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On the impedance matching of left-handed materials to free-space

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Abstract— Using an original approach, this work shows that the impedance of left-handed materials (LHMs) is negative and a method to match left-handed materials to free space is proposed. A full-wave technique is used to validate our analysis and proposed scheme.

1. INTRODUCTION

Veselago¹ explored the proprieties of isotropic media where both the permittivity and the permeability are negative. He observed that in such media, the wave vector k , the electric field vector \vec{E} , and the magnetic field vector \vec{H} form a left-handed system. For this reason, these materials are called left-handed materials (LHMs). He deduced that these media possess negative refractive indexes and proposed to use these materials to design flat lenses, by taking benefits of the negative index of refraction. This idea has been recently reintroduced and further analyzed by Pendry². Different methods have been proposed to design LHMs by using artificial periodic structures and experimental prototypes have been fabricated^{3,4}. These artificial materials are also called metamaterials, because they have characteristics that can not be found in materials of the nature.

Even if a homogeneous non-dispersive LHM has never been observed in the nature, to facilitate the analysis, one should be able to consider an artificial LHM as a homogeneous non-dispersive material, around a frequency of interest. However, several authors have noticed that the theoretical analysis of a homogeneous non-dispersive LHM slab excited by a plane wave or a line source leads to a divergent solution⁵⁻⁷, even if the relative permittivity and permeability are negative but different from -1.

In this work, we present a new analysis of media with negative permittivity and permeability. According to our analysis, it is shown that, contrary to the usual assumption, the characteristic impedance of an LHM is negative. From this, when a plane wave illuminates an LHM slab in free space, the wave is reflected and transmitted with transmission and reflection coefficients that tend to infinity, which is not physically meaningful. This can be explained only if someone knows that surface waves are excited at the interface between air and LHM. To resolve the problem, it is shown that by using a sheet of resistors and a sheet of amplifiers, it is possible to match the LHM to air. To validate our approach, we present the analysis of the transmission and reflection coefficients of an LHM slab illuminated by a plane wave at normal incidence, by using a transmission line model, a Fabry-Perot cavity model and a full-wave time-domain technique.

2. CHARACTERISTIC IMPEDANCE OF A MEDIUM HAVING NEGATIVE PERMITTIVITY AND PERMEABILITY

The objective of this section is to determine the characteristic impedance of a medium having, at a given frequency, the following constitutive parameters : $\epsilon = -\epsilon_0$ and $\mu = -\mu_0$. ϵ_0 and μ_0 are the permittivity and permeability of air, respectively.

Let us assume a medium, called Medium A, that has the permittivity ϵ_0/p and the permeability $\mu_0 p$, where p is a real. For this medium, Maxwell's equations can be written

$$\begin{aligned}\nabla \times \vec{E}(\vec{r}, t) &= -p\mu_0 \frac{\partial \vec{H}(\vec{r}, t)}{\partial t}, \\ \nabla \times \vec{H}(\vec{r}, t) &= \frac{\epsilon_0}{p} \frac{\partial \vec{E}(\vec{r}, t)}{\partial t}\end{aligned}\tag{1}$$

Because p is a scalar, it is obvious that (1) can be written as follows

$$\begin{aligned}\nabla \times E(\vec{r}, t) &= -\mu_0 \frac{\partial p\bar{H}(\vec{r}, t)}{\partial t}, \\ \nabla \times p\bar{H}(\vec{r}, t) &= \epsilon_0 \frac{\partial \bar{E}(\vec{r}, t)}{\partial t}\end{aligned}\quad (2)$$

Solutions obtained for air can be used for Medium A by using $p\bar{H}$ instead of \bar{H} . The impedance of Medium A is obtained by calculating the ratio between an electric field component E_z and a magnetic field component H_x . From the previous statement, $\frac{E_z}{pH_x} = Z_0 = 120\pi$, which leads to

$$Z_{MediumA} = \frac{E_z}{H_y} = pZ_0 = p120\pi \quad (3)$$

This equation can be tested for positive values of p . For example, for $p = 0.5$ ($\epsilon = 2\epsilon_0$ and $\mu = 0.5\mu_0$), we obtain $Z_{MediumA} = p120\pi = 60\pi$ ($= \sqrt{\mu/\epsilon}$).

