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Link between the free field and the elements of frontiers in a complex structure

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Abstract

In a complex structure, interactions between various elements that must be compute to resolve EMC problem, can exists through radiated or conducted fields. In this second case, the description of the field propagation is easier in the frequency domain. We present in this paper an analytical approach to resolve the field in a large and complex structure and to incorporate its description in a tensorial analysis of network developed in a previous work on the base of Kron's formalism [1][2]. We discuss first of the various kind of field describe in the Poynting's equation. From this equation we discuss of two states of the field: the trans electromagnetic guided waves and the stationary one. From this second kind we study the construction of 3D modes. This allows us to introduce a model for guided waves in both domains: frequency and temporal one. To conclude we show how to determine the transmission of energy in any structure using a technique near to Branin's one, considering 3D TE modes of propagation in the frequency domain and wavelets for the time domain and non linear impedances.

1. Introduction

Previous works have clearly identified the various kind of fields involved in the EMC of complex system [1][3]. These various fields are represented by various elements in the topology. So, to have a good understanding on the representation of the field in closed volumes and to see the various possibilities to manage the link between the field and the currents on the surface around it, we try to construct simple images of these mechanisms in order to join the formulas of guided waves in the frequency domain.

Once the formulas are based in the frequency domain, we try to make the link with the expression of the same field in the temporal domain. We show that this link is complex and justify the fact to use the frequency description of the field even in a temporal process. The frequency description allows to understand how the field energy is guided through various modes. This understanding is essential to improve efficiently the EMC interactions computed with the tensorial analysis of network method [1].

2. Poynting's energy, field nature and TEM waves

The field is a quantity very difficult to understand. The free field is quite clearly understood starting from the force concept[4]. But when we want to make a link between the classical field and the quantum one, it stills difficult. The free field diffusion is well represented by the Green's function [5][6]. Event in quantum mechanics, the Green's function can be seen as the probability to measure a photon on a trajectory. We still in a Maxwell's description of the field, where the field in an abstract mathematical expression very directly similar with this Green's function[7]. But further than the field, another difficulty appears in closed structures: the link with the charges and currents. As Einstein's said[8]: "Maxwell's equations does not say to us exactly how the charges are". There is for that one reason: the self interaction of the electron is a very difficult problem[9]. It can be obtained at a macroscopic level through the self interaction between the various element of an antenna, or an electronic circuit. This global computation is very interesting and is included in the Poynting's theorem. To have good patterns of the interactions in closed structures where there is a hard link between the field and the charges and currents in the structure, a solution consists in using energy approaches. Using potentials rather than fields, as Maxwell's has done[10], it's easier to find patterns efficient to compute the fields and the currents in complex structures. Following Jackson[6] we can recall the link between the field and the currents. Poynting's theorem is the equation of conservation of energy W for the electromagnetic field. In a volume v , we can write that the energy dissipated in the field-currents interaction is:

$$W = \int_v d^3x \vec{J} \cdot \vec{E} \quad (1)$$

Using the Ampere-Maxwell law we can write:

$$W = \int_v d^3x \left[\vec{E} \cdot (\vec{\nabla} \times \vec{H}) - \frac{\vec{E} \cdot \partial \vec{D}}{\partial t} \right] \quad (2)$$

We have the vectorial identity:

$$\vec{\nabla} \cdot (\vec{E} \cdot \vec{H}) = \vec{H} \cdot (\vec{\nabla} \cdot \vec{E}) - \vec{E} \cdot (\vec{\nabla} \cdot \vec{H}) \quad (3)$$

Using this identity and the Faraday's law we obtain:

$$\int_{\forall} d^3 x \vec{J} \cdot \vec{E} = - \int_{\forall} d^3 x \left[\vec{\nabla} \cdot (\vec{E} \times \vec{H}) + \frac{\vec{E} \cdot \partial \vec{D}}{\partial t} + \frac{\vec{H} \cdot \partial \vec{B}}{\partial t} \right] \quad (4)$$

If, as usually, we note u the total energy density with:

$$u = \frac{1}{2} (\vec{E} \cdot \vec{D} + \vec{B} \cdot \vec{H}) \quad (5)$$

We can write:

$$- \int_{\forall} d^3 x \vec{J} \cdot \vec{E} = \int_{\forall} d^3 x \left[\frac{\partial u}{\partial t} + \vec{\nabla} \cdot (\vec{E} \times \vec{H}) \right] \quad (6)$$

This relation leads to the known Poynting's equation given by:

$$\frac{\partial u}{\partial t} + \vec{\nabla} \cdot \vec{S} = - \vec{J} \cdot \vec{E} \quad (7)$$

With \vec{S} being the Poynting's vector. This expression shows the three kinds of electromagnetic energy: the conductive one ($\vec{E} \cdot \vec{J}$), the propagative one ($\vec{\nabla} \cdot \vec{S}$) and the stored one (u). Each energy speaks of a particular kind of field. The flux \vec{S} represents the flow of radiated energy including the radiation resistance while \vec{J} represents the field in the matter under its particular aspect. u gives the quantity of electromagnetic energy contains in equivalent capacitance and inductance, i.e. in the stationary fields in the volume. This energy is strictly the same as the one we compute in case of LC circuit. That's why modes can be schematically represented with LC circuits too, which is a technique often used in high frequency[11]. There is here an important point. In Maxwell's equations, a radiated field can be created by a static field. The equation that Maxwell has added demonstrates that a Coulomb's field generates a magnetic field. The Faraday's law then show that this magnetic field creates after an electric one, and the field wave process is understood from this basic mechanism. Effectively, in the case of a simple plate capacitor we have:

$$C = \epsilon_0 \frac{S}{h} \quad (8)$$

The current in this capacitor is given by the law:

$$i_c = C \frac{\partial V}{\partial t} \quad (9)$$

By replacement and with Ampere's law we obtain:

$$\epsilon_0 \frac{S}{h} \frac{\partial}{\partial t} \vec{E} \cdot \vec{h} = \iint_s \vec{d}s \cdot \vec{H} \Rightarrow \vec{\nabla} \times \vec{H} = \epsilon_0 \frac{\partial \vec{E}}{\partial t} \quad (10)$$

In this well known relation \mathbf{E} can be a Coulombian field, a static one derived from a scalar potential in the coulomb's gauge. The inverse process results from the stationary field construct with a radiated one. When we take a look to the charges and currents distribution on the walls of a cavity, we see that the charges still static, exactly like in a capacitor. The "capacitor" field coming from a scalar potential is a longitudinal one, while the radiative one comes from the vector potential. These transformations can be represented by the symbolic diagram given figure 1.

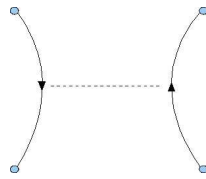


Figure 1: TEM to static field process

The dashed line is the TEM wave associated with the radiated or propagated transverse field while the full line is the longitudinal field. the difference of nature between the two fields (the longitudinal one and the transverse one) is deep. In quantum mechanics, Feynman shows that the Coulombian interaction can be seen as a virtual photon[12].

In complex structure, the problem will consists in computing the various transformation of the field in losses in the walls, stored energy in the stationary waves and propagation in the TEM waves. But, as we will show it, it is possible to have another kind of field where the stored energy moves like a TEM wave. In this model, the field is seen like a "2D particle". This kind of fields are commonly experiments in waveguides. The advantage of this model is that it includes efficiently the relations between the field in the free space of guided wave and the charges and currents in the wall of the structure.

2. The TE wave

In a non homogeneous structure, TEM wave cannot exists. If we take a look to a structure of guided waves, the waves propagates in the direction x , with the electric field along z and a width in the direction y . If the structure is homogeneous on the width Y , a TEM wave can exists with a Poynting's vector s_x . Now if we imagine a change of height in the direction y , a part of field should be reflected to equilibrate the energy. The potential stills the same at the equilibrium state all along the y direction. So, the field which is a gradient of the potential must be different. But if the field is different, Ampere's law tells us that an x component of the magnetic field must appears. The electric field is then associated with two magnetic fields: one in the x direction and one in the y direction. The non

homogeneous structure creates a wave of transverse electric field. If we consider a distribution of the field depending on y like:

$$\vec{E}_z(y) = E_0 \sin(k_y y) \vec{u}_z \quad (11)$$

where E_0 is a given amplitude of the field. This field can propagate in the x direction. The field in both x and y directions has this definition:

$$\vec{E}_z(x, y) = E_0 \sin(k_y y) \vec{u}_z e^{i(\omega t - k_x x)} \quad (12)$$

By replacement of this field in the equation of Helmholtz we find:

$$k_x = \sqrt{\frac{\omega^2}{c^2} - k_y^2} \quad (13)$$

Under the cutoff frequency $\pi c/Y$ if Y is the structure width, the wave vector is evanescent. The dependency of the field in the y direction is obtained for a short-circuit limit condition. If we suppose $X \gg Y$ with X the structure length, the field defined in 11 can propagate in the x direction. Before to study how the total field will be distributed, we recall the topological mechanism of a line structure.

3. Line structure

Using the Branin's modeling, a line structure is represented topologically by two edges and four nodes, two nodes for one port. From each port, the impedance seen is the backward impedance determined from the reflection coefficient at the other port. The figure 2 shows one simple case of two edges connected with a line: a generator and an impedance.

