

On the Impedance Matching of Left-Handed Materials to Free-Space

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- II. Index of a Left-Handed Medium (LHM)**
- III. Intrinsic impedance of a LHM**
- IV. Interpretation of the results**
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- VI. Potential applications (absorbers, reconfigurable antennas)**
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❖ Objectives

- Revisiting the characteristic parameters (index and impedance) of left-handed media
- Explaining the problem encountered when one simulates a homogeneous left-handed medium with a full-wave electromagnetic calculator
- Proposing a method to match a left-handed medium to free-space for forward waves
- Proposing new applications of left-handed materials

❖ Generalities

- The signs of the index and of the intrinsic impedance of a medium depend on the convention that is chosen
- In a right-handed medium (for example, air), we use a convention such that the signs of the index and of the intrinsic impedance are positive
- To avoid errors, one should use the same convention that is used for a right-handed medium for determining the characteristic parameters of a left-handed medium

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❖ Introduction

To determine the sign of the index of a LHM, one should use Maxwell's equations because the wave-equation leads to an ambiguity, that is mathematically impossible to resolve.

We consider a LHM that has the following parameters :

$$\varepsilon = -\varepsilon_0$$

$$\mu = -\mu_0$$

❖ For uniform plane waves in air, Maxwell's equation can be written

$$\vec{k} \wedge \vec{E} = -\mu_0 \omega \vec{H} \quad \vec{k} \wedge \vec{H} = \varepsilon_0 \omega \vec{E} \quad (1)$$

❖ **Usual definition**

$$\vec{k}_{LHM} = n_{LHM} \overset{\text{Index}}{\vec{k}} \quad \Rightarrow \quad \vec{k} = \frac{\vec{k}_{LHM}}{n_{LHM}} \quad (2)$$

❖ By using (2) in (1), we obtain

$$\vec{k}_{LHM} \wedge \vec{E} = -\mu_0 n_{LHM} \omega \vec{H} \quad \vec{k}_{LHM} \wedge \vec{H} = \varepsilon_0 n_{LHM} \omega \vec{E} \quad (3)$$

❖ We have obtained

$$\vec{k}_{LHM} \wedge \vec{E} = -\mu_0 n_{LHM} \omega \vec{H} \quad \vec{k}_{LHM} \wedge \vec{H} = \varepsilon_0 n_{LHM} \omega \vec{E} \quad (3)$$

❖ We also can deduce easily the following equations

$$\vec{k}_{LHM} \wedge \vec{E} = -(-\mu_0) \omega \vec{H} \quad \vec{k}_{LHM} \wedge \vec{H} = (-\varepsilon_0) \omega \vec{E} \quad (4)$$

❖ By indentifying (4) and (3), we can conclude

$$n_{LHM} = -1$$

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III. Intrinsic impedance of a LHM

- ❖ Let us assume a Medium called Medium A that has the following parameters :

$$\varepsilon = \frac{\varepsilon_0}{p} \quad \mu = p\mu_0$$

Where p is a real

- ❖ For this Medium, Maxwell's equations can be written

$$\nabla \times \vec{E} = -p\mu_0 \frac{\partial \vec{H}}{\partial t} \quad \nabla \times \vec{H} = \frac{\varepsilon_0}{p} \frac{\partial \vec{E}}{\partial t}$$

- ❖ Because p is a real we can write

$$\nabla \times \vec{E} = -\mu_0 \frac{\partial(p\vec{H})}{\partial t} \qquad \nabla \times (p\vec{H}) = \varepsilon_0 \frac{\partial\vec{E}}{\partial t}$$

- ❖ From this, the same results that those obtained for air can be used for Medium A, by using pH instead of H .

- ❖ **Usual definition**

$$\eta = \frac{E}{H} \qquad \text{Intrinsic impedance}$$

III. Intrinsic impedance of a LHM

- ❖ The same results that those obtained for air can be used for Medium A, by using pH instead of H .

$$\Rightarrow \frac{E}{pH} = \eta_0 = 120\pi$$

- ❖ We can conclude that the intrinsic impedance of Medium A is

$$\eta = \frac{E}{H} = p120\pi$$

Note: p can be positive or negative. One can easily check the validity of this equation for positive values of p .