If we consider $p = -1$, Medium A is a LHM ($\epsilon = -\epsilon_0$ and $\mu = -\mu_0$) and the impedance can be written $Z_{LHM} = -Z_0 = -120\pi$.

Note that the negative characteristic impedance of the left-handed medium does not mean necessarily the use of active elements to design it. It is due to the fact that the signs of current and charge change in a left-handed material⁸.

3. METHOD TO MATCH A LHM SLAB TO FREE SPACE

Let us consider a slab of the previously discussed LHM in free space. We assume that a plane wave is illuminating this slab at normal incidence. If nothing is added, the transmission and reflection coefficients at the interfaces would be infinite. This is not physically meaningful, of course.

If we introduce loss inside the LHM, by considering non-null imaginary parts of the permittivity and permeability, we obtain also a divergent solution. This is due to the sum of the successive transmitted plane waves that is infinite⁵. This solution is also confirmed by Finite Difference Time Domain (FDTD) simulations that we had carried out. Note that this result and the results presented in this paper can be obtained by any researcher active in Electromagnetic domain and it is not necessary to develop new numerical methods.

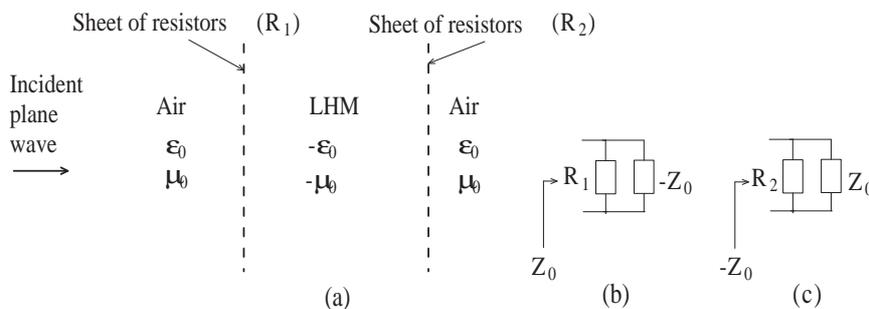


Figure 1: Principle of the matched left-handed material. (a) An incident plane wave illuminates a matched LHM. (b) Matching condition at the left-side interface. (c) Matching condition at the right-side interface.

Now we consider that two sheets of resistors are placed in both sides of the LHM as illustrated in Fig. 1. According to Figs. 1(b) and (c), the impedance matching conditions at the left and right interfaces, respectively, can be written as

$$\frac{-Z_0 R_1}{-Z_0 + R_1} = Z_0 \quad (4)$$

and

$$\frac{Z_0 R_2}{Z_0 + R_2} = -Z_0 \quad (5)$$

Which result in

$$R_1 = \frac{Z_0}{2} = 60\pi \quad (6)$$

and

$$R_2 = -\frac{Z_0}{2} = -60\pi \quad (7)$$

The matching sheets can be designed by using 2-D periodic structures and are assumed to be characterized in the air. From this, R_1 corresponds to an absorption of the wave and R_2 corresponds to an amplification of the wave, which can be obtained in practice by using linear amplifiers.

4. TRANSMISSION AND REFLECTION CHARACTERISTICS OF A MATCHED OR IMPERFECTLY MATCHED LHM SLAB IN AIR

In this section, we use a transmission line model to calculate the transmission and reflection coefficients of the LHM slab with matching sheets. The effect of imperfectly matching is also analyzed.

$k = 2\pi f/c = \omega/c$ is the propagation constant of air, f is the frequency and c is the speed of light in air. The propagation constant of the left-handed medium can be expressed by

$$k_{LHM} = n_{LHM}k = -k \quad (8)$$

In this paper, we use the convention $\exp(-j\omega t)$ to express the propagation of a wave with time. From this, the term $\exp(-jkx)$ expresses the propagation of a plane wave in positive x direction in air. If we consider a plane wave propagating from $x = 0$ to $x = x_1$, the transmission coefficient of this line section of width x_1 can be written $\exp(-jkx_1)$ and x_1/c express the time that the wave makes for travelling from $x = 0$ to $x = x_1$.

In addition, it should be noted that if we obtain the expression $\exp(jkd)$ for the transmission coefficient of a slab of width d and made of a material, in air, for a monochromatic plane wave, it means that this slab introduces a time advance. This is, of course, not in accordance with Causality neither with Einstein's theory of Relativity (the free space speed of light would be surpassed when the slab is present by comparison with the slab not present).