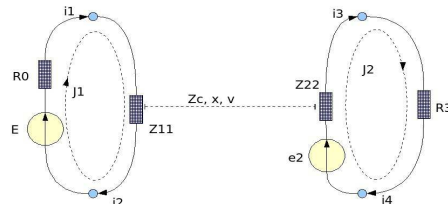


Figure 2: simple circuit with a line

The backward impedances Z_{11} and Z_{22} are given by the ratio of the sum of the incident and reverse voltage and current seen from a port:

$$Z_{11} = Z_c \frac{V_i + V_R}{i_i - i_R} = Z_c \left(\frac{1 + s_2 e^{-2ikX}}{1 - s_2 e^{-2ikX}} \right) \quad (14)$$

s_2 is the reflexion coefficient computed at the opposite port:

$$s_2 = \frac{R_3 - Z_c}{R_3 + Z_c} \quad (15)$$

Z_{22} and s_1 are computed by the same way. The backward impedance takes into account the reflected wave of voltage. But the transmitted one must be added to the second mesh in order to transmit the energy coming from the generator E[13].

$$e_2 = \frac{Z_{11} E (Z_{22} + R_3)}{[R_0 + Z_{11}] (1 + s_2 e^{-2ikX})} \frac{e^{-ikX} (1 - s_2)}{Z_c} \quad (16)$$

This expression, quite complicated, is extracted from the mesh current J2 equation. Writing:

$$2 \frac{e_2}{(Z_{22} + R_3)} = J^2 \quad (17)$$

we obtain relation 16. The guided wave interaction is finally computed using for one side the backward impedances Z_{11} and Z_{22} , and adding in the source vector the transmit generator $e_2[1]$.

4. TE wave in a closed structure

The previous approach can be extend to the case where the propagation is not in a TEM mode. Under the cutoff frequency of the structure, the transmitted energy vanishes. Up to this frequency, a wave can propagates with a field organized following a TE mode. If we imagine a closed structure, this field is completely reflected at the end of the structure. At the beginning, we observe a wave given by:

$$E_z(x=0) = E_0 \sin(k_y Y) [1 - e^{-2ik_x X}] = E_0 \sin(k_y Y) i 2 e^{ik_x X} \left\{ \sin(k_x X) \right\} \quad (18)$$

The modes link with the closed structure appear if we write:

$$E_z(x=0) = i 2 e^{ik_x X} E_0 \sin\left(n \frac{2\pi}{\lambda_x} X\right) \sin\left(m \frac{2\pi}{\lambda_y} Y\right) = i 2 e^{ik_x X} E_0 TE_{(n,m)}^1 \quad (19)$$

TE^1 being the normalized trans electric mode of propagation. This mode is obtained for a given polarization of the source of field. Figure 2 shows that the topology of the problem cannot be separate from the source of the field. The polarization and the location of this source cause the field organization in the volume at the various frequency band. The first source of field coming from the generator is the scalar potential. Equation 19 can be integrated along z to obtain the transmitted voltage to the structure of propagation. This transmitted voltage is the one created by the generator at the point of excitation. The voltage amplitude in the x,y surface follows

the electric field amplitude for the z polarization. The gradient of the voltage determine the currents in the surface of the structure. But these currents radiate a refracted transverse field. This field can store energy, up to a given cutoff frequency, in stationary waves – secondary modes. This energy must be subtract from the one contained in the first modes. If the receptor doesn't care to these secondary modes, a part of the stored energy is not use. All the energy of the source must be include in the various modes excited. We can notice that the potential propagation is an efficient base to determine the currents and the field in all the surface or volume of the structure. Once the surface currents are known, the fields can be compute in all the directions and locations of the volume. We will show that using virtual frontiers, the 2D TE mode determine the 2D one in a cavity (more usually the field is determine from its longitudinal part[14]: the frontier is an edge).

5. 3D organization of the field deduced from the TE mode

In a structure where a scalar potential $V_z(y)$ propagates along x , no electric field exists in the x direction until the wavelength is large compare to the height z of the structure. As said before, the currents follow the gradient of $V_z(x,y)$. The figure 3 shows this process.

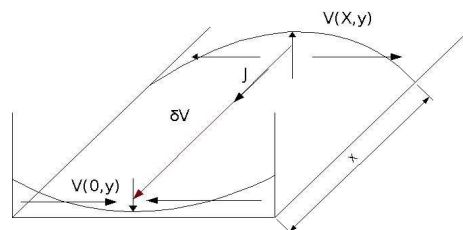


Figure 3: current lines versus voltage distribution

The distribution of the field is completely defined under the assumption that there is no propagation in the z direction. The modes have their third number always equal to zero. If the wavelength begin shorter, until to be shorter than the height, a new field distribution in the z direction will appear. We can detail the mechanism of interactions between the currents figure 4.

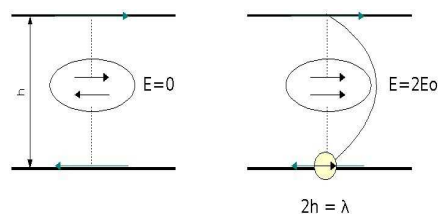


Figure 4: Stationary field along z

We see that when the wavelength begin short enough, each current creates an opposite current on the other side. The reduction of current translates the energy stored in the stationary field in the direction z. This transfer appears for the principal wave as losses. The distribution of the field in the volume follows in this case a law where appear the three directions x, y and z:

$$E(x, y, z) = \alpha \sin\left(m \frac{2\pi}{\lambda_x} X\right) \sin\left(n \frac{2\pi}{\lambda_y} Y\right) \sin\left(q \frac{2\pi}{\lambda_z} Z\right) \quad (20)$$

Where α is some function of amplitude and phase. Under the z stationary wave cutoff frequency, the field propagates for part continuously and for part like stationary wave in the x and y directions. The standing wave ratio gives the proportion of propagating and standing waves. Figure 5 shows this process.

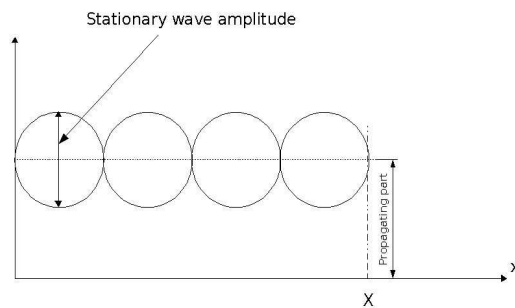


Figure 5: Stationary wave ratio

Where only 2D wave exists, the stationary amplitude changes in the z direction only and the field vector stills in this direction as shown figure 3. Equation 19 describes completely the field. When the z mode appears, the field doesn't change only following a $A_z(x,y)$ law of amplitude, but with a $A_x(x,y)$ component too. Figure 6 details the potential evolution in the case of a 2D stationary wave.

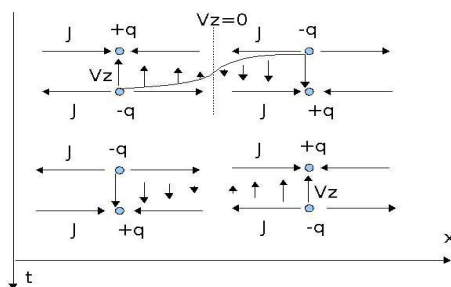


Figure 6: Vz evolution for a 2D stationary wave

In case where the z mode exists, the field has an x component and turn from one direction to another from one location of the maximum of the potential to the next one[15]. Figure 7 details this process.

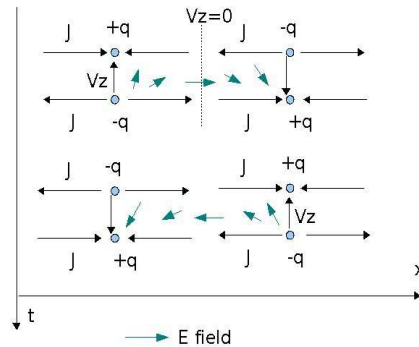


Figure 7: Wave process with 3D standing waves

Having an understanding of how the field works in the volume, the question now is to compute this field at any frequency.

6. Propagation of the scalar potential

We can give four evidences: the maximum of the x component of the electric field can exist only where the B_y field is maximum, i.e. it cannot be located where the current are null and where the charges are maximum. The divergence of the currents are given by the time evolution of the charges. The distribution of the charges follows a diffusion process and the interactions guided by the scalar potential without delays in the Coulomb's gauge. Depending on the excitation, the voltage transmitted from the generator is the incident potential for the wave. It is possible to define a reflection coefficient only link with the impedance of the medium, independently of the fields. The voltage distribution respects this coefficient whatever is its amplitude on y for example. How this works? The generator applies a force on two separate domains. We can consider the waves link with the diffusion of these charges. The diffusion of the force inside the metallic surface can be seen as a Huygens process repeated from each location where the generator vibration is transmitted. From the positive charge to the negative one at the other side of the structure, we can compute the scalar potential, not delayed. Under the hypothesis that the interaction inside the metal is the major one compare to the interaction between the positive and negative charges through the structure, we can project all the points at the same time distance from the source on each side of the structure and compute the scalar potential in the free volume between these sides. Each part of the structure excited by a pole of the generator (one by the positive pole, the other by the negative one) is an image of the other. Each charge as its image in the opposite set. This rule doesn't depend on the structure geometry. The potential energy induced by the generator diffuse in the two set (the scalar potential is a bijective application from the positive set of charges to the negative one). Figure 8 try to represent the potential propagation.

