



HAL
open science

Crackling dynamics in material failure as the signature of a self-organized dynamic phase transition

Daniel Bonamy, Stéphane Santucci, Laurent Ponson

► **To cite this version:**

Daniel Bonamy, Stéphane Santucci, Laurent Ponson. Crackling dynamics in material failure as the signature of a self-organized dynamic phase transition. *Physical Review Letters*, 2008, 101, pp.045501. 10.1103/PhysRevLett.101.045501 . hal-00260014v1

HAL Id: hal-00260014

<https://hal.science/hal-00260014v1>

Submitted on 2 Mar 2008 (v1), last revised 22 Sep 2008 (v2)

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Crackling dynamics in material failure as the signature of a self-organized dynamic phase transition

D. Bonamy,^{1,*} S. Santucci,^{2,3} and L. Ponson^{1,4}

¹*Fracture Group, Service de Physique et Chimie des Surfaces et Interfaces, DSM/DRECAM/SPCSI, CEA Saclay, F-91191 Gif sur Yvette, France*

²*Fysisk Institutt, Universitetet i Oslo, P.O. Boks 1048 Blindern, N-0316 Oslo 3, Norway*

³*Present address: Physics of Geological Processes, Universitetet i Oslo, P.O. Boks 1048 Blindern, N-0316 Oslo 3, Norway*

⁴*Present address: Division of Engineering and Applied Science, California Institute of Technology, Pasadena, CA 91125, USA*

(Dated: March 3, 2008)

We derive here a linear elastic stochastic description for slow crack growth in heterogeneous materials. This approach succeeds in reproducing quantitatively the intermittent crackling dynamics observed recently during the slow propagation of a crack along a weak heterogeneous plane of a transparent Plexiglas block [Mly *et al.*, PRL **96** 045501]. In this description, the quasi-static failure of heterogeneous media appears as a self-organized critical phase transition. As such, it exhibits universal and to some extent predictable scaling laws, analogue to that of other systems like for example magnetization noise in ferromagnets.

PACS numbers: 62.20.mt, 46.50.+a, 68.35.Ct

Driven by both technological needs and the challenges of unresolved questions in fundamental physics, the effect of materials heterogeneities onto their failure properties has been extensively studied in the recent past (see [1] for a recent review). So far, many efforts have been focusing on the morphology of fracture surfaces (see [2] for a review). In particular, crack surface roughness was recently shown to exhibit anisotropic morphological scaling features [3, 4] reminiscent of interface growth problems [5] that could be understood for brittle materials [4].

Here, we will focus our study on the dynamics of cracks. In heterogeneous materials under slow external loading, this propagation displays a jerky dynamics with seemingly random sudden jumps spanning over a broad range of length-scales. Such a complex dynamics also called “Crackling Noise” [6] has been observed directly in various fracture experiments [7], and also suggested from the acoustic emission accompanying the failure of various materials [8] up to much larger scale during seismic activity accompanying earthquakes [9]. Recently, this intermittent dynamic has been locally characterized using a high speed and high resolution camera following directly the crack growth along a weak heterogeneous plane within a transparent Plexiglas block [10]. The crack was observed to progress through irregular avalanches with a variety of sizes. Till now, this complex dynamics could not be captured by the continuum theory classically proposed in Linear Elastic Fracture Mechanics (LEFM) [11].

We will demonstrate here that this crackling dynamics can be fully reproduced through a stochastic description derived from LEFM extended to disordered materials. In this description, quasi-static failure of heterogeneous brittle elastic media can be interpreted as a self-organized critical dynamic phase transition and - as such - exhibits

universal behaviors. We will then show how one can use Universality and previous calculations performed on different systems belonging to the same universality class, here the motion of domain walls in disordered ferromagnets, to derive predictive laws for the failure of materials.

Theoretical description: LEFM is based on the fact that – in an elastic medium under tensile loading – the mechanical energy G released as fracture occurs is entirely dissipated within a small zone at the crack tip. Defining the fracture energy Γ as the energy needed to create two crack surfaces of unit area, under quasi-static condition, we assume that the local crack velocity v is proportional to the excess energy locally released:

$$\frac{1}{\mu}v = G - \Gamma \quad (1)$$

where μ is the effective mobility of the crack front.