The ABCD matrix (or chain matrix) of a line section of air of width d can be written by

$$\begin{pmatrix} \cos(kd) & j\sin(kd) \\ j\sin(kd) & \cos(kd) \end{pmatrix} \quad (9)$$

The ABCD matrix of a line section of LHM of width d can be written as

$$\begin{pmatrix} \cos(kd) & -j\sin(kd) \\ -j\sin(kd) & \cos(kd) \end{pmatrix} \quad (10)$$

The ABCD matrix of a sheet of resistors R can be written as

$$\begin{pmatrix} 1 & 0 \\ \frac{Z_0}{R} & 1 \end{pmatrix} \quad (11)$$

The chain matrix of the system composed of the LHM and the two sheets of resistors can be expressed as follows :

$$\begin{aligned} \begin{pmatrix} A & B \\ C & D \end{pmatrix} &= \begin{pmatrix} 1 & 0 \\ \frac{Z_0}{R_1} & 1 \end{pmatrix} \begin{pmatrix} \cos(kd) & -j\sin(kd) \\ -j\sin(kd) & \cos(kd) \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{Z_0}{R_2} & 1 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} \cos(kd) & -j\sin(kd) \\ -j\sin(kd) & \cos(kd) \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} \\ &= \begin{pmatrix} \cos(kd) + 2j\sin(kd) & -j\sin(kd) \\ 3j\sin(kd) & \cos(kd) - 2j\sin(kd) \end{pmatrix} \end{aligned} \quad (12)$$

The transmission and reflection factors of the *matched* LHM slab, excited from the left, T_{MLHM} and R_{MLHM} , respectively, are given by

$$T_{MLHM} = \frac{2}{A + B + C + D} = \exp(-jkd) \quad (13)$$

$$R_{MLHM} = \frac{A + B - C - D}{A + B + C + D} = 0 \quad (14)$$

which are the same results as those obtained for a wave propagating in air. It can be concluded that the time for the wave to pass through the matched LHM slab is the same as that for the wave to propagate if the LHM is replaced by air. The normalized transmission coefficient of the slab is then written as $T = T_{LHM}/T_{AIR} = 1$.

Let us analyze the case where the matching conditions are not perfectly achieved. We consider that the equivalent impedance of the sheets are not equal to the ideal case due to an additional imaginary admittance, $j\delta/Z_0$, which leads to the followings values of the equivalent impedances of the sheets, Z_1 and Z_2 :

$$Z_1 = \frac{Z_0}{2 + j\delta} \quad (15)$$

and

$$Z_2 = -\frac{Z_0}{2 + j\delta} \quad (16)$$

where δ is a real close to zero.

The chain matrix of the system composed of the LHM and the two imperfect matching sheets can be written by

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 2 + j\delta & 1 \end{pmatrix} \begin{pmatrix} \cos(kd) & -j\sin(kd) \\ -j\sin(kd) & \cos(kd) \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -2 - j\delta & 1 \end{pmatrix} \quad (17)$$

After some algebraic manipulation, one can obtain the transmission and reflection coefficients of the *imperfectly matched* LHM slab, T_{IMLHM} and R_{IMLHM} :

$$T_{IMLHM} = \frac{2}{A + B + C + D} = \frac{2}{2\exp(jkd) + (4\delta j - \delta^2)j\sin(kd)} \quad (18)$$

$$R_{IMLHM} = \frac{A + B - C - D}{A + B + C + D} = \frac{-(2\delta + \delta^2)j\sin(kd)}{2\exp(jkd) + (4\delta j - \delta^2)j\sin(kd)} \quad (19)$$

Note that $\delta = 0$ corresponds to the perfectly matched LHM slab. These results will be used to explain the numerical results obtained with the FDTD method in Section 6.

5. FABRY-PEROT CAVITY MODEL AND ANALYSIS OF CONVERGENCE CONDITIONS FOR IMPERFECTLY MATCHED LHM

In this section, we use the Fabry-Perot cavity model to analyze the transmission coefficient of the imperfectly matched LHM slab. In other words, we analyze the multiple wave-reflections between the two interfaces. This model shows more clearly the physical insight of the characteristics of the slab and allows to determine the conditions that must be verified to ensure the convergence of the solution.