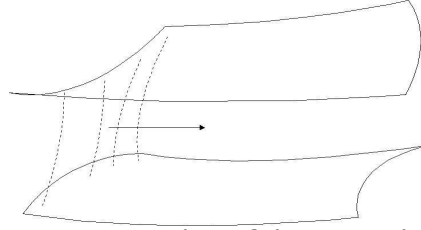


Figure 8: propagation of the potential lines

Now we can write the physical relations that translates our discussion. In the nodes space[1] the Lagrangian is (q being the electric charge):

$$L = \frac{1}{2} (\dot{q}^a m_{ab} \dot{q}^b) - U_a q^a = T - U \quad (21)$$

The equation of the problem are given by:

$$\frac{\partial}{\partial t} \frac{\partial T}{\partial \dot{q}^a} - \frac{\partial U}{\partial q^a} = 0 \quad (22)$$

By replacement we obtain:

$$\frac{\partial}{\partial t} (m_{ab} \dot{q}^b(s, s')) = U_a(s, s') \quad (23)$$

In this expression, s and s' are the limit surfaces of the volume, s being the one of the positive charges and s' of the negative ones. m_{ab} is the metric between the charges and the voltage developed between the nodes. U_a is the potential energy given to the network, i.e. the voltage generator reported to each node a . This equation demonstrates that if we are able to compute the scalar potential at each time step, we can know all the charge values at each node of the network. The law of conservation of the charge can be written using the connectivity between charges and currents:

$$\dot{q}^a = N_b^a J^b \quad (24)$$

J^b being the conductive or displacement currents. We can put this relation in 23 to obtain:

$$\frac{\partial}{\partial t} (m_{ab} N_k^b J^k(s, s')) = U_a(s, s') \quad (25)$$

The currents J^k are conductive or displacement ones:

$$J^k = J^c \quad J^k = C^{kd} \frac{\partial V_d}{\partial t} \quad (26)$$

Using these expressions in 25 we have:

$$\frac{\partial}{\partial t} \left(m_{ab} N_c^b J^c \left(\begin{matrix} s \\ s' \end{matrix} \right) + m_{ab} N_k^b C^{kd} \frac{\partial V_{d\bullet}(s, s')}{\partial t} \right) = U_a(s, s') \quad (27)$$

U_a is a function of the time, we can so compute its Fourier transformation to write in the frequency domain:

$$\left(i\omega m_{ab} N_c^b J^c \left(\begin{matrix} s \\ s' \end{matrix} \right) - \omega^2 m_{ab} N_k^b C^{kd} V_{d\bullet}(s, s') \right) = \hat{U}_a(s, s') \quad (28)$$

m_{ab} being the self or mutual inductances, this equation shows all the roots (proper values) of the network corresponding to the modes and to the terms EJ or du/dt of equation 7. The propagation is done here by the infinite succession of inductive and capacitive components associated with the network of infinite order of the latticed structure. The conductive currents can be linked with the potential writing:

$$J^c = \sigma^{cd} E_{d\bullet} = \sigma^{cd} \frac{\partial V_{d\bullet}}{\partial(s, s')} \quad (29)$$

Finally:

$$\left(i\omega m_{ab} N_c^b \sigma^{cd} \frac{\partial V_{d\bullet}}{\partial(s, s')} - \omega^2 m_{ab} N_k^b C^{kd} V_{d\bullet}(s, s') \right) = \hat{U}_a(s, s') \quad (30)$$

Equation 30 shows that the propagation of the wave can be computed considering only the propagation of the potential between the two layers of the structure.

We can integrate equation 7 on the volume v of the structure (with Y admittance of the source):

$$\frac{\partial u}{\partial t} + \vec{\nabla} \cdot \vec{S} = -\vec{J} \cdot \vec{E} \Rightarrow \frac{1}{2} \frac{\partial}{\partial t} (\epsilon v E^2 + \mu v H^2) + \phi_{\vec{s}} = \frac{1}{2} \frac{\partial}{\partial t} (Cv^2 + Li^2) + \phi_{\vec{s}} = -YU^2 \quad (31)$$

We don't take care in this relation of the losses in the walls. Now integrating over the time we have:

$$\frac{1}{2} \left(\epsilon \Gamma \left(\begin{matrix} s \\ s' \end{matrix} \right) (t) E^2 + \mu \Gamma \left(\begin{matrix} s \\ s' \end{matrix} \right) (t) H^2 \right) + \int_0^t dt \phi_{\vec{s}} = - \int_0^t dt Y U^2(t) \quad (32)$$

Where IU is nothing else than the generator energy, the covered volume at the time t being the surface of flux Γ defined by the potential lines described figure 8 multiplied by the curvilinear coordinates s or s' . When a

mode is filled, the Poynting flux can go only in the direction far from the source. Looking at figure 7 it means that once the z mode is occupied, the filed continue to propagates in the x direction. So, as we don't care of the losses, the source energy minus the Poynting flux must be the stored energy. This appears writing:

$$\frac{1}{2} \left(\epsilon \Gamma \begin{matrix} s \\ s' \end{matrix} \right) (t) \mathbf{E}^2 + \mu \Gamma \begin{matrix} s \\ s' \end{matrix} \right) (t) \mathbf{H}^2 = - \int_0^t dt Y U^2(t) - \int_0^t dt \phi_{\tilde{s}} \quad (33)$$

We know that the electric energy stored must equal to the magnetic one[11]. 33 can be written:

$$\left(\epsilon \Gamma \begin{matrix} s \\ s' \end{matrix} \right) (t) \mathbf{E}^2 = - \int_0^t dt Y U^2(t) - \int_0^t dt \phi_{\tilde{s}} \quad (34)$$

Looking 27 and 34 we understand that U_a must be the transmitted potential from nodes to nodes. In the harmonic domain the time to fill the stored energy disappears and we see that the modes are link with the transmitted voltage spectrum. In the process of energy transmission some modes are lost like the z mode in figure 7, but they increase the time rise and time fall of the impulse response of the structure (playing the role of a dispersive medium for the TE wave of figure 3) as they appear only up to their cutoff frequency. The wave vector for the s curvilinear direction of propagation with a wave surface developed in the t direction and a polarization in the z direction begun something like:

$$k_s = \sqrt{\frac{\omega^2}{c^2} \left[1 + \frac{Q_z Z}{X \left(1 + \frac{\lambda}{\lambda_z} \right)^2} \right]^2 - k_t^2} \quad (35)$$

Q_z is the quality coefficient and λ_z the cutoff wavelength for the z mode (given by the curvilinear distance between the s and s' domains). As the limit condition and the reflection coefficients can be determined by the classical rules of continuity of the field at the frontier between the mediums[16], the Branin's model can still be employed even for 3D waves, using the wave vector defined in 35.

7.Link between the temporal and the frequency mechanism

We consider a simple line of length L, with a coefficient of reflection g_s and g_L and a characteristic impedance Z_c . In the temporal domain, we can compute the transmitted voltage at the line far end:

$$V(L) = (1 + g_L) \left[\frac{Z_c}{Z_c + Z_s} V_s \delta_{\frac{L}{c}} \right] + g_L g_s \left[\frac{Z_c}{Z_c + Z_s} V_s (1 + g_L) \right] \delta_{\frac{3L}{c}} + \dots \quad (36)$$

In this expression $\delta_{L/c}$ is the delay operator, and the coefficients g_s and g_L are computed from purely real impedances. It respects:

$$\delta_a \delta_b = \delta_{a+b} \quad (37)$$

So:

$$V(L) = (1 + g_L) \left[\frac{Z_c}{Z_c + Z_s} V_s \delta_{\frac{L}{c}} \right] \left\{ 1 + g_s g_L \delta_{2\frac{L}{c}} + \dots \right\} \quad (38)$$

In the frequency domain we define:

$$\hat{V}_L = \left(\frac{1 + g_L}{1 - g_s g_L e^{-2ikL}} \right) \left[\frac{Z_c}{Z_c + Z_s} V_s e^{-ikL} \right] \quad (39)$$

As:

$$\frac{1}{(1 - g_s g_L e^{-2ikL})} = 1 + g_s g_L e^{-2ikL} + \dots \quad (40)$$

We obtain:

$$\hat{V}_L = (1 + g_L) \left[\frac{Z_c}{Z_c + Z_s} V_s e^{-ikL} \right] \left\{ 1 + g_s g_L e^{-2ikL} + \dots \right\} \quad (41)$$

We can compare equation 38 and 41 to see that the Laplace's transform of 38 gives 41.

Now if the impedance at the extremity is not real but complex, like an inductance, the reflection coefficient is complex too and the development 41 makes appear a series of added delays coming from the coefficient phase:

$$\text{if } g_L = g_L e^{i\phi_L}, \text{ etc.: } \hat{V}_L = (1 + g_L e^{i\phi_L}) \left[\frac{Z_c}{Z_c + Z_s} V_s e^{-ikL} \right] \left\{ 1 + g_s g_L e^{-i(2kL + \phi_L)} + \dots \right\} \quad (42)$$

In the temporal domain this should be translated by a series of added Dirac's pulses. Effectively, an inductance can be seen as an impedance which varies with time. At the beginning, it behaves like an open circuit. For long time, it behaves like a short circuit. So, the inductance in the time domain can be seen as a series of real impedance delayed. This series must be linked with the one obtained in equation 42. Figure 9 shows the correspondence between the frequency and the time domain in the case of real impedances.