III. Intrinsic impedance of a LHM

❖ If $p = -1$, Medium A is a LHM

$$\epsilon = -\epsilon_0 \quad \mu = -\mu_0$$

❖ As a result, the Intrinsic impedance of a LHM is

$$\eta_{LHM} = \frac{E}{H} = -120\pi = -\eta_0$$

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IV. Interpretation of the results

- ❖ We have found that the intrinsic impedance of a LHM is negative (this is validated by numerical analysis using the FDTD method and a commercial software, HFSS, as it will be shown later)
- ❖ This does not mean that the medium is active : in the same way that the intrinsic impedance of a right-handed medium does not correspond to a loss, the intrinsic impedance of a LHM does not correspond to a gain
- ❖ We have obtained this result because the intrinsic impedance is usually defined for a forward wave (a wave that goes from the generator to the load)

IV. Interpretation of the results

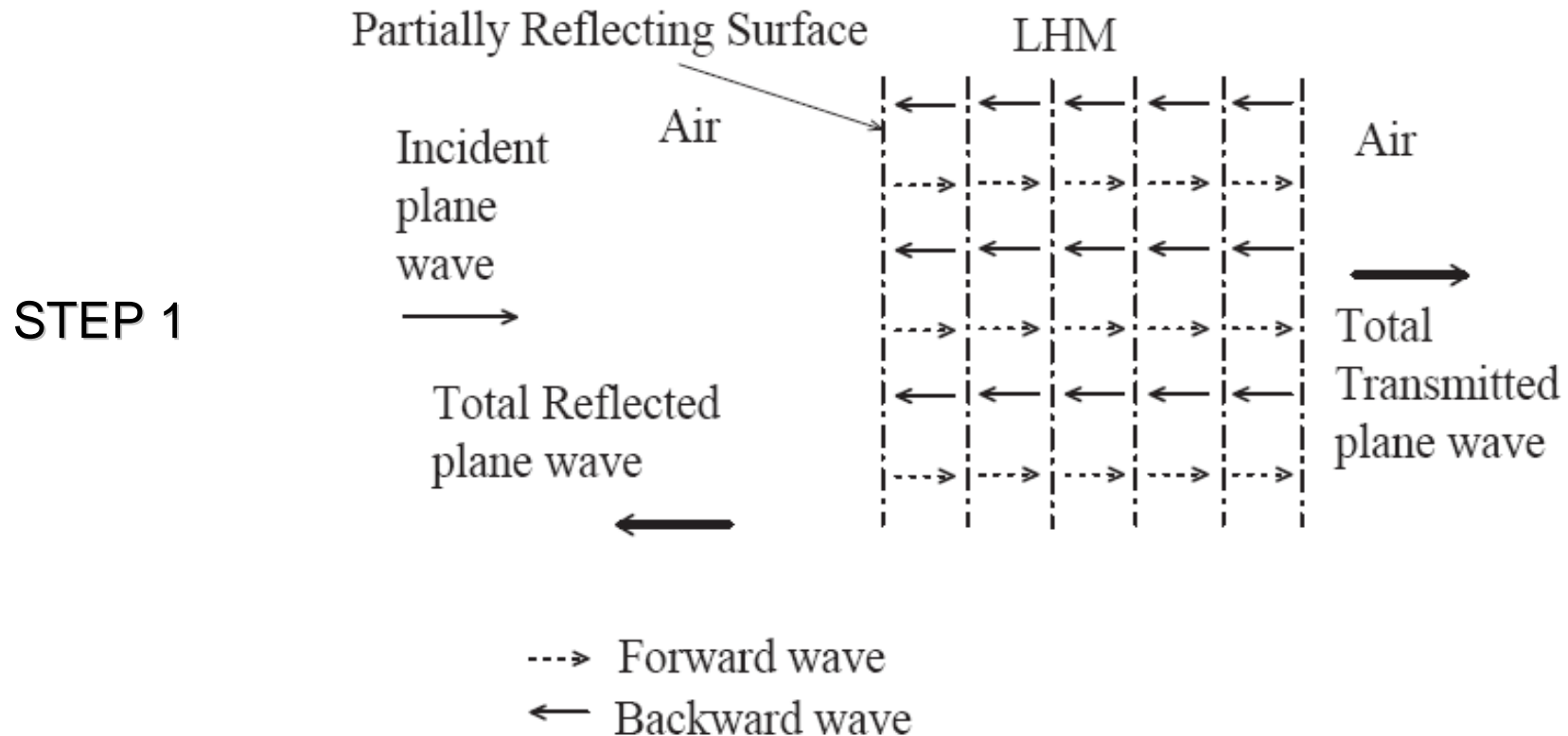
❖ Taking the convention for current flow to be from the generator end to the load, we can make the following remarks :