We consider that an imaginary admittance $j\delta/Z_0$ is added to the equivalent admittance of the matching sheets. The reflection coefficients at the left and right sides of the slab, respectively, can be written as

$$r = -\frac{\delta j + 4}{\delta j + 2} \quad (20)$$

and

$$r' = -\frac{\delta j}{\delta j + 2} \quad (21)$$

The transmission coefficients at the left and right interfaces are

$$t = t' = \frac{2}{\delta j + 2} \quad (22)$$

By considering the multiple reflections of the plane waves between the two interfaces, and by adding the successive transmitted rays, the transmission coefficient of the imperfectly matched LHM slab

can be written by

$$\begin{aligned} T_{IMLHM} &= tt' T_{MLHM} \sum_{i=0}^{\infty} (rr' T_{MLHM}^2)^i \\ &= \frac{4}{(\delta j + 2)^2} \exp(-jkd) \sum_{i=0}^{\infty} \left(\frac{\delta j(\delta j + 4)}{(\delta j + 2)^2} \exp(-2jkd) \right)^i \end{aligned} \quad (23)$$

To ensure the convergence of the sum of geometrical series in (23), *i.e.* to ensure that the sum of the successive transmitted rays is finite, we need to verify the following relation

$$\left| \frac{\delta j(\delta j + 4)}{(\delta j + 2)^2} \right| < 1 \quad (24)$$

When this relation is satisfied, (23) can be written

$$T_{IMLHM} = \frac{\frac{4}{(\delta j + 2)^2} \exp(-jkd)}{1 - \frac{\delta j(\delta j + 4)}{(\delta j + 2)^2} \exp(-2jkd)} = \frac{2}{2 \exp(jkd) + (4\delta j - \delta^2) j \sin(kd)} \quad (25)$$

This is the same result as (18). However, by using the Fabry-Perot cavity model we have now also the information on the convergence condition. If one want to simulate this problem with a numerical method, it is clear that the convergence condition (24) must be satisfied, independently of the numerical technique that is used.

Note that $j\delta = -2$ corresponds to the LHM without matching sheets. If $j\delta$ tends to -2 , it can be seen from (24) that the solution diverges. This is in accordance with the result obtained by Hooft⁵. We can conclude that the divergence problem observed in ⁵ can be resolved by adding a sheet of resistors and a sheet of amplifiers. In the next Section, the analytical formulas are validated by using a numerical method.

6. NUMERICAL RESULTS USING THE FDTD TECHNIQUE

In this section, the full-wave simulation results of a *homogeneous non-dispersive* LHM slab, excited by a plane wave in air, are presented for the first time.

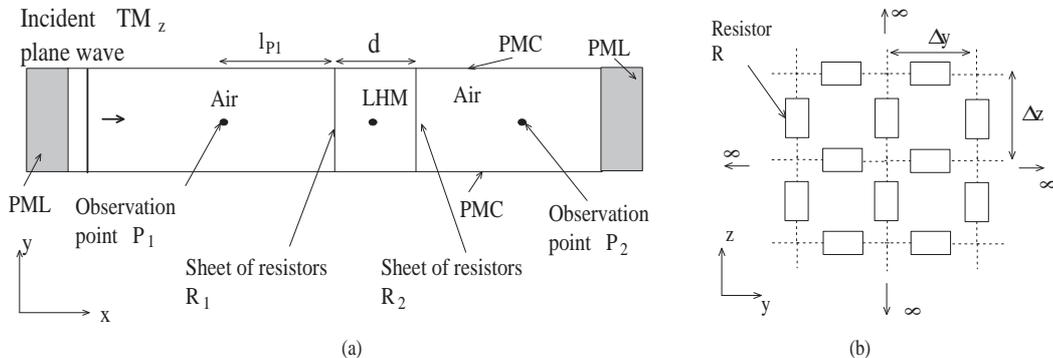


Figure 2: (a) Schematic for the calculation of the transmission and reflection coefficients in the FDTD method. (b) Model of the resistors sheet in the FDTD method. Note that for a TM_z wave excitation, only the resistors in z -direction are required.

We use a Finite Difference Time Domain (FDTD) code, which has been well validated for modeling and simulating a wide range of electromagnetic problems. This method is based on a direct discretization in space and time domains of Maxwell's equations.

Note that no arithmetic mean was used to force the permittivity at the interface. This introduces a space shift between the sheets of resistors and the actual interfaces between the two media.