In the harmonic domain, the voltage reflected is included in the reverse impedance relation (impedance on the port 1: Z_{11}):

$$Z_{11} = Z_c \frac{Z_L + iZ_c \tan(kL)}{Z_c + iZ_L \tan(kL)} = Z_c \frac{1 + g_L e^{-2ikL}}{1 - g_L e^{-2ikL}} \quad (43)$$

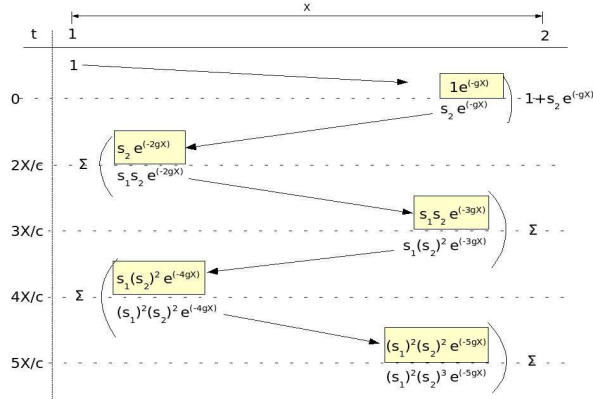


Figure 9: the delayed process

Equation 43 can be found thinking in transmitted current and voltage. For a line we can obtain the input impedance Z_{11} writing (L is the line length):

$$Z_{11} = \frac{V_i + V_R}{I_i - I_R} = \frac{V_i}{I_i} \left(\frac{1 + s_2 e^{-2ikL}}{1 - s_2 e^{-2ikL}} \right) = Z_c \left(\frac{1 + s_2 e^{-2ikL}}{1 - s_2 e^{-2ikL}} \right) \quad (44)$$

The transmitted current on the port 2 is obtained writing (we suppose all the quantities to be in the frequency domain):

$$I_t(L) = I_i (1 - s_2) e^{-ikL} = \frac{V_i}{Z_c} (1 - s_2) e^{-ikL} \quad (45)$$

V_i is the incident voltage. We know that the transmitted voltage at $x=0$ is the sum of the incident and the reflected ones:

$$(1 + s_2 e^{-2ikL}) V_i = V(0) \quad (46)$$

$V(0)$ is the transmitted voltage created by the source V_0 . So:

$$V(0) = V_0 \frac{Z_{11}}{(Z_{11} + R_0)} \quad (47)$$

Finally:

$$I_t(L) = \frac{V_0}{(1 + s_2 e^{-2ikL})} \frac{Z_{11}}{(Z_{11} + R_0)} \frac{(1 - s_2)}{Z_c} e^{-ikL} \quad (48)$$

A generator can be computed using:

$$I_t(L) = \frac{e_2}{Z_{22} + Z_L} \quad (49)$$

Using 48 and 49 we obtain:

$$e_2 = (Z_{22} + R_3) \frac{Z_{11}}{(R_0 + Z_{11})} \frac{1}{(1 + s_2 e^{-2ikL})} \frac{e^{-ikL}(1-s_2)}{Z_c} \quad (50)$$

We can write this equation quite differently, making to appear $I(0)$:

$$e_2 = (Z_{22} + R_3) \left(\frac{1-s_2}{Z_c} \right) e^{-r} \frac{Z_{11}}{(1 + s_2 e^{-2r})} I(0) \quad (51)$$

With $r=ikL$. Topologically $I(0)$ is the current of the mesh one I^1 , and e_2 the emf of the mesh two. We can define a function of cross talk:

$$G_{21} = \frac{e_2}{I^1} = \left(\frac{1-s_2}{Z_c} \right) \left[\frac{(Z_{22} + R_3) Z_{11}}{(1 + s_2 e^{-2r})} \right] e^{-r} \quad (52)$$

Developing this expression, we obtain something like:

$$G_{21} = K_2 e^{-r} \left(\frac{\sum_h N_h e^{-hr}}{\sum_b D_b e^{-br}} \right) \quad K_2 = \frac{1-s_2}{Z_c} \quad (53)$$

The function \mathbf{G} is a part of the metric in the mesh space. The inverse Fourier's transformation of \mathbf{G} is the filter that can be used in the time domain, what we will detailed in chapter 12. The characteristic impedance of the medium of propagation, in this definition does not depends on the modes. Like in a line with standing waves, the z_c parameter stills the same – with the usual changes in its self parameters like the losses, etc. The process by which the infinite series of temporal impulsion between two perfectly reflective limit conditions became a finite expression involving only one traveling and back wave can be well understood looking at the figure 10. This figure shows, in case where the limit conditions are perfectly reflective ones, that the incident and reflected waves are the same every $2L/c$ times. So, the temporal mechanism of the waves continue until the infinite (we suppose the medium without losses). As the temporal process is periodic we can compute the Fourier's series of one time evolution of the waves from 0 to $2L/c$. The time response of this system is $\delta_0 - \delta_{2L/c}$ and the Fourier's transformation of this function is simply $1 - e^{-2ikL}$.

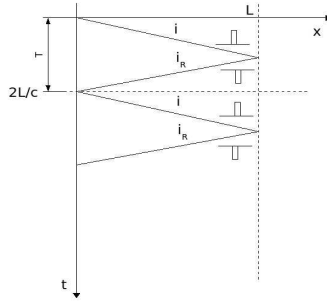


Figure 10: temporal process between two short circuits

We can recall Paul's rules[13] to construct a guided wave interaction:

1. determine the reflection coefficient $s_2 = \frac{Z_L - Z_c}{Z_L + Z_c}$
2. determine the input reflection coefficient $s(0) = s_2 e^{-i2kL}$
3. determine the input impedance to the line $[Z_{ii}]_{ii} = Z_c \frac{1 + s(0)}{1 - s(0)}$
4. determine the input voltage determine the input voltage

$$V(0) = \frac{[Z_{ii}]_{ii}}{Z_0 + [Z_{ii}]_{ii}} V_0$$

5. determine the incident wave $V^+ = \frac{V(0)}{1 + s(0)}$
6. determine the source voltage $I(L) = \frac{V^+}{Z_c} e^{-ikL} [1 - s_2]$

From these rules we determine easily the reported emf:

$$[e_j]_j = ([Z_{ii}]_{ii} + Z_L) I(L)$$

and the expression of the cross talk impedance in the mesh space:

$$G = \frac{e_2}{I(0)} = \left(\frac{Z_{22} + Z_L}{Z_c} \right) Z_{11} e^{-ikL} \frac{(1 - s_2)}{(1 + s_2 e^{-i2kL})} \quad (54)$$

this relation gives the key to resolve in the frequency domain the global problem without knowing by advance the emf induced.

8. How to compute the reflection coefficients?

If the propagation structure is connected to other complex structures, a problem is to determine the impedance of the sources and loads to compute the reflection coefficients. To reduce a network to an edge is simply the Thèvenin's process. Computing the open circuit voltage and

short circuit current between the two input and output nodes of the structure, it is possible to construct the input and output edges of the circuit represented figure 2. But this method requests to compute the circuit before to have connected the line. It implies that a characterization of each network should be made before to realize any connections. Once the impedance is known, the connection of the guided wave structure is very simple as the emf e_2 and the current I^1 are defined in the mesh space. The coupling impedance \mathbf{G}_{21} has just to be added in the mesh space in the metric of the network connected.

The reflection coefficient can be computed from the fields at the frontiers between the medium, the integration of the field giving the transmitted voltage.

When we consider a TE wave which defines the wave vector in a z direction, the values of the electric field in the y direction, perpendicular to z , is the one obtained when the energy is established. For example in the case of a guided wave, the $\sin(kx)$ distribution of the field is obtained once the field has made various paths. It is the same for the potentials and the currents. Like a line under a standing wave working, until the field stills only in the direction of the potential, the iterative impedance of the structure is a constant value (except for the losses). This stills true for the direction s of propagation even when the z mode appears. We can use Collin's approach to justify this. The characteristic impedance of a line can be written:

$$Z_c = (\mu \epsilon)^{\frac{1}{2}} C^{-1} \quad (55)$$

Whatever are the modes, until no modes exists in the z direction, the capacitance of the propagation structure stills the same in the s direction. So, for an homogeneous structure without modes in the z direction (but potentials only in the s and t directions), the characteristic impedance still continuous in frequency. Now when the z modes appear, the added energy stores in the z mode can be seen in an extended telegraph's equation has shown figure 11.

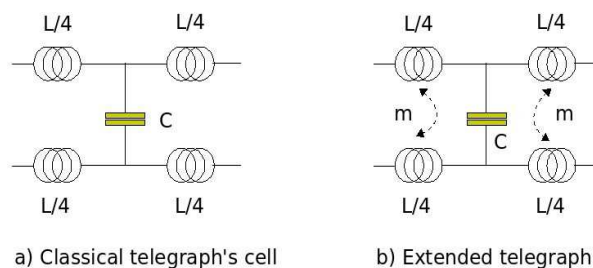


Figure 11: Classical and extended telegraph's cell

The energy stores in the z modes cannot come from charges as it is stored in the wide space between the plates. This energy can only come from the time derivation of the vector potential created by the currents. So, it can be seen as a mutual coupling between the plate. Figure 11 a) shows the classical telegraph's cell where the element L and C are linked with the

magnetic and the electric energy stored (u term of the Poynting's equation). Now when the z mode appears, the mutual coupling between the plate must be added as represented on figure 11 b). To compute the expression of the characteristic impedance Z_c , we can start from the energy study of the matched cell. The metric in the mesh space[1] of this first network is:

$$Z = \begin{bmatrix} \frac{iL}{2}\omega + Z_c + \frac{1}{iC\omega} & \frac{-1}{iC\omega} \\ \frac{1}{iC\omega} & \frac{iL}{2}\omega + Z_c + \frac{1}{iC\omega} \end{bmatrix} \quad (56)$$

From this metric we can deduced the magnetic T and electric p energies:

$$T = \begin{bmatrix} \frac{L}{2} & 0 \\ 0 & \frac{L}{2} \end{bmatrix} \quad p = \begin{bmatrix} \frac{1}{C} & \frac{-1}{C} \\ \frac{-1}{C} & \frac{1}{C} \end{bmatrix} \quad (57)$$

The characteristic impedance is given by the equation:

$$\det |pT - Zc^2 I| = 0 \quad (58)$$

Using the expressions of 51.3 we obtain:

$$\left(\frac{L}{2C} - Zc^2 \right)^2 - \frac{L^2}{4C^2} = 0 \Rightarrow Zc = \sqrt{\frac{L}{C}} \quad (59)$$