In **air**, a **forward wave** has a **positive** intrinsic impedance η_0 and the **backward wave** has a **negative** intrinsic impedance $-\eta_0$

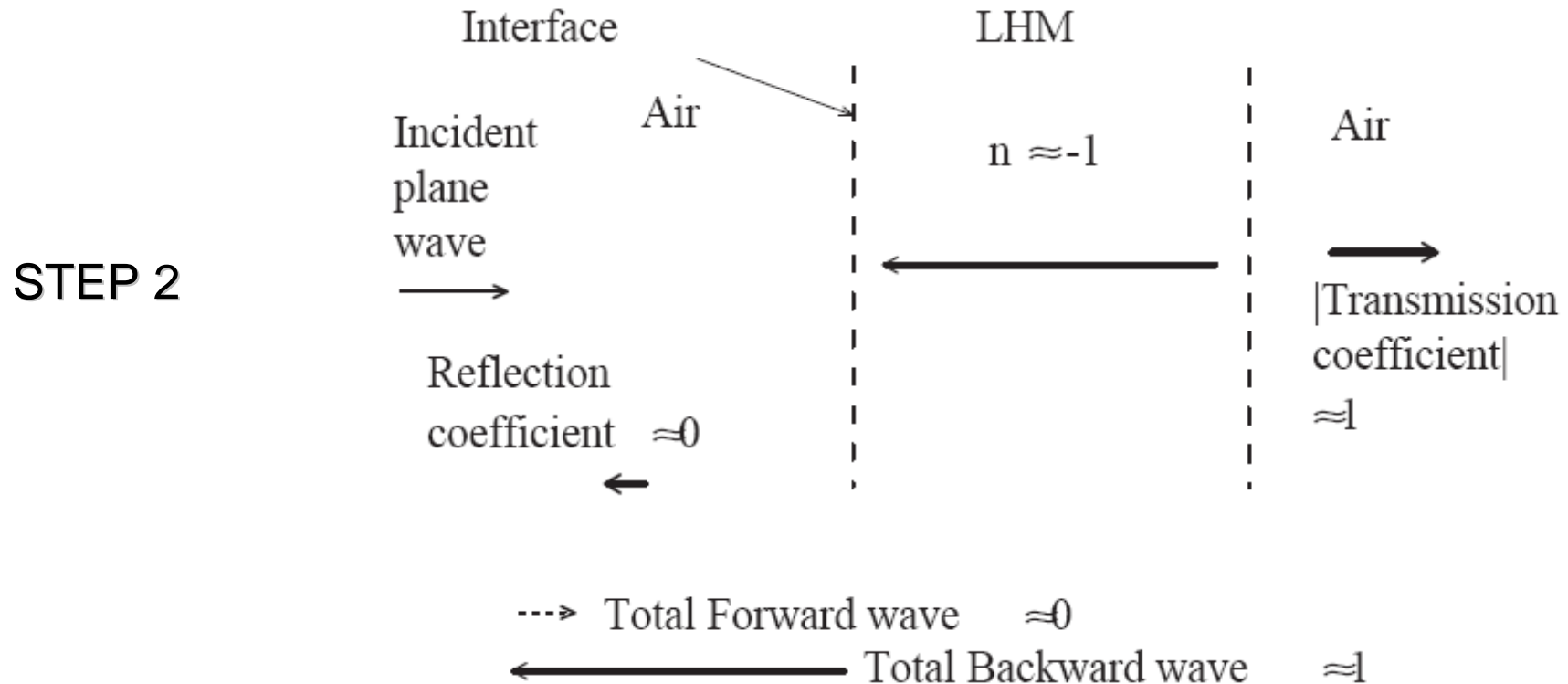
In **LHM**, a **forward wave** has a **negative** intrinsic impedance $-\eta_0$ and the **backward wave** has a **positive** intrinsic impedance η_0