In the FDTD method, we use a TM_z polarized plane wave and a gaussian pulse in the frequency band $0 - 5$ GHz. Figure 2(a) shows the computational domain. Perfect Electric Conductors, Perfect Magnetic Conductors, and Perfect Matched Layers⁹ are used at the boundaries to limit the

computational domain. Two observation points, P_1 and P_2 , before and after the slab, respectively, are used for the calculation of the reflection and transmission coefficients. At the interfaces, matching resistors are located in each cells in y - z plane, as illustrated in Fig. 2(b). Simulations without the LHM slab were also carried out for normalization. The space mesh in all the directions is $\Delta = 5\text{mm}$. The LHM is made by using a medium with the following parameters : $\epsilon = -\epsilon_0$ and $\mu = -\mu_0$.

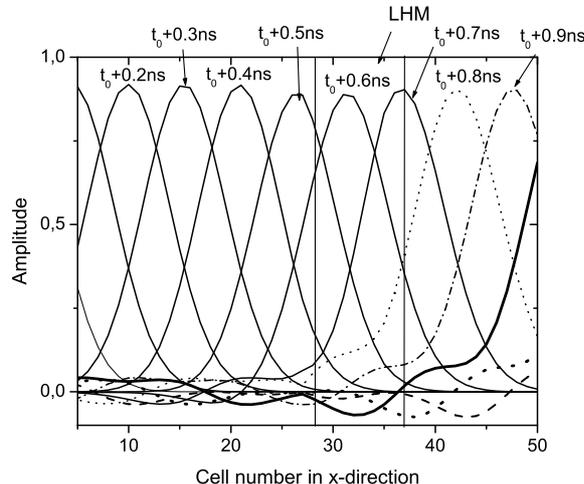


Figure 3: Amplitude of the field versus x -direction at different time instants. The slab width is $d = 40\text{ mm}$. A plane wave with a gaussian pulse comes from the left side. The position of the LHM is indicated in the figure. Note that the wave appears to propagate in the LHM as in the air (except for a small reflected energy part that is due to imperfection of the matching).

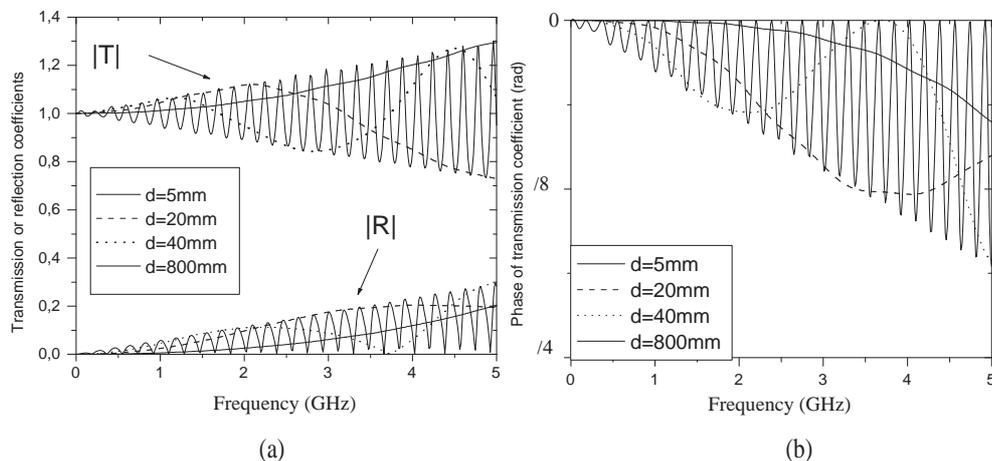


Figure 4: (a) Computed amplitudes of the transmission and reflection coefficients, T and R , of the matched LHM slab *vs.* frequency, for different values of the slab width (FDTD). (b) Phase of the transmission coefficient *vs.* frequency, for different values of the slab width.

Figure 3 shows the amplitude of normalized field E_z versus x -direction at different time instants. This figure demonstrates that the results converge with a wide frequency band excitation. These curves show also that the direction of the wave propagation does not change inside the LHM slab. Inside the LHM slab, the wave propagates like in free space, except for a small reflected energy part that is due to the imperfections in impedances values at the interfaces, as it will be developed

later.

