How long can excess pollution persist? The non-cooperative case

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May 14, 2007

Appendix

A The perfect information case

The parameters of preference for the environment of the two countries θ_i et θ_j are common knowledge. Let us suppose $\theta_i > \theta_j$: country i is the greenest and it knows, in a non-cooperative framework, that it will bear the whole burden of stabilization (if it is sufficient), or the greatest part of it (if it is not). The only variable that must be determined is the date of stabilization.

A.1 Stabilization before T_m

If country i decides to complete stabilization at $T \leq T_m$, it must reduce its pollution to the level $P(T)-p = \delta q_{\text{BAU}}(T)-p$. Its utility is then $u_i^{L1}(t) = (P(T)-p)+\theta_i q_{\text{BAU}}(T) = (\delta+\theta_i)q_{\text{BAU}}(T)-p = u_i^{L1}(T)\ \forall t \geq T$. Its intertemporal welfare from $T \leq T_m$ is

$$V_i^{L1}(T) = \int_T^\infty e^{-\gamma(t-T)} u_i^{L1}(t)dt = \frac{u_i^{L1}(T)}{\gamma} = \frac{(\delta+\theta_i)q_{\text{BAU}}(T)-p}{\gamma}.$$

Its intertemporal welfare from the origin is then

$$W_i^1(T) = \int_0^T e^{-\gamma t} u_i(t)dt + e^{-\gamma T} V_i^{L1}(T).$$

We have

$$\begin{aligned}\frac{dW_i^1(T)}{dT} &= e^{-\gamma T}\left[u_i(T) - \gamma V_i^{L1}(T) + \frac{dV_i^{L1}(T)}{dT}\right] \\ &= e^{-\gamma T}\frac{\gamma-\delta-\theta_i}{\gamma}\left(2p - \delta q_{\text{BAU}}(T)\right).\end{aligned}$$

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Intertemporal welfare is then a strictly decreasing function of the date of stabilization if the parameter of preference for the environment is high enough compared to impatience corrected for natural regeneration ($\theta_i > \gamma - \delta$); in this case, country i must stabilize at once ($T = 0$). Its intertemporal welfare is then $W_i^1(0) = V_i^{L1}(0) = \frac{(\delta+\theta_i)q_0-p}{\gamma}$. If, on the contrary, the preference for the environment of country i is low ($\theta_i < \gamma - \delta$), it will not stabilize before T_m. Finally, country i is indifferent to any stabilization date between 0 and T_m if $\theta_i = \gamma - \delta$.

A.2 Stabilization after T_m

If $\theta_i < \gamma - \delta$, country i which has the highest preference for the environment and then is the loser does not stabilize before T_m.

Let us suppose that it decides to cut its pollution to zero at $\widetilde{T} \in]T_m, T_x]$ and that the winner then stabilizes at $\tau \geq \widetilde{T}$. Between $\widetilde{T}$ and τ, the utility of the winner is $(p + \theta_j q(t))$ as far as it goes on polluting p, $q(t)$ being given by equation (6). After τ, the utility of the winner is $u_j^{W2}(\tau) = (\delta + \theta_j)q(\tau)$ and its intertemporal welfare after stabilization has occured is

$$V_j^{W2}(\tau) = \frac{(\delta + \theta_j)q(\tau)}{\gamma}.$$

From $\widetilde{T}$ on, the intertemporal utility of the winner which stabilizes at τ is then given by

$$W_j^2(\widetilde{T}, \tau) = \int_{\widetilde{T}}^{\tau} e^{-\gamma(t-\widetilde{T})}(p + \theta_j q(t))dt + e^{-\gamma(\tau-\widetilde{T})}V_j^{W2}(\tau)$$

with $q(t)$ given by equation (6).