IV. Interpretation of the results

❖ Principle of homogenization of a LHM



❖ Principle of homogenization of a LHM



IV. Interpretation of the results

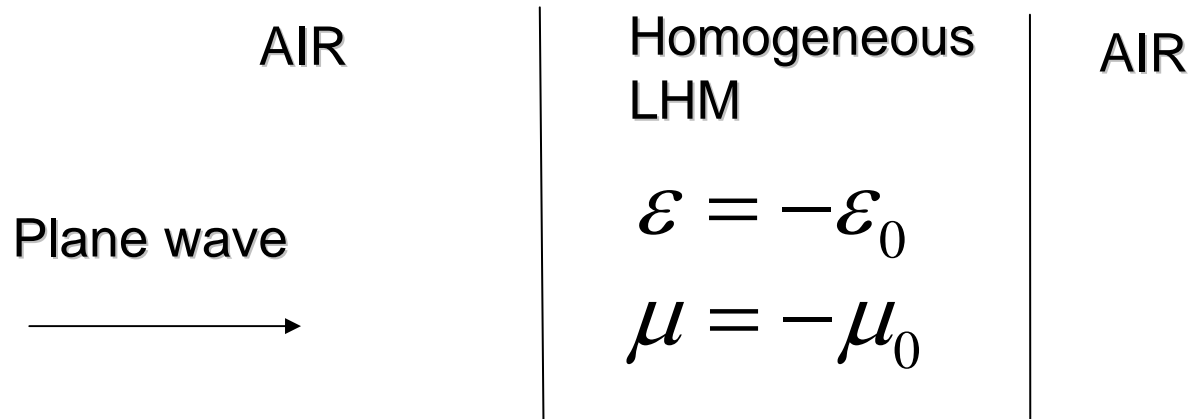
❖ In the LHM made from a periodic structure, the total backward wave is predominant as compared to the total forward wave

❖ From this, the LHM is **matched** to free space, because the intrinsic impedance of the LHM for backward wave and the intrinsic impedance of air for forward wave have same sign and same value

❖ However, it is not possible to confirm this matching by using a homogeneous LHM, because the backward wave is not excited for this case

One can try to simulate a homogeneous LHM slab in free space by using usual available home-made or commercial software to check our statement

❖ Numerical Analysis



We have tested this problem with a home-made FDTD code and with Ansoft HFSS

Results \Rightarrow The FDTD program becomes unstable and HFSS results give values of S11 and S21 very large

Explanation \Rightarrow the intrinsic impedance of the LHM is negative and it is not possible to excite the backward wave for a homogeneous LHM

IV. Interpretation of the results

❖ In the LHM, the negative total power

$$P = \varepsilon |E|^2 + \mu |H|^2$$

is another confirmation that the intrinsic impedance is negative

❖ The negative total power and the negative intrinsic impedance inside the LHM means that the wave goes to the generator (backward wave)

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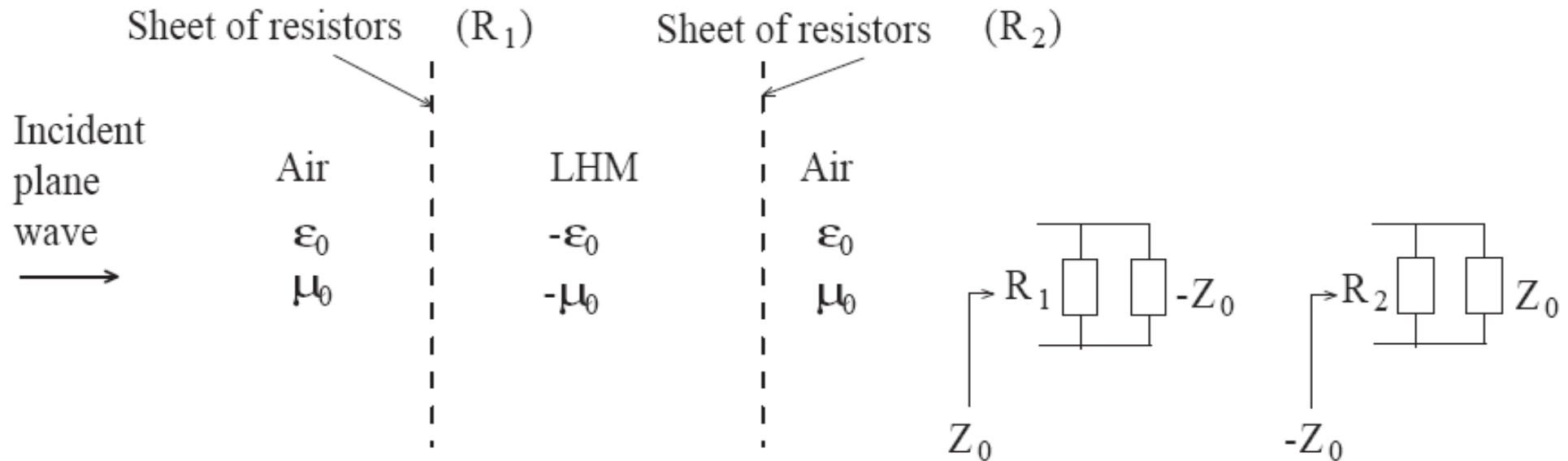
III. Intrinsic impedance of a LHM