In case of the cell b), the metric becomes:

$$Z = \begin{bmatrix} \left(\frac{L}{2} - 2m \right) \frac{1}{C} - Zc^2 & - \left(\frac{L}{2} - 2m \right) \frac{1}{C} \\ - \left(\frac{L}{2} - 2m \right) \frac{1}{C} & \left(\frac{L}{2} - 2m \right) \frac{1}{C} - Zc^2 \end{bmatrix} \quad (60)$$

Which gives the energy operator:

$$T = \begin{bmatrix} \frac{L}{2} - 2m & 0 \\ 0 & \frac{L}{2} - 2m \end{bmatrix} \quad p = \begin{bmatrix} \frac{1}{C} & \frac{-1}{C} \\ \frac{-1}{C} & \frac{1}{C} \end{bmatrix} \quad (61)$$

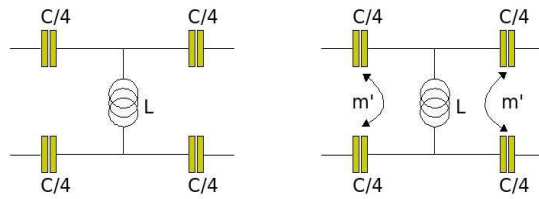
Resolving 58 we obtain:

$$Zc = \sqrt{\frac{2}{C} \left(\frac{L}{2} - 2m \right)} \quad (62)$$

We see that if $m=0$ we retrieve the relation 59. Computing the energy stored in the z mode gives the characteristic impedance through the element m (which depends on the frequency) knowing:

$$W_H = \frac{1}{2} J^{*a} m_{ab} J^b \quad (63)$$

The equivalent cell shown figure 11 is a good base to compute the characteristic impedance expression depending on the frequency. The approach can be extended to the magnetic modes. In this case, the telegraph's cell is given figure 12.



a) Telegraph's cell for TM mode b) Telegraph's cell for TM mode

Figure 12: Telegraph's cell for the TM mode

Each element, as for the TE mode representation, is linked with the energy stored by the field in electric and magnetic forms. The Poynting's equation 7 stills true whatever is the microscopic form of u . Figure 12 b) shows an equivalent process to the one observed for the electric field. When the delay between the plates becomes to long, a standing displacement current appears in the z direction, being modeled by the m' coupling term. The creation of the m and m' coupling by the currents (of conduction or displacement) makes hybrid modes coupled with the original TE and TM modes. We could think in this case to draw a generalized cell allowing to represents classical or hybrid modes. This special cell is represented figure 13 where we retrieve with bold lines the known TE and TM extended cells. This graph was suggested by Sir Gabriel KRON in a similar form fifty years ago. The z modes are include in these cells. And all coupling between the elements are authorized. The cells can be reduced to any sampled space depending on the wavelength. Between each "globule", we place a resistor that takes into account the losses of the structure.

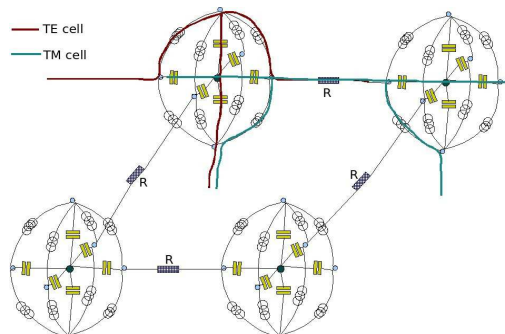


Figure 13: the generalized cells for all modes

The source determine which mode is excited first. The modes are created by the antenna and/or the coupling process between the structure and the source. Then, these first modes can create secondary modes.

The schematics figures 11, 12 and 13 can be used too to understand and modeling non homogeneous structures. In these structures, an electric field in the s direction can create an electric field in the z direction through the non homogeneous permittivity of the medium. In this case, ϵ and μ are dyadic. We have, if e and d are the microscopic fields:

$$\begin{cases} d_s = \epsilon_{ss} e_s + \epsilon_{st} e_t + \epsilon_{sz} e_z \\ d_t = \epsilon_{ts} e_s + \epsilon_{tt} e_t + \epsilon_{tz} e_z \\ d_z = \epsilon_{zs} e_s + \epsilon_{zt} e_t + \epsilon_{zz} e_z \end{cases} \Leftrightarrow \mathbf{d}_u = \epsilon_{uv} \mathbf{e}_v. \quad (64)$$

9. Connection of various zone of propagations

Any non homogeneous guided wave structure can be divide in series parallel structures. Studying the propagation from the source in the various directions of the structure, once determine the graph equivalent to the problem in order to resolve the currents and voltages in the 3D volume of the structure. Once these voltages and currents are known, the field can be determined in all the volume and the various coupling involved can be computed too. At each change of the geometry, a new structure of propagation appears. Between each guided wave structure, the frontiers can radiates diffracted fields. Finally the complete metric of a complex guided wave structure is a series of elementary ones with coupling impedance between each elementary structure. The example given at the end of the article shows how to proceed.

Working with the potential allows to use simple models. For a wide frequency band, the elementary propagation cell is our extended telegraph's cell. For each frequency of mode we will compute the characteristic impedance and the propagation speed. After what we can compute the standing waves depending on the limit conditions.

Basically, the principle consists in verifying on the surfaces of separation the equations of continuity in the fields. These surfaces can be seen as impedances between two regions of propagation and the sources of diffractions. Relation 52 must be constructed from surfaces to surfaces where R3 became an impedance computed from the input impedances of the previous part of the network. Figure 14 represents a typical case of connection of three primitive networks by two guided wave structures.

The impedance and edges 1 (R1), 4 (R4) and 7 (R7) are the primitive edge. Between these edges we connect two lines. The impedances b2, b3, b5 and b6 are the backward impedances. In this graph the G_{ij} impedances are defined in the edge space.

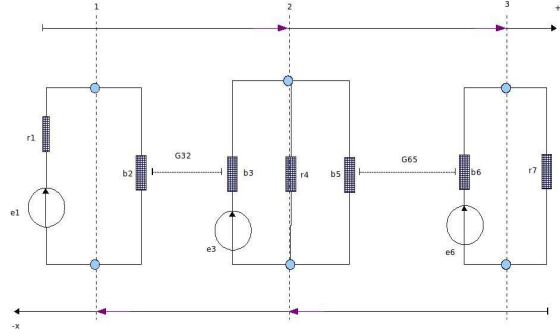


Figure 14: Three networks connected

For the three frontiers we define three reflection coefficients S1 to S3 for the direction of propagation (+x) and (-x). We compute:

$$\begin{aligned}
 b_5 &= \beta_c \left(\frac{r_7 - \beta_c \text{Th}(\gamma x)}{\beta_c - r_7 \text{Th}(\gamma x)} \right) & b_2 &= \alpha_c \left(\frac{(r_4 b_5 (r_4 + B_5)^{-1}) - \alpha_c \text{Th}(\gamma x)}{\alpha_c - (r_4 b_5 (r_4 + B_5)^{-1}) \text{Th}(\gamma x)} \right) \\
 b_3 &= \alpha_c \left(\frac{r_1 - \alpha_c \text{Th}(\gamma x)}{\alpha_c - r_1 \text{Th}(\gamma x)} \right) & b_6 &= \beta_c \left(\frac{(r_4 b_3 (r_4 + B_3)^{-1}) - \beta_c \text{Th}(\gamma x)}{\beta_c - (r_4 b_3 (r_4 + B_3)^{-1}) \text{Th}(\gamma x)} \right)
 \end{aligned} \tag{65}$$

α_c and β_c are the characteristic impedances of propagation. Once the impedances and reflection coefficients are all defined, the impedances G_{ij} can be computed, where R3 in equation 50 is the input impedance of a connected part of the network. We can write:

$$e_{3\bullet} = G_{32} i^2 \quad e_{6\bullet} = G_{65} i^5 \tag{66}$$

Noting with a Greek index the meshes currents and emf, we have:

$$e_{6\bullet} = G_{65} L_v^5 J^v = G_{65} L_v^5 Y^{v3} e_3 = G_{65} L_v^5 Y^{v3} G_{32} i^2 \tag{67}$$

We can define:

$$L_v^5 Y^{v3} = X^{53} \tag{68}$$

We obtain:

$$\frac{e_{6\bullet}}{i^2} = G_{62} = G_{65} X^{53} G_{32} \tag{69}$$

Equation 69 is the equation of the connexion. When the structures seen by the source are in parallel, each layer has its own \mathbf{G} function, but the source potential is the same for both. Equation 69 should allow to compute any organization of structures.

To the final composition of sub structures, the diffraction of the little element can be added.

10. How to determine the frequencies of the modes?

The Fourier transformation of the geometric profiles of each direction of propagation gives the modes for each of these directions. These profiles are constructed by some operations of scanning. The functions $p(s,t,z)$ of scanning are the evolution of height z of the structure for each (s,t) directions followed by the wave surface of potential excited by one source. The wave vector 35 is defined for one excitation. Once the function p is known, its Fourier's transformation gives the modes for one polarization and under the hypothesis that no modes appear in the direction of the potential. To these modes must be added the ones coming from the longitudinal components of the field. These modes are extracted from the length of the potential lines used for the scan. By replacing all the walls of the structure by short circuits, we can determine all the modes of the structure. After what depending on the direction and type of excitation, we can identify the longitudinal and transverse modes to the propagation. This technique is detailed in [7].