We then have

$$\begin{aligned}
\frac{dW_j^2(\widetilde{T}, \tau)}{d\tau} &= e^{-\gamma(\tau-\widetilde{T})}\left[p + \theta_j q(\tau) - \gamma V_j^{W2}(\tau) + \frac{dV_j^{W2}(\tau)}{d\tau}\right] \\
&= e^{-\gamma(\tau-\widetilde{T})}\left[p - \delta q(\tau) + \frac{\delta + \theta_j}{\gamma}\left(\delta q(\tau) - p\right)\right] \\
&= \left(\delta q(\tau) - p\right) e^{-\gamma(\tau-\widetilde{T})}\frac{\theta_j + \delta - \gamma}{\gamma}
\end{aligned}$$

Intertemporal welfare is a strictly increasing function of the date of stabilization τ decided by the winner, knowing that the loser cuts its pollution to zero at $\widetilde{T}$, because the winner's parameter of preference for the environment is low enough compared to impatience corrected for natural regeneration (we are by assumption in the case $\theta_j < \gamma - \delta$), and the winner pollutes more than natural regeneration. The winner will not perform stabilization before $\widetilde{t}(\widetilde{T})$ defined by equation (7).

Knowing that the winner will not make any concession to avoid the exhaustion of environmental quality, what will the loser do?

The intertemporal welfare of country i from the origin is

$$W_i^2(\widetilde{T}) = \int_0^{\widetilde{T}} e^{-\gamma t}u_i(t)dt + \int_{\widetilde{T}}^{\widetilde{t}(\widetilde{T})} e^{-\gamma t}\theta_i q(t)dt,$$

environmental quality being equal to $q_{\text{BAU}}(t)$ (equation (3)) in the first integral, and being given by equation (6) in the second one, and we have

$$\frac{dW_i^2(\widetilde{T})}{d\widetilde{T}} = e^{-\gamma\widetilde{T}}\left[u_i(\widetilde{T}) - \theta_i q_{\text{BAU}}(\widetilde{T})\right] + e^{-\gamma\widetilde{t}}\theta_i q(\widetilde{t})\frac{d\widetilde{t}(\widetilde{T})}{d\widetilde{T}}.$$

As far as $q(\widetilde{t}) = 0$, we then have

$$\frac{dW_i^2(\widetilde{T})}{d\widetilde{T}} = e^{-\gamma\widetilde{T}}p > 0.$$

So the optimal behaviour of the loser is to do nothing and let the environmental quality be exhausted at T_x, as the benefit of its concession would be wholly captured as an extra rent by the winner.

B Proof that θ_m is increasing in γ

θ_m is given by equation (17), with T_m independent of γ (equation (5)). A tedious derivation then allows us to show that

$$\frac{\partial\theta_m}{\partial\gamma} = \frac{\left(\overline{\theta}-\underline{\theta}\right)\left[\left(\overline{\theta}-\underline{\theta}\right) + e^{(\underline{\theta}+\delta-\gamma)T_m}\left(\left(\overline{\theta}+\delta-\gamma\right)\left(\underline{\theta}+\delta-\gamma\right)T_m - \left(\overline{\theta}-\underline{\theta}\right)\right)\right]}{\left[\left(\overline{\theta}+\delta-\gamma\right)e^{(\underline{\theta}+\delta-\gamma)T_m} - \left(\overline{\theta}-\underline{\theta}\right)\right]^2},$$

and has the sign of the second term of the numerator.

Let us note $x = \left(\underline{\theta}+\delta-\gamma\right)T_m$. x is strictly positive by assumption 2. We have $\overline{\theta}+\delta-\gamma = \overline{\theta}-\underline{\theta}+\frac{x}{T_m}$, and $\frac{\partial\theta_m}{\partial\gamma}$ has the sign of $\left[\left(\overline{\theta}-\underline{\theta}\right) + e^x\left(\left(\overline{\theta}-\underline{\theta}+\frac{x}{T_m}\right)x - \left(\overline{\theta}-\underline{\theta}\right)\right)\right]$ i.e. the sign of $\left[1 + e^x\left(\left(1+\frac{x}{(\overline{\theta}-\underline{\theta})T_m}\right)x - 1\right)\right]$, which is obviously always strictly positive for $x > 0$.

C Proof of Proposition 3

The intertemporal expected utility of country i in the first game is given by equation (11), with $u_i(t)$ given by equation (2), $q_{\text{BAU}}(t)$ by equation (3), $V_i^{L1}(T)$ by equation (9) and $V_i^{W1}(T)$ by equation (10):

$$\begin{aligned} EU_i^1(T) &= \left[1-H_j(T)\right]\left[\int_0^T e^{-\gamma t}\left(p+\theta_i q_{\text{BAU}}(t)\right)dt + e^{-\gamma T}\frac{(\delta+\theta_i)q_{\text{BAU}}(T)-p}{\gamma}\right] \\ &\quad + \int_0^T h_j(t)\left[\int_0^t e^{-\gamma s}\left(p+\theta_i q_{\text{BAU}}(s)\right)ds + e^{-\gamma t}\frac{p+\theta_i q_{\text{BAU}}(t)}{\gamma}\right]dt. \end{aligned}$$