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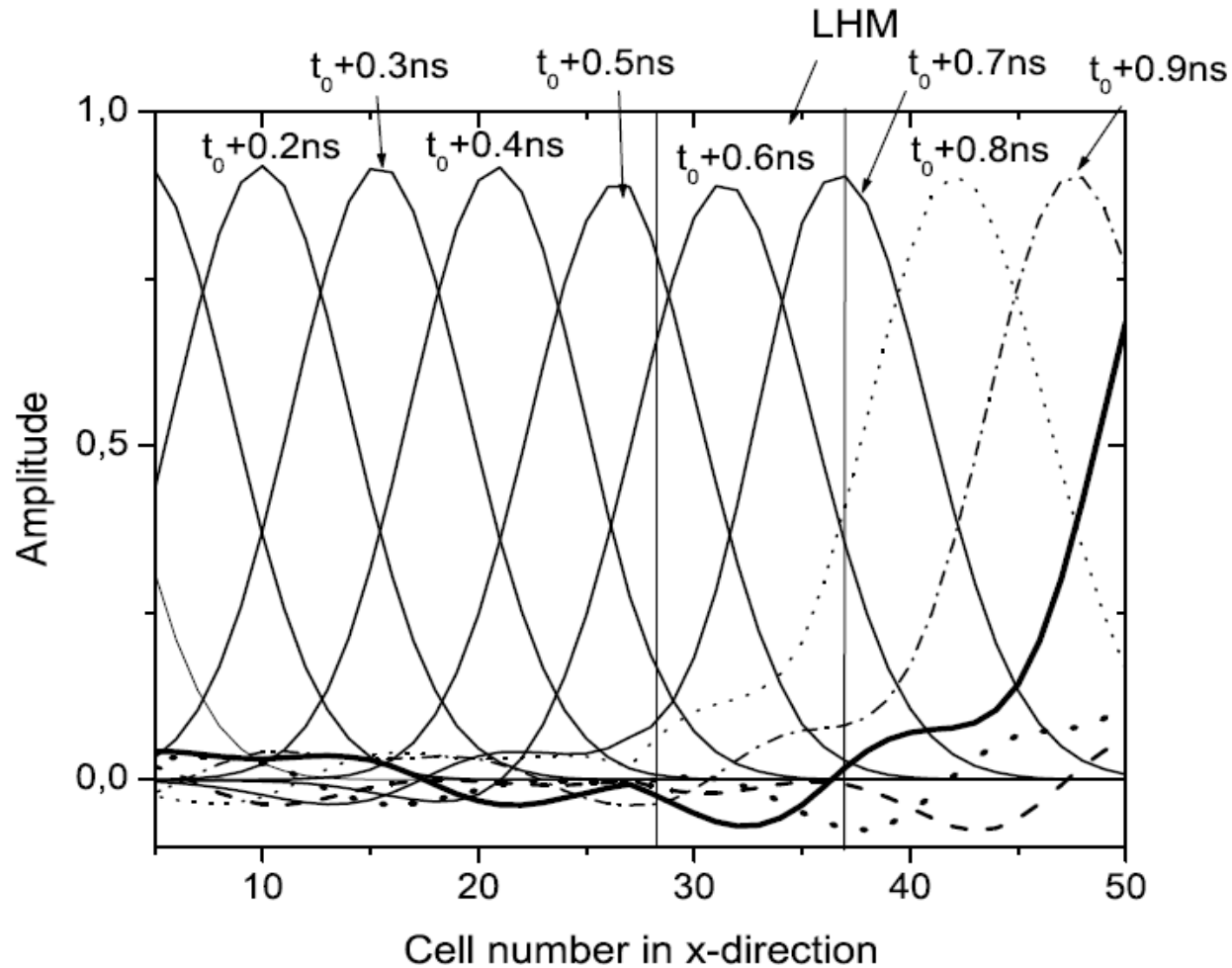
VII. Conclusion