In a homogeneous medium, $\Gamma = \Gamma_0$ is constant and an initially straight crack front will be translated without being deformed. LEFM allows the determination of the energy released G in any geometry. In particular, for the experimental configuration chosen in [10], considered here and depicted in Fig.1 where a crack grows at the interface between two elastic plates by lowering the bottom plate at constant velocity V , G is given by [12]:

$$G^0(f(t)) = \frac{1}{3}E\delta^3 \frac{(\Delta_0 + Vt)^2}{(c_0 + f(t))^4} \quad (2)$$

where $c_0 + f(t)$ is the instantaneous crack length, E the Young modulus, δ the thickness of the lowered plate, Δ_0 the initial opening and c_0 the initial crack length. Then, considering a slow driving velocity such as $Vt \ll \Delta_0$, one can show from Eqs. 1,2 that after a short transient regime, the crack front propagates at a constant velocity:

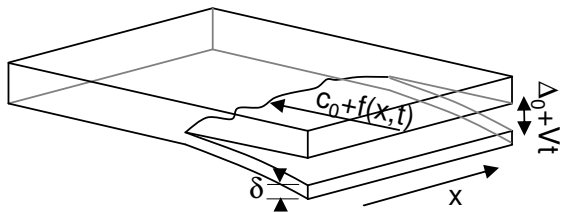


FIG. 1: Crack growth along the inhomogeneous interface between two elastic plates loaded according to the geometry used in [10]: Sketch and notations.

$$f(t) \simeq v_m t \text{ with } v_m = \frac{Vc_0}{2\Delta_0} \text{ and } c_0 = \left(\frac{E\delta^3\Delta_0^2}{3\Gamma_0} \right)^{1/4} \quad (3)$$

In a heterogeneous material, defects induce fluctuations in the local toughness: $\Gamma(x, y) = \Gamma^0(1 + \eta(x, y))$ where the noise term $\eta(x, y)$ captures these fluctuations, and x and y denotes the coordinates in the propagation and crack front directions respectively. These fluctuations induce local distortions of the crack front $f(x, t)$, which in turn generate local perturbations in G [13]. In a weakly heterogeneous material with $\eta(x, y) \ll 1$, and therefore a weak front roughness $\partial f/\partial x \ll 1$, to linear order in $f(x, t)$, one can show that [14]:

$$G(x, f(x, t)) \approx G^0(\langle f(x, t) \rangle_x) + \frac{1}{2\pi} G^0(\langle f(x, t) \rangle_x) \int_{-\infty}^{\infty} \frac{f(x', t) - f(x, t)}{(x' - x)^2} dx' \quad (4)$$

And finally, by replacing this expression in Eq. 1, using the expression of G^0 for the homogeneous case, and by introducing the quantities defined in Eq. 3, we get the following equation of motion for the quasi-static crack growth in heterogeneous medium:

$$\frac{1}{\mu\Gamma^0} \frac{\partial f}{\partial t} = F(t, \{f\}) + \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{f(x', t) - f(x, t)}{(x' - x)^2} dx' - \eta(x, f(x, t)) \text{ where } F(t, \{f\}) = \frac{4}{c_0} (v_m t - \langle f(x, t) \rangle_x) \quad (5)$$

Strictly speaking, this equation describes the interfacial crack growth according to the geometry depicted in Fig. 1. But the very same equation – with different prefactors in the expression of $F(t, \{f\}) = \frac{4}{c_0} (v_m t - \langle f(x, t) \rangle_x)$ – would have been obtained for the quasi-static stable crack growth of the in-plane component of the crack front within a three-dimensional solid independently of the tensile loading conditions and the system geometry.

Variants of this equation with constant force F have been extensively studied in the past to model crack propagation in solids [13], but also to describe other systems as diverse as interfaces in disordered magnets [15, 16] or contact lines of liquid menisci on rough substrates [17, 18]. In this case, the interface remains stationary, pinned by the heterogeneities, unless this constant force

F is sufficiently strong, larger than a threshold value F_c . A key feature of these systems is that the so-called depinning transition at F_c belongs to the realm of collective critical phenomena characterized by universal scaling laws [19]. In particular, at F_c , the interface moves through scale free avalanches, both in space and time.