11. To construct a base of modes

Starting from the reflection coefficient and from a binomial series expansion we have obtained:

$$\hat{V}_L = (1 + g_L) \left[\frac{Z_c}{Z_c + Z_s} V_s e^{-ikL} \right] \left[1 + g_s g_L e^{-i2kL} + \dots \right] \quad (70)$$

k is given by equation 35. We can define a functional space[5] noting that the wave vector is a function of ω :

$$|\psi_{\omega=1}\rangle = \begin{bmatrix} e^{-ik_{(\omega=1)}L} \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad |\psi_{\omega=2}\rangle = \begin{bmatrix} 0 \\ e^{-2ik_{(\omega=2)}L} \\ \vdots \\ 0 \end{bmatrix} \quad \dots \quad |\psi_{\omega=n}\rangle = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ e^{-nik_{(\omega=n)}L} \end{bmatrix} \quad (71)$$

On this base we can define the transmitted voltage by:

$$\hat{V}_L = W_s (1 + g_L) \left[\sum_{\omega} C_{\omega} |\psi_{\omega}\rangle \right] \quad (72)$$

Expressed here on the base of modes $|\psi_{\omega}\rangle$ with W_s the sources and C_{ω} the coefficients of the modes. The 3D properties of the waves are included in the ω index of ψ which is a normalized one, i.e. the harmonic order of a

fundamental cutoff frequency ω_0 .

The direction of propagation s is perpendicular to the scalar potential in the direction of polarization z . In the direction transverse t , the modes can cut the wave under a given cutoff frequency before which the wave are purely evanescent. The modes in the z direction doesn't cut the wave, and do not carry losses. So, as we have seen, they change only the speed of propagation. For large structures, the approach can be the same in a spheric description of the propagation.

The connection consists in making the intersection between the bases of two connected medium. Starting from equation 52 we can write:

$$G_{21} = H_{21} \left[\left(R3 + (1 + s_1 e^{-2ikL}) \sum_q (s_1 e^{-2ikL})^q \right) (1 + s_2 e^{-2ikL}) \sum_{q'} (s_2 e^{-2ikL})^{q'} \right] \quad (73)$$

with:

$$H_{21} = \frac{1 - s_2}{Z_c} \frac{e^{-r}}{1 + s_2 e^{-2r}} \quad (74)$$

G_{21} can be written:

$$G_{21} = S_{21}^{*k} | \psi_k \rangle_{21} \quad k = q + q' \quad \text{and} \quad S_{21}^{*k} \quad \text{including} \quad H_{21} \quad (75)$$

Equation 69 can be generalized to all the space writing:

$$G_{iy} = G_{ij} X^{ju} G_{uy} \quad (76)$$

Using 75 we write:

$$G_{iy} = S_{ij}^{*k} | \psi_k \rangle_{ij} X^{ju} S_{uy}^{*k'} | \psi_{k'} \rangle_{uy} \quad (77)$$

In the tensorial notation this can be written:

$$G_{iy} = S_{ij}^{*k} \psi_{k \cdot | (ij)} X^{ju} S_{uy}^{*k'} \psi_{k' \cdot | (uy)} \quad (78)$$

By the Hilbert's transformation, this becomes:

$$G_{iy} = \psi_{k \cdot | (ij)}^* (S_{ij}^k)^* X^{ju} S_{uy}^{*k'} \psi_{k' \cdot | (uy)} = \psi_{k \cdot | (ij)}^* S_{iy}^{kk'} \psi_{k' \cdot | (uy)} \quad (79)$$

which can be written more usually:

$$G_{iy} = (\psi_k | S_{iy}^{kk'} | \psi_{k'} \rangle_{(ij)}) \quad (80)$$

This is our generalized law for the connections of various homogeneous structures.

12. How to treat non linear impedances?

In the temporal domain, it is difficult to compute the modes and their factor of quality, for example because of the skin effect in the walls.

When the impedances are non linear, the harmonic domain is not sufficient. It is necessary to use the time domain in order to take into account these non linearities. Nevertheless, the harmonic domain stills the best one to translate the physical process of the guided waves. In the same idea, we can think of the radiation process as a purely frequency one, link with the photons. If the impedance change with time, the function G (and its associated function f) will change too. How to proceed?

The basic idea is quite simple. We will separate the time in intervals where the impedances are constants. In these intervals, the inverse Fourier's transformation of the harmonic spectrum of propagation is the time filter that must be convolved with the temporal signal of the circuit connected to this structure of propagation. The harmonic expression of G must gives through an inverse Fourier's transform the coefficients of the filter. Some frequencies seen their phases increased by the z modes, depending on the wave vector values. From the frequency where the modes appears (for given limit conditions) the speed of propagation is a function of the frequency. The dispersion properties of the medium comes from the fact that the z modes does not create losses but only a delay due to the necessary time to fill the energy in this modes.

The Fourier's inverse transformation is quite easy to realize, and still a robust process. The phases of each harmonic must be ik_x and their amplitudes the one of $|V_L(f)|$ of equation 72.

One technique consists in finding a function F which respects this property:

$$f(t) = \sum_n F_n(t) f(t) \quad (81)$$

If we find such a function, we can after write:

$$f_{(n)}(t) = F_n(t) f(t) \quad (82)$$

The function F is chosen to assure that $f(t)$ is constant on the domain of F . Under this hypothesis, the computation of the problem can be made on each function f_n with an update of the coefficient of the filter at each time step.

Equation 66 in the time domain becomes:

$$e_{3\bullet}(t) = G_{32} * (i^2(t)) \quad (83)$$

Using 74, this equation becomes:

$$e_{3\bullet}(t) = G_{32} * \left(\sum_n F_n(t) i^2(t) \right) \quad (84)$$

We chose a F function such that the time domain Ω of F of order n is the sample used to compute the convolution product of G by i. It means that we compute:

$$\begin{cases} e_{3\bullet}(0) = G_{32}(0, e_{t<0}) F_0 i^2(t) \\ e_{3\bullet}(1) = G_{32}(1, e_0) F_0 i^2(t) + G_{32}(0, e_0) F_1 i^2(t) \\ e_{3\bullet}(2) = G_{32}(2, e_1) F_0 i^2(t) + G_{32}(1, e_1) F_1 i^2(t) + G_{32}(0, e_1) F_2 i^2(t) \\ \dots \end{cases} \quad (85)$$

The function $\mathbf{G}(i, e_{i-1})$ is updated at each time step depending on the impedances values at the time i-1, these impedances being changed by the emf e_{i-1} at the same instant.

Another technique is to constructed a function that includes all the transfer functions between two frontiers for various impedance values. The non linear impedances are computed through a variable impedance depending on voltage or current values. In a first step we construct for example the admittance Y^{ij} giving the current J^i for the excitation e_j . This admittance is obtained for a first value of impedance of a non linear element. If p is the parameter that indicates the non linear state, we can notice $Y^{ij}|_p$ the tensor Y^{ij} obtained for the parameter value p. After what in the time loop, the parameter p is adjust at each time step depending on a criterion, a voltage value for example. Once $Y^{ij}|_p$ is constructed, the time evolution of a current J^i can be entirely determined.

13. Example

A method can be proposed in order to guide our computations. We can follows the ones given by Paul [18] for the computation of the lines, adding some extensions for 3D waves. The rules are:

1. to identify the homogeneous structures seen from the source location in the direction of the spherical or rectangular waves going far from the source;
2. To integrate the field to compute the characteristic impedance of each structure of propagation;
3. to determine the reflection coefficient at each frontiers;
4. to determine the input impedance to each structure, starting from the end of propagation;
5. to use equation 80 or a similar one to add the G interaction in the metric.

We want to resolve the problem presented figure 15. We consider an open structure with one discontinuity and a single wire line with a resistive impedance and a diode. The structure is excited by a source at the coordinates $x=0$ and $y=Y/2$. The structure is made of five primitive networks or subnetworks. These primitive elements are shown figure 15.

Each of these primitive networks has its own equivalent circuit. We will compute the problem in a rectangular space. The first work consists in drawing the equivalent schematic of each primitive element.

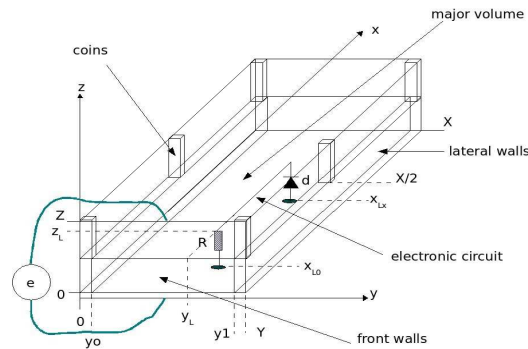


Figure 15: primitive elements

The models attached to the elements are defined for a given frequency band. For this example we will stay in a frequency band such that the coins stills linked together with simple resistors. The major volume is a line made of two parallel plates. The figure 16 shows the five primitive elements and their possible equivalent circuits.

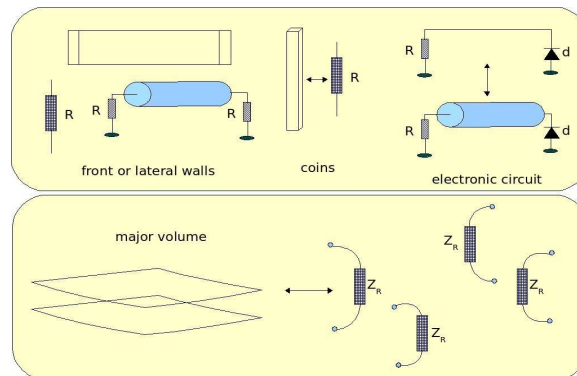


Figure 16: primitive elements

From these primitive elements we will construct three subsystems and after we will construct the interactions between these subsystems. The construction we present are dependent on the frequency. In order to clarify the method, we limit our frequency band to have quite simples patterns for the primitive elements. But the technique can be extended to any frequencies.

13.2 Construction of the subsystems

First we can make the principal structure. It is made of the major volume and of the two front walls. In the x direction, the structure behave like a

line. Connected with the two walls we obtain the substructure which graph is presented figure 17. In this graph we recognize the front wall, the open major structure and the generator which is used to create the excitation of the system.

