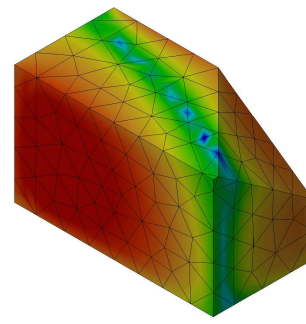


Fig. 8: Reference results (pressure) at 500 Hz

The hybrid method has been applied on a parallelepipedic structure whom a corner is removed as shown in Fig. 3. Results of the Substructure Deletion Method are compared to reference results obtained with Sysnoise Direct BEM.

Fig. 4 to 6 show the pressure at different points on the wetted surface. SDM results appear to be in good agreement with Sysnoise results. The results on the entire model at 500 Hz (Fig. 7 and 8) are also quite good even if some inaccuracies remain.

### 4 CONCLUSION

In this article, new hybrid method has been described. Results on a geometrically simple model reveal a good agreement with reference BEM calculations.

This type of method may be useful in prospecting phase of the vehicle unit design for example. Indeed, if an invariant box is considered to model the unit global size, only one BE calculation would have to be carried out on the box. Thanks to the present method, Finite Element analysis on the volume left would be sufficient to evaluate different architectural concepts according to acoustical criterions.

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