In the case of a quasi-static crack growth in heterogeneous media as described by Eq. 5, the situation is different since the effective force acting on the front is not constant anymore, but given by the difference between the mean front position and the one that would have been observed within the homogeneous case: $F(t, \{f\})$. Thus, when $F(t, \{f\}) < F_c$, the front remains pinned and $F(t, \{f\})$ increases with time. As soon as $F(t, \{f\}) > F_c$, the front propagates, $F(t, \{f\})$ increases, and, as a consequence, $F(t, \{f\})$ is reduced until the front is pinned again. This retroaction process keeps the crack growth close to the depinning transition *at each time* and, within the limit $v_m \rightarrow 0$ and $c_0 \rightarrow \infty$ the system remains at the critical point during the whole propagation, as for self-organized-critical systems [20]. The crack growth is then expected to exhibit crackling dynamics characterized by sudden jumps, – avalanches –, spanning over a wide range of time and length-scales.

Spatio-temporal intermittent dynamics – Predictions of this stochastic description are now confronted to the experimental observations reported in [10]. Using a fourth order Runge-Kutta scheme, Eq. 5 is solved for a front $f(x, t)$ propagating in a 1024×1024 uncorrelated 2D random gaussian map $\eta(x, y)$ with zero average and unit variance. The parameter $\mu\Gamma_0$ was set to unity while the two parameters c_0 and v_m were varied from 2.5 to 250, and 10^{-2} to 5×10^{-1} , respectively. In order to characterize the scaling features of the crack front local dynamics, we adopt the analysis procedure recently proposed in [10] and compute at each point (x, y) of the recorded region the time $w(x, y)$ spent by the crack front within a small $1 \times 1 \text{ pixel}^2$ region as it passes through this position. A typical gray-scale image of this so-called waiting time map are presented in Fig. 2a. The numerous and various regions of gray levels reflect the intermittent dynamics, and look very similar to those observed experimentally (Fig. 1b of [10]). From the inverse value of waiting-time map, we compute a spatial map of the local normal speed velocity $v(x, y) = 1/w(x, y)$ of the front as it passes through the location (x, y) . Avalanches are defined as clusters of velocities v larger than a given threshold $v_c = C\bar{v}$ where \bar{v} denotes the crack velocity averaged over both time and space within the steady regime. Their area is power-law distributed with an exponent $\alpha_0 = -1.65 \pm 0.05$ (Fig. 2c). These clusters exhibit morphological scaling features since their width, – measured along the direction of crack growth –, is shown to go as a power-law with respect to their length – measured along the direction of crack front – with an exponent $H = 0.65 \pm 0.05$ (Fig. 2d). All these results are in perfect agreement with those measured ex-

perimentally in [10]. It is worth to mentionned that contrary to what conjectured in [10], H is *significantly different* from the value of the roughness exponent $\zeta \simeq 0.39$ expected for the interface at the depinning point of Eq. 5 with a constant force $F = F_c$ [13, 21]. Let us mention that recent experimental results sheds light on the apparent disagreement between this prediction on ζ and the value experimentally measured [23], showing at small length-scales a roughness exponent $\zeta \simeq 0.55$ close to the value reported till now [23] and a cross-over to another scaling regime at large length-scale characterized by a smaller ζ [22], in qualitative agreement with scenario proposed in [4] invoking damage screening effects at small scales.

To complete the characterization of the avalanche statistics, we measure a new observable, the avalanche duration, – defined as the difference between the times when the crack front leaves and arrives to the considered avalanche cluster (Fig. 2b). The distribution of the avalanche duration is plotted on Fig. 2e. For durations D smaller than the average avalanche duration $\langle D \rangle$, this distribution clearly depends on v_m , c_0 and the clipping threshold C while for $D \gg \langle D \rangle$ all the distributions seem to collapse onto a single power-law behaviour characterized by an exponent $\tau_D = -2.1 \pm 0.2$ (Fig. 2e). Finally, the mean avalanche duration is found to go as a power law with the mean avalanche size, characterized by an exponent $\gamma = 0.4 \pm 0.05$ (Fig. 2f).