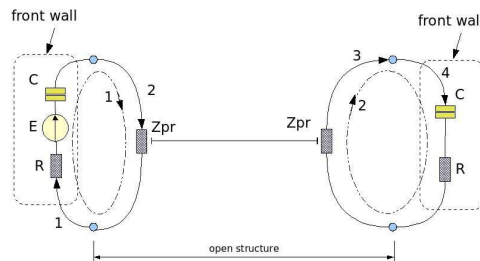


Figure 17: graph of the major structure with the front walls

The lateral walls appear like line too, but with a characteristic impedance lower than the one of the major structure. Between each coins, we construct a line but with limit conditions different from the ones we saw on the major structure. Figure 18 shows the graph of the lateral walls.

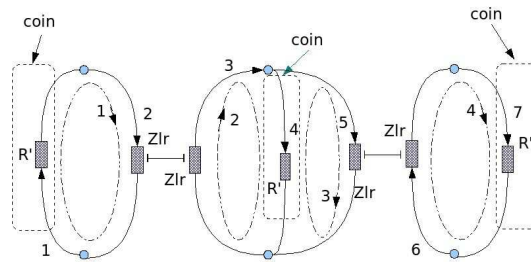


Figure 18: lateral structure

There is two of these structures in the system. To make this system we have to connect these three subsystems (we will consider the electronic circuit after).

13.2 Connection of the subsystems

Each subsystem has its own connectivity between the edge space and the mesh one[1]. For the major structure this connectivity is:

$$L(\text{major}) = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} \quad (86)$$

For the lateral ones the connectivity is given formula 87.

Having define the metric in the edge space, these connectivities define completely the metric in the mesh space where the problem is solved. If we connect the three subsystems, the numbering of the meshes of the first subsystem will kept its values, for example from 1 to N, while the numbering of the second will start at 1+N until M+N and the third from 1+N+M to P+N+M.

$$L(\text{lateral}) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (87)$$

But due to the fact that we connect the nodes of edge 1 of subsystem 1 to the nodes of the edge 1 of the subsystem 2 we create a new mesh that doesn't exist before. So the connection of the three connectivities of the three subsystems follows the procedure shown figure 19.

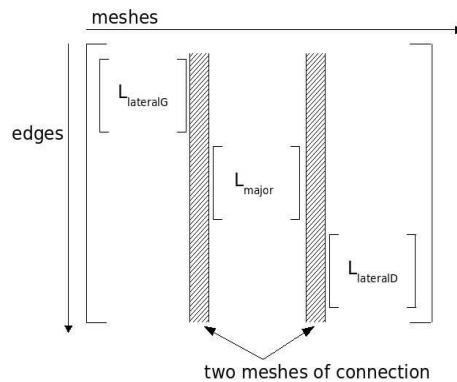


Figure 19: organization of the partial connectivities

The added connectivities are shown figure 20.

Between the last numbering of edge of the first subsystem and the first numbering of the second one we must add the number of meshes added connecting the two subsystems. Here two meshes are added, one to each extremity of the structure.

For the electronic circuit, the coupling is made only through radiated field between edges or meshes. There is no edge or mesh to add.

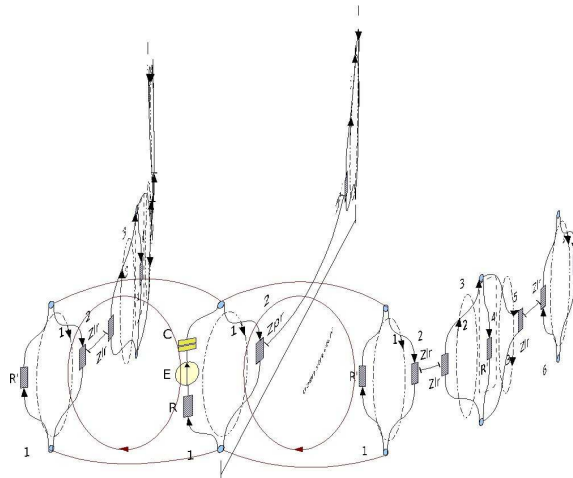


Figure 20: added connectivities

Two actions still to be done to end the connection: the coupling between edges must be added to the metric in the mesh space and the modes in the t direction for the wave in the major structure must be defined (in the case where some electrostatic interactions appears, C edge must be added in the edge space. see [1][2] to make this operation, strictly similar for one only network construction).

Each subsystem has its own edges. When we connect one to another, a field emitted by an edge can create an emf on an edge of the second subsystem. This coupling must be added in the global metric of the edge space.

To construct this metric the first action is to put each metric of each subsystem in the global one like shown figure 21.

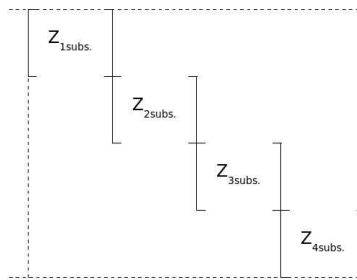


Figure 21: construction of the metric in the edge space

To the first construction we must add the coupling created during the connection. These terms are extra-diagonal ones. Figure 22 shows some terms added between symmetric structures. In the graph, the interactions added must be the same that the one added in the global metric.

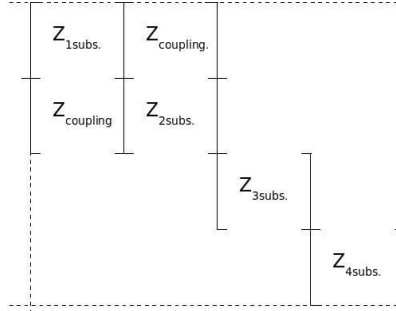


Figure 22: to add interactions

The major section is a propagative one. As we have seen, these structures are defined for modes of propagation in the t and z directions, i.e. here for the y and z directions. Connecting the lateral walls, the t modes can be modified, that's what we have to verify.

The free major structure has, seen from its center, a law of potential given by:

$$V(y=0) = V_0 (1 + \Gamma_y e^{-ikY}) \quad (88)$$

The connection of the major structure to the lateral walls modify the reflection coefficient Γ_y . For the free structure, its value is 1. When we connect the lateral walls, this value becomes:

$$\Gamma_y = \frac{R_w + \frac{1}{iC_w \omega} + \eta_0 \operatorname{th} \left[\frac{kY}{2} \right]}{\eta_0 + \left(R_w + \frac{1}{iC_w \omega} \right) \operatorname{th} \left[\frac{kY}{2} \right]} \quad (89)$$

With this parameter and knowing the frequency of the generator we can compute the y distribution of the voltage and the t mode of the wave in the major structure. Another method can be to directly compute the wave vector of the structure from the coefficients of propagation:

$$k = \frac{\omega}{\sqrt{2}} \left(C \sqrt{\frac{R^2}{\omega^2} + L^2 + LC} \right)^{\frac{1}{2}} \quad (90)$$

Once the tensor of the impedances in the edge space is modified, one operation to do is to add the coupling between meshes due to the connection in the tensor of impedance in the mesh space. Once these coupling done, the last operation to do is to compute the global moment of the connected network for far field interaction with the environment. These two operations are strictly similar to the one done for one network[1][2].

The connection of the subsystems is so resume by the six actions:

1. to associate the primitive tensors to numbering the new ones;
2. to add the capacitive edge due to electrostatic interactions and to compute and add the coupling interactions in the edge space;
3. to associate the connectivities meshes to edges and to add the meshes of the edges connected;
4. to modify the k_s and k_t wave vector of the guided wave structures connected;
5. to add the mutual coupling in the mesh space of the computed tensor of the impedance transformed in the mesh space;
6. to compute eventually the new global moment of the new network created.

13.3 Resolution

The resolution of our problem will be made in two phases. In a first step we will compute only the structure in the frequency domain. Then we can add the electronic circuit and to compute it in the time domain. The lateral walls have the metric and connectivity written relation 91.

$$Z_L = \begin{bmatrix} R1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & ZR1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & ZR1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & R4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & ZR2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & ZR2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & R7 \end{bmatrix} \quad L_L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (91)$$

For the central volume we make the same approach to obtain:

$$Z_c = \begin{bmatrix} \frac{1}{sC} + Rc & 0 & 0 & 0 \\ 0 & ZRc & 0 & 0 \\ 0 & 0 & ZRc & 0 \\ 0 & 0 & 0 & \frac{1}{sC} + Rc \end{bmatrix} \quad L_c = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} \quad (92)$$

We can connect the three networks following our rules with the edge numbering beginning by the first lateral network and ending by the second one. If $Z1$, $Z2$, $Z3$ are the three metrics of the three networks, we can construct the global one making in a first step:

13.4 To add the electronic

For this first example we consider an electronic circuit short compared to the wavelength. We consider too that the disturbance of the waveguide structure by the electronic mesh is weak. The direction of the single mesh of the electronic circuit is in front of the excitation mesh, i.e. that these two meshes are the only one in interaction (this is just to simplify our demonstration). It is quite simple to add this circuit. We just have to add two edges in the metric and two connectivities. But if we accept some approximation, there is another way to do it which is a simplest one. Directly in the mesh space we can create a cross talk between the mesh of the electronic circuit and the meshes 7 and 8 of the main structure. Once this new metric is created, the technique here consists in computing the transfer function $Y^{15,7}$ for each impedance value of the diode. In a first step we compute $Y^{15,7}|_0$ for the blocked diode, then for the passed one $Y^{15,7}|_1$. The transfer function is updated each step time. The algorithm has the following sequence:

```
parameter p initialization: p=0
for t=1:1000
  J=0
  for q=1:t
    J=J+ Y15,7|p(q)*e7(t-q+1)
  next q
  J15(t)=J
  if (diode(p).J>0,6) then p=1;end,
  if (p==1 && J<J0) then p=0; end,
next t
```

The listing of the program is given annexe II. The voltage through the non linear element is shown figure 25.