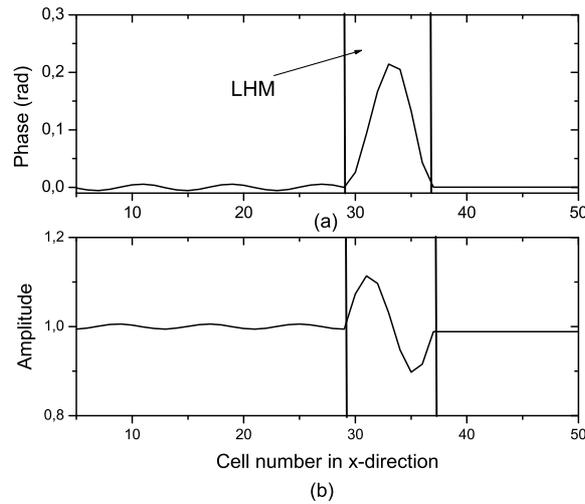


Figure 5: Distributions of the phase and the amplitude of the normalized field in x-direction for $d = 40 \text{ mm}$ at 3.7 GHz ($d \approx \lambda/2$). At this frequency the LHM is matched ($T = 1$, $R = 0$). The position of the LHM is indicated in the figures. (a) Distribution of the phase. (b) Distribution of the amplitude. Note that the amplitude varies inside the LHM, which indicates that a stationary waves regime appears.

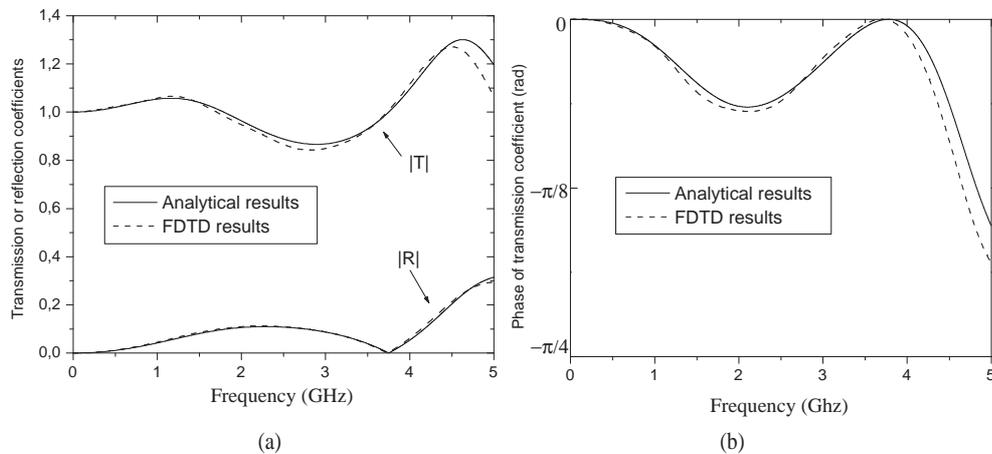


Figure 6: Comparison between analytical and FDTD results for the characteristics the LHM slab with matching sheets *vs.* frequency, for $d = 40 \text{ mm}$ and $\Delta = 5 \text{ mm}$: (a) amplitudes of the transmission and reflection coefficients, T and R (b) phase of the transmission coefficient

Figure 4(a) presents simulated amplitudes of the transmission and reflection coefficients, T and R , for different slab widths. One can note that the transmission coefficient is not equal to one for all frequencies. This is due to the fact that the sheets are not at the same positions as the interfaces, which introduces additional terms in the equivalent admittance values of the interfaces. However, the transmission coefficient is equal to one at resonant frequencies at which the slab widths are equal to a multiple of the half-wavelength.

In Fig. 4(b), is plotted the phase of the transmission coefficient for different slab widths. From these curves, it can be seen that the phase is never positive, which indicates that the slab does not accelerate the propagation of wave, but can only introduce a time-delay. The fact that the speed of light can not be surpassed is in accordance with the theory of relativity¹⁰. When the LHM

is matched ($T = 1$ and $R = 0$), the time-delay is null. On the other hand, if we look into the distribution of the phase of the normalized field, at a resonant frequency, as shown in Fig. 5(a), the phase can be positive inside the slab. This is due to the stationary waves regime that occurs inside the slab, as it can be seen from the distribution of amplitude of the normalized field in Fig. 5(b).