$$R_1 = \frac{Z_0}{2} = 60\pi$$

$$R_2 = -\frac{Z_0}{2} = -60\pi$$

❖ FDTD results



❖ HFSS results show also that by using the adequate surface impedance boundaries (R_1 and R_2) in both sides of the LHM slab, S_{11} becomes null and S_{21} becomes equal to one

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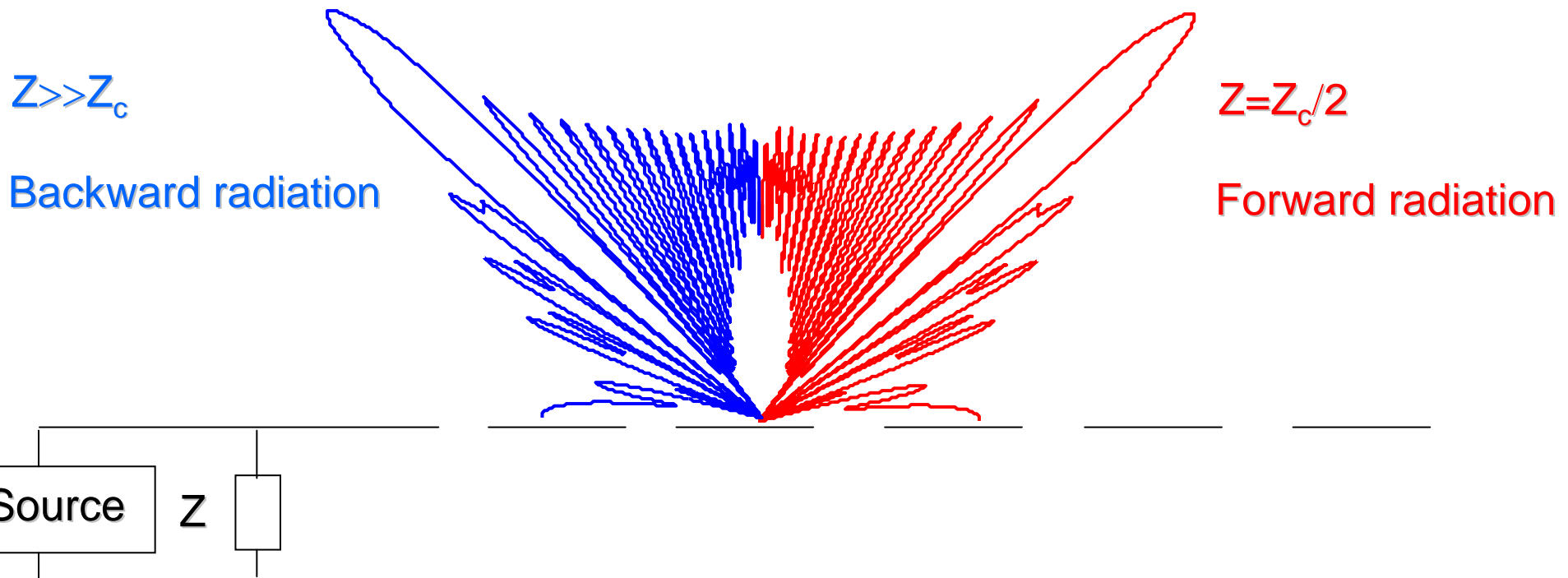
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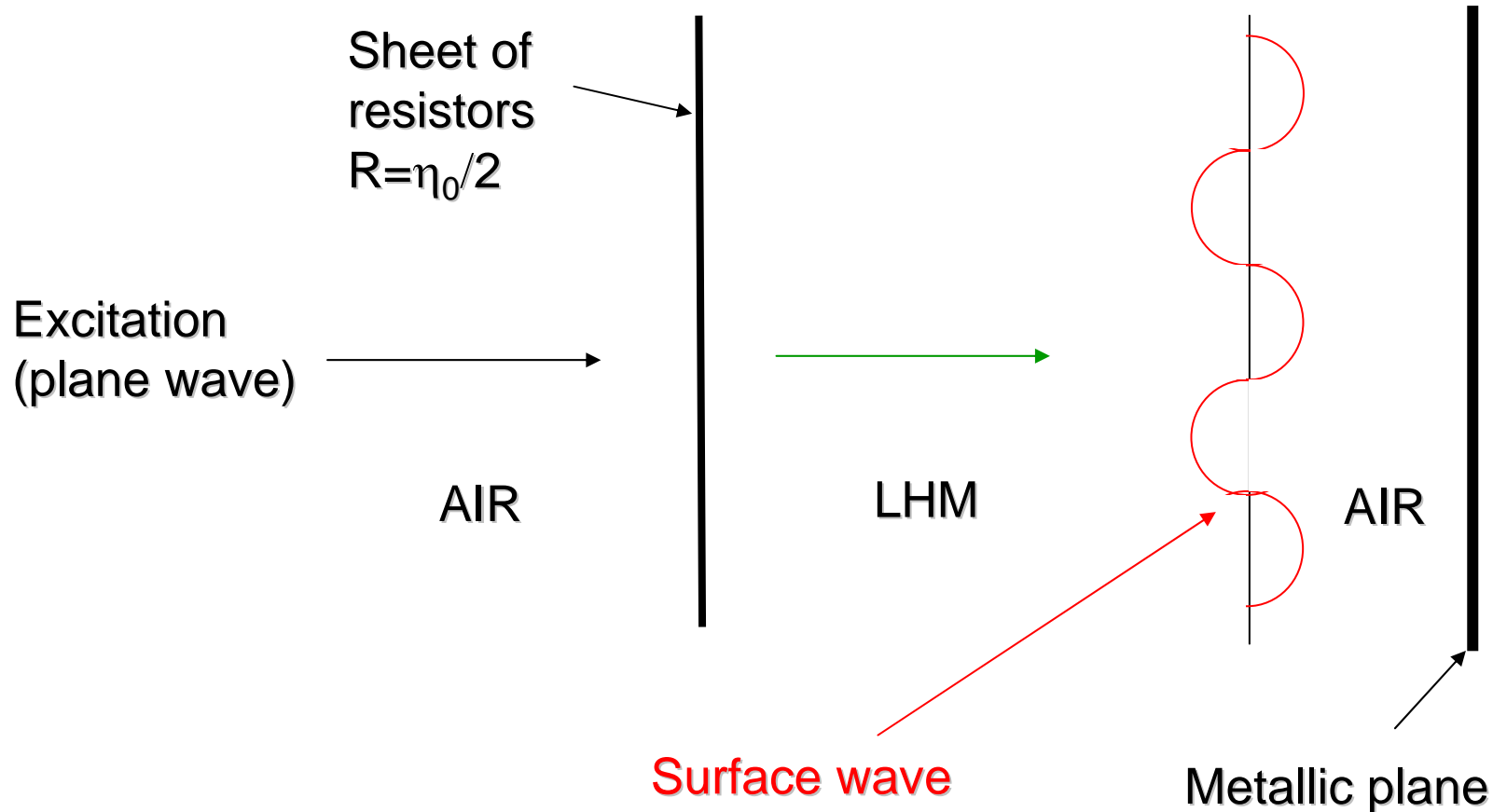
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❖ Reconfigurable backward-radiation leaky-wave antenna



❖ Absorbers



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VII. Conclusion

- ❖ By using Maxwell's equations, we have shown that $n_{\text{LHM}} = -1$ and $\eta_{\text{LHM}} = -\eta_0$
- ❖ The negative intrinsic impedance is due to the definition of the intrinsic impedance (for forward waves) and the backward wave that is predominant inside a LHM made of a periodic structure
- ❖ It is not possible to excite the backward wave of a homogeneous LHM. FDTD results and HFSS results give transmission and reflection coefficients for a LHM slab that tend to infinity
- ❖ The problems encountered with the numerical simulation of homogeneous LHMs are due to the negative intrinsic impedance of the LHM
- ❖ It is possible to match the LHM for forward waves
- ❖ We have proposed new schemes and applications of LHMs
- ❖ The numerical results presented can be tested with any full-wave electromagnetic software