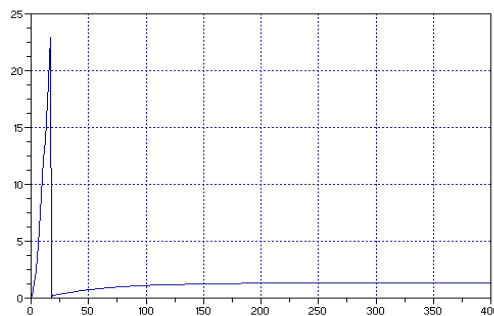


Figure 25: time result through the diode

This last result shows that the method allows to treat non linear elements in the network.

14 References

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```

0 0
0 0
0 -1
-1 0
0 0
0 0
0 0
0 0
0 0
0 0
0 1];

Lc=[1 0
1 0
0 1
0 1];

LG=[[LL;Lgb],LCg,[zeros(7,2);Lc;zeros(7,2)],LCd,[Lgb;LL]];

//tabs
J=[]; //meshes currents
res=[]; //one particular result wanted
Efv=[]; //field in the volume for one frequency
sveG=[];
Zm3D=[];

//frequency loop
disp('start of computation');

fo=1e6;
for n=[1:1000]
f=n*fo;
w=2*pi*f;
s=i*w;
//metric definition
Zc1=n*Z/2/1e-2;
ZR2g=Zc1*(R7+Zc1*tanh(s*(X/2)/c))/(Zc1+R7*tanh(s*(X/2)/c));
RT=R4*ZR2g/(R4+ZR2g);
ZR1g=Zc1*(RT+Zc1*tanh(s*(X/2)/c))/(Zc1+RT*tanh(s*(X/2)/c));
ZR1d=Zc1*(R1+Zc1*tanh(s*(X/2)/c))/(Zc1+R1*tanh(s*(X/2)/c));
RT2=R4*ZR1d/(R4+ZR1d);
ZR2d=Zc1*(RT2+Zc1*tanh(s*(X/2)/c))/(Zc1+RT2*tanh(s*(X/2)/c));

Zc2=n*Z/(Y-2e-2);
kx=w/sqrt(2)*(sqrt(u*eps)/Zc2*sqrt(Ro^2/w^2+(Zc2/(sqrt(u*eps)*c^2))^2)+1/c^2)^(0.5);

//svgde pour trace 3D
//$$$$$$$$$$$$$$$$$$$$
fchoi=400;
if (n==fchoi) then sveG(4)=kx;end,

Zoc=1/(s*C8);
ZRCg=Zc2*(Zoc+Zc2*tanh(i*kx*(X)))/(Zoc+Zc2*tanh(i*kx*(X)));
ZRCd=ZRCg;

ZL=[R1 0 0 0 0 0
0 ZR1g 0 0 0 0
0 0 ZR1d 0 0 0
0 0 0 R4 0 0
0 0 0 0 ZR2g 0
0 0 0 0 0 ZR2d 0
0 0 0 0 0 0 R7];

ZC=[Zoc 0 0 0
0 ZRCg 0 0
0 0 ZRCd 0
0 0 0 Zoc];

ZG=[[ZL;zeros(11,7)],[zeros(7,4);ZC;zeros(7,4)],[zeros(11,7);ZL]];

//to add the edges cross talk
ZG(12,4)=-s*Z/2*Z/2*u/(4*pi*(Y-1e-2))*exp(-i*w/c*(Y-1e-2));
ZG(4,12)=ZG(12,4);

//svve 3D
if (n==fchoi) then Zm3D=ZG;end,

Zm=LG'*ZG*LG;
//Zm=[Zm;zeros(1,14)];
//Zm=[Zm,zeros(15,1)]; //1 mesh added in Zm

//first sources
Em=[0 0 0 0 0 1 0 0 0 0 0 0];

```



```

//to add the guided wave sources in the mesh space

RTT1=R4*ZR2g/(R4+ZR2g);
GL1=(RTT1-Zc1)/(RTT1+Zc1);
GL2=(R7-Zc1)/(R7+Zc1);
GLc=(Zoc-Zc2)/(Zoc+Zc2);

//svgde pour trace 3D
if (n==fchoi) then sveG(1)=GL1;sveG(2)=GL2;sveG(3)=GLc;end,

Zm(2,1)=(ZR1d+RTT1)*exp(-%i*w/c*(X/2-1.5e-2))/(1-GL1*exp(-2*%i*w/c*(X/2-1.5e-2)))*(1-GL1);
Zm(12,11)=Zm(2,1);
Zm(4,3)=(ZR2d+R7)*exp(-%i*w/c*(X/2-1.5e-2))/(1-GL2*exp(-%i*w/c*(X/2-1.5e-2)))*(1-GL2);
Zm(14,13)=Zm(4,3);

Zm(8,7)=(ZRcd+Zoc)*exp(-%i*kx*(X-1e-2))/(1-GLc*exp(-%i*2*kx*(X-1e-2)))*(1-GLc);

//résolution des courants
J=pinv(Zm)*Em';
res=[res;J(8)];

//svgde pour trace 3D
if (n==fchoi) then Efv=J;end,

end, //end of frequency loop

//curves
n=[1:1000];
subplot(2,1,1);plot2d(n*fo,abs(res),style=9,logflag='l');xgrid(9);xtitle('Output current J(8) [A] versus frequency [Hz]');

disp('Start of the 3D computation');

//3D plot of the field for one frequency
//coordinates of the emitter edges: connectivity edge - space(x,y,z=Z/2), for the diffraction part
COO=[0 0
X/2-0.5e-2 0
X/2+0.5e-2 0
X 0
Y/2 0
Y/2 X
0 Y
X/2-0.5e-2 Y
X/2+0.5e-2 Y
X Y];

EV=zeros(300,30);
//the repartition is EV(300,[1:2])->lateral walls. EV(300,[2:28])-> central structure. EV(300,[29:30])

uk=2*%pi*fchoi*fo/c;
//Efv(11)=Efv(1);Efv(12)=Efv(2);Efv(13)=Efv(3);Efv(14)=Efv(4);

for x=[2:149]
EV(x,1)=Z/2*Zc1*((Efv(1))/(1-sveG(1)*exp(-2*%i*uk*X/2))*exp(-%i*uk*x*1e-2))-((Efv(2))/(1-sveG(2)*exp(-2*%i*uk*X/2))*exp(-%i*uk*(X/2-x*1e-2))));
EV(x,30)=EV(x,1);
end,

for x=[151:299]
EV(x,1)=Z/2*Zc1*((Efv(3))/(1-sveG(1)*exp(-2*%i*uk*X/2))*exp(-%i*uk*x*1e-2))-((Efv(4))/(1-sveG(2)*exp(-2*%i*uk*X/2))*exp(-%i*uk*(X/2-x*1e-2))));
EV(x,30)=EV(x,1);
end,

for x=[2:299]
for n=[2:29]
EV(x,n)=Z*Zc2*((Efv(7))/(1-sveG(3)*exp(-2*%i*sveG(4)*X))*exp(-%i*sveG(4)*x*1e-2))-((Efv(8))/(1-sveG(3)*exp(-2*%i*sveG(4)*X/2))*exp(-%i*sveG(4)*(X-x*1e-2))));
end,
end,
EV(1,[2:29])=Efv(7)*Zm3D(9,9);
EV(300,[2:29])=Efv(8)*Zm3D(10);
EV(1,1)=Efv(1)*Zm3D(2,2);
EV(300,1)=Efv(4)*Zm3D(6,6);
EV(1,30)=Efv(11)*Zm3D(13,13);
EV(300,30)=Efv(14)*Zm3D(17,17);
EV(150,1)=(Efv(2)-Efv(3))*Zm3D(4,4);
EV(150,30)=(Efv(12)-Efv(13))*Zm3D(15,15);

```

```

//adding diffraction
//-----
j=[1 2 3 4 7 8 11 12 13 14];
for n=[2:299]
    for m=[2:29]
        xn=n*1e-2;
        ym=m*1e-2;
        EVnm=0;
        for p=[1:8]
            d=sqrt((xn-COO(p,1))^2+(ym-COO(p,2))^2);
            //if (d==0) then d=1e-3;end,
            EVnm=EVnm-2*pi*fchoi*fo%i*u*LG(j(p,.)).*Efv(:)/(4*pi*(d))*exp(-%i*sveG(4)*d);
        end,

        EV(n,m)=EV(n,m)+Z*EVnm;
    end,
end,
//-----

disp('end of the 3D computation');

n=[1:300];m=[1:30];

//subplot(2,1,2);plot3d(n,m,20*log10(1e6*(abs(EV)+1e-6)));xtitle('3D curve of the vertical electric field[dBμV/m] at 458
MHz');
subplot(2,1,2);plot3d(n,m,abs(EV));xtitle('3D curve of the vertical electric field[dBμV/m] at 458 MHz');

```

Annexe II

```

//program of time computation

Y=[];
Y1=read('hbas.csv',-1,1);
Y2=read('hhaut.csv',-1,1);

Y=[Y1,Y2];
diode=[1 10e3];

//temporal source
e=[];
for t=[1:400]
    e=[e;500*(exp(-t/50)-exp(-t/10))];
end

//plot2d(e);
res=[];

J=[];
p=2;

for t=[1:400]
    Jj=0;
    for q=[1:t]
        Jj=Jj+Y(q,p)*e(t-q+1);
    end,
    J=[J;Jj];
    if (abs(diode(p)*Jj)>0.6) then p=1;end,
    if (abs(Jj)<0.0025) then p=2;end,

    res=[res;diode(p)*Jj];

end,

n=[1:400];
plot2d(n,res,style=9);xgrid(9);

```