The previous numerical results show that the matching sheets are not perfect. This is due to the fact that the distance between the sheets and the boundary between the two media are spaced by half space mesh, *i.e.* $\Delta/2$, due to the FDTD technique. Physically, this distance represents the electrical distance between a matching sheet and the LHM.

At the interface between the two media, the matching conditions impose that the admittance is null. Therefore, the equivalent admittance of the interface at the sheet position is given by

$$j\delta/Z_0 = j\tan(k\Delta/2)/Z_0 \quad (26)$$

where $\Delta = 5\text{mm}$. $j\delta/Z_0$ represents the additional term that appears in the equivalent admittances of the sheets. Equation (26) is introduced in Eqs. (18)-(19). In Fig. 6(a), are plotted the amplitude of the transmission and reflection coefficients and in Fig. 6(b) is plotted the phase of the transmission coefficient. In the same Figures, the FDTD results are also added. These curves show that the numerical and analytical calculations yield the same results. We have also validated the analytical results for other values of d (not shown here). From this, we can conclude that our theory is validated.

If the mesh Δ tends to zero, *i.e.* if the space between the sheet and the interface tends to zero, then δ tends to zero and the LHM slab becomes perfectly matched to air.

7. CRITICS ON CLASSICAL THEORY OF LHMS AND CONSEQUENCES OF THE PROPOSED THEORY

In this work, we have presented an analyze that shows that the characteristic impedance of a LHM (*i.e.* a material having the the following constitutive parameters : $\epsilon = -\epsilon_0$ and $\mu = -\mu_0$) is negative. An important number of papers can be found in the literature about LHMs. All have demonstrated or considered that the characteristic impedance of the LHM is positive. For this reason, we believe that is important to review some ideas proposed in these papers in order to show where we believe that an error has been made.

Several researchers have considered that there is an ambiguity in the sign of the impedance (or the index of refraction) and that the positive sign should be “chosen” to satisfy physical principles such as conservation of energy or Causality. However, in Maxwell’s equations there is no ambiguity in the signs of the impedance or of the index and the ambiguities can be observed only if one use the wave equation.

From the plane-wave equation, we obtain easily $(k_{LHM})^2 = (-\mu_0)(-\epsilon_0)\omega^2$. Because of this, several authors have proposed to use the expression $k_{LHM} = \sqrt{(-\mu_0)}\sqrt{(-\epsilon_0)}\omega$, to determine the sign of n_{LHM} . This demonstration is not satisfactory because it is mathematically wrong. A similar method is often used to demonstrate that the impedance of the LHM is positive. It is obvious that *it is mathematically impossible to determine the signs of the index or of the impedance by using the wave equation*. As stated, the negative sign of the index can also be obtained from Maxwell’s equations and the ambiguity in the sign is only observed when the wave equation is used to obtain the solution. For uniform plane waves, Maxwell’s equations in LHM can be written by

$$\begin{aligned} \bar{k}_{LHM} \wedge \bar{E} &= -(-\mu_0)\omega\bar{H} \\ \bar{k}_{LHM} \wedge \bar{H} &= (-\epsilon_0)\omega\bar{E} \end{aligned} \quad (27)$$

By doing the identification with the following relations

$$\begin{aligned} \bar{k}_{LHM} \wedge \bar{E} &= -\mu_0 n_{LHM} \omega \bar{H} \\ \bar{k}_{LHM} \wedge \bar{H} &= \epsilon_0 n_{LHM} \omega \bar{E} \end{aligned} \quad (28)$$

it is found $n_{LHM} = -1$.

At an interface, the resonance equation $Z^+ + Z^- = 0$ (Fig. 7) is often used to determine the frequency of a surface wave or a leaky wave¹¹⁻¹³. Our theory, which shows and considers that the impedance of air and LHM have opposite signs, reveals that surface wave is excited at their

interface. The presence of a surface wave is not obvious if we use the classical theory on LHM. However, from the literature, several simulations that consider the flat lens made of a dispersive LHM slabs show clearly that the surface wave is excited and important¹⁴. The surface wave is often more important than the image and the source itself. A consequence of our work is that it is possible to reduce or suppress this surface wave by adding matching sheets.