A straightforward (but tedious) calculation of the two integrals in this expression allows us to obtain

$$\begin{aligned} EU_i^1(T) &= \left[1-H_j(T)\right]\left[K + e^{-\gamma T}\frac{\theta_i+\delta-\gamma}{\gamma(\gamma-\delta)}\left(2p-\delta q_{\text{BAU}}(T)\right)\right] \\ &\quad + \int_0^T h_j(t)\left[K + e^{-\gamma t}\frac{\theta_i}{\gamma(\gamma-\delta)}\left(2p-\delta q_{\text{BAU}}(t)\right)\right]dt, \end{aligned}$$

with

$$K = \frac{\theta_i}{\gamma - \delta} q_0 + \frac{\gamma - \delta - 2\theta_i}{\gamma(\gamma - \delta)} p.$$

So we have, using $q_{\text{BAU}}(T_m) = \frac{p}{\delta}$,

$$EU_i^1(T_m) = [1 - H_j(T_m)] \left[K + e^{-\gamma T_m} \frac{\theta_i + \delta - \gamma}{\gamma(\gamma - \delta)} p \right] + \int_0^{T_m} h_j(t) \left[K + e^{-\gamma t} \frac{\theta_i}{\gamma(\gamma - \delta)} \left(2p - \delta q_{\text{BAU}}(t)\right) \right] dt.$$

The intertemporal expected utility of country i in the second game is

$$\begin{aligned} EU_i^2(T) &= [1 - H_j(T)] \left[\int_0^T e^{-\gamma t} \left(p + \theta_i q_{\text{BAU}}(t)\right) dt + e^{-\gamma T} \frac{\theta_i q_{\text{BAU}}(T)}{\gamma} \right] \\ &\quad + \int_0^T h_j(t) \left[\int_0^t e^{-\gamma s} \left(p + \theta_i q_{\text{BAU}}(s)\right) ds + e^{-\gamma t} \frac{(\delta + \theta_i) q_{\text{BAU}}(t)}{\gamma} \right] dt, \end{aligned}$$

or, after calculation of the two integrals,

$$\begin{aligned} EU_i^2(T) &= [1 - H_j(T)] \left[K + e^{-\gamma T} \left(-\frac{p}{\gamma} + \frac{\theta_i}{\gamma(\gamma - \delta)} \left(2p - \delta q_{\text{BAU}}(T)\right) \right) \right] \\ &\quad + \int_0^T h_j(t) \left[K + e^{-\gamma t} \left(\frac{p}{\gamma} + \frac{\theta_i + \delta - \gamma}{\gamma(\gamma - \delta)} \left(2p - \delta q_{\text{BAU}}(t)\right) \right) \right] dt. \end{aligned}$$

So we get

$$\begin{aligned} \lim_{T \searrow T_m} EU_i^2(T) &= [1 - H_j(T_m)] \left[K + e^{-\gamma T_m} \frac{\theta_i + \delta - \gamma}{\gamma(\gamma - \delta)} p \right] \\ &\quad + \int_0^{T_m} h_j(t) \left[K + e^{-\gamma t} \left(\frac{p}{\gamma} + \frac{\theta_i + \delta - \gamma}{\gamma(\gamma - \delta)} \left(2p - \delta q(t)\right) \right) \right] dt. \end{aligned}$$

We obtain finally

$$\begin{aligned} &\lim_{T \searrow T_m} EU_i^2(T) - EU_i^1(T_m) \\ &= \int_0^{T_m} h_j(t) e^{-\gamma t} \left(\frac{p}{\gamma} + \frac{\theta_i + \delta - \gamma}{\gamma(\gamma - \delta)} \left(2p - \delta q_{\text{BAU}}(t)\right) - \frac{\theta_i}{\gamma(\gamma - \delta)} \left(2p - \delta q_{\text{BAU}}(t)\right) \right) dt \\ &= \int_0^{T_m} h_j(t) e^{-\gamma t} \frac{\delta q_{\text{BAU}}(t) - p}{\gamma} dt. \end{aligned}$$