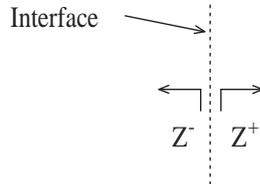


Figure 7: At an interface, the resonance equation $Z^+ + Z^- = 0$ reveals the presence of a surface wave or a leaky wave

Tab. 1: Comparison between the proposed theory and the classical theory on LHMs

	Classical theory	Our theory
Sign of Z_{LHM}	“must be” positive	< 0 from Maxwell’s equations
Sign of n_{LHM}	negative	negative
Interface air-LHM char.	We “have” to use Z_{LHM} ¹⁴	We can use Z_{LHM} or n_{LHM}
LHM slab char.	Time advance	Matched LHM: time delay
$\epsilon_r = -1$ and $\mu_r = -1$	Theoretically not possible ⁷	Theoretically possible
Surface wave: theory	no evidence of existence	exists for unmatched LHM
Surface wave: simulation	always exists	doesn’t exist for matched LHM

A different demonstration that Z_{LHM} is positive has been presented in¹⁵ (Note that the authors use the wave equation). However, their demonstration is not satisfactory. Indeed, they assume *a priori* that Z_{LHM} is positive and their final conclusion is that Z_{LHM} is positive: below Eq. (5)¹⁵, the authors use the equation $Z_{LHM} = \mu_{LHM}/n_{LHM}$. If μ_{LHM} and n_{LHM} are negative, it is obvious that Z_{LHM} is positive. If we replace the equation by $Z_{LHM} = -\mu_{LHM}/n_{LHM}$, Eq. (13) of¹⁵ becomes $Re(Z_{LHM}) < 0$.

In¹⁴, it is stated that to calculate the characteristic of the interface LHM-air we “must” use the impedance and not the index. However, if one considers that the impedance is positive and uses the proposed method, one obtains that the LHM slab introduces a time advance. In addition, this is in contradiction with the work in², where the author considers the multiple wave-reflections between the interfaces, because, for the matching case, no reflection between interfaces should occur.

In², the author uses a sum of geometrical series that does not converge, as it has been observed also in^{5,6}. Nothing can be deduced from this sum. According to⁷, this series can converge by using the “notion of analytic continuation”, even if it “does not converge in the current sense”. However, what does it mean physically? Does it mean that the amplitude of the successive transmitted waves increases and that we have to wait during an infinitely long time to obtain the transmission of all waves? In our work, the convergence of the sum of the geometrical series (*i.e.* the sum of the successive transmitted waves) is verified (Section 5).

In⁷, the authors conclude that a “left-handed material with both relative permittivity and permeability equal to -1 cannot exist”. In our work, there is theoretically no contradiction with the existence of a LHM. There is no problem for simulating a LHM by using available electromagnetic simulators. One encounters a problem when an interface between air and the LHM is introduced. Thanks to the proposed matching sheets, this problem can be resolved.

In³, the authors present an experimental demonstration of the negative index of refraction, by measuring the angle of the refracted field on a wedge shaped body made of an artificial metamaterial. They also measured the refracted field for a Teflon wedge ($\epsilon \approx 2.1$) for comparison. As noticed by Munk¹⁶ and other researchers, an important feature of the presented results is that “the refracted field for the synthesized material is 20 dB or more below the refracted field from the Teflon case. Such a large loss can not be attributed to either ohmic or dielectric loss for frequencies below

100 GHz". We believe that this low transmission level is due to the fact that the material is not matched to free space, according to our theory.

Table 1 summarizes the results obtained with our theory and the classical theory.

8. CONCLUSION

In this work, a new theory on the characteristic impedance of media with simultaneously negative permittivity and permeability, called left-handed materials (LHMs), has been proposed. This theory shows that the characteristic impedance of these media is negative. A method has been proposed to match a LHM slab to free space, by using sheets of resistors and amplifiers. We have also considered the effect of an imperfectly matching. The transmission and reflection characteristics of an imperfectly matched LHM slab has been calculated by using analytical methods (transmission line and multiple-reflections methods). For the first time, the full-wave simulation results (FDTD) of a homogeneous non-dispersive LHM slab excited by a plane wave in air have been presented. The agreement between analytical and FDTD results confirms our theory that the characteristic impedance of a medium with simultaneously negative permittivity and permeability is negative. Future work will concentrate on the analysis of the excitation by plane wave at oblique incidence of an LHM slab and on the flat lens. For oblique incidence, the matching conditions should be modified. We will also verify our theory with artificial LHMs made with periodic structures.

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