Spatially-averaged dynamics – As for other critical systems, very different physical systems that belong to the same universality class will display similar scaling behaviours with same scaling exponents. In particular, it was shown that the Barkhausen noise [15, 16] accompanying the motion of domain wall driven by external magnetic field through soft disordered ferromagnets can be described by an equation with shape similar to Eq. 5. In this respect, it is interesting to analyse the global crack front dynamics by computing the velocity of the crack interface $v(t) = \langle \frac{\partial f}{\partial x}(x, t) \rangle_x$ spatially averaged along the front direction x , as in [15, 16]. We observe once again a jerky signal, characterized by sudden bursts, signature of an intermittent “crackling noise” dynamics [6]. As expected, this signal looks really similar to Barkhausen noise [15, 16]. Therefore, we develop the classical analysis performed on Barkhausen experiments [15, 16]: We impose a given reference level $v_c = C \max(v)$ and define bursts as zones where $v(t)$ is above v_c . The duration T of a given burst is defined as the interval within two successive intersections of $v(t)$ with v_c while the size S is defined as the integral of $v(t)$ between the same points. For the Barkhausen noise, this analysis leads to power-law distributions $P(T) \propto T^{-\tau}$ and $P(S) \propto S^{-\alpha}$ [15, 16], and a power-law relation between the duration and size of the avalanches $T \propto S^a$, with critical exponents that can be predicted using functional renormalisation group calculations [16, 17] leading to $\alpha \approx 1.25$, $\tau \approx 1.43$, and $a \approx 0.58$. We show on Fig. 3b, c and d. that all these scal-

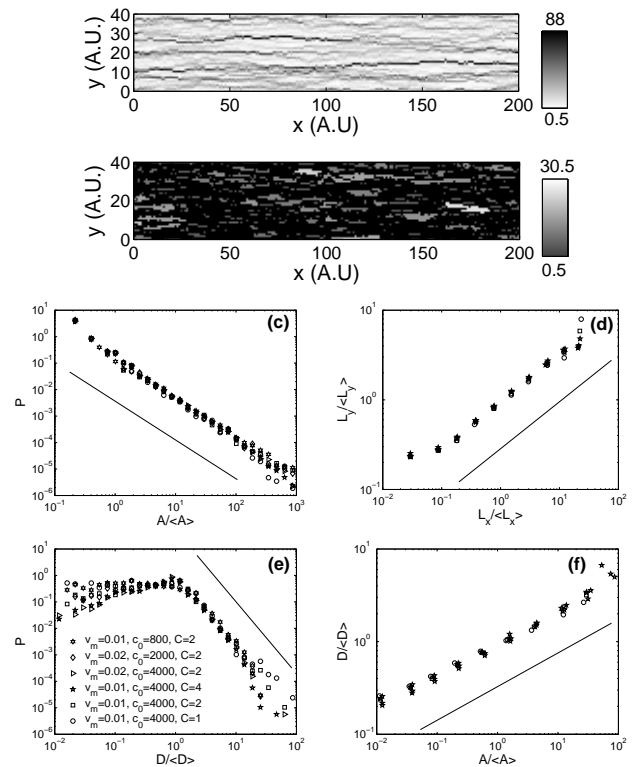


FIG. 2: (a) Typical gray scale map of the waiting time matrix $w(x, y)$ obtained from the solution of Eq. 5 with $c_0 = 4000$ and $v_m = 0.01$. (b) Spatial distribution of clusters corresponding to velocities $v(x, y) > C \bar{v}$ with $C = 4$. The quantity \bar{v} refers to the crack velocity averaged over both time and space within the steady regime. The clusters duration is given by the clusters color according to the colorscale given in inset. The distribution of the cluster area A , the scaling between the width L_y and the length L_x of clusters, the distribution of the cluster duration D , and the scaling between D and A are plotted in (c), (d), (e) and (f), respectively. The various symbols corresponds to various values of v_m , c_0 and C as specified in the inset of Fig. e. The straight lines correspond to $P(A) \propto A^{-\alpha_0}$, $L_y \propto L_x^H$, $P(D) \propto D^{-\tau_D}$ and $D \propto A^\gamma$ with $\alpha_0 = -1.65$, $H = 0.65 \pm 0.05$, $\tau_D = -2.1$ and $\gamma = 0.4$.

ing relations as well as the values of the exponents are found to be in good agreement with the ones observed here for the average interfacial crack growth.

Conclusion – We have derived a description for planar crack growth in a disordered brittle material which succeeds to capture the statistics of the intermittent crackling dynamics recently observed experimentally [10]. In particular, we have shown that material failure appears as a critical system, where the crack front progresses through scale-free avalanches signature of a dynamic depinning phase transition. As for other critical systems, microscopic and macroscopic details do not matter at large length and time scales and this simple Linear Elastic Stochastic description contains all the ingredients needed to capture the scaling statistical properties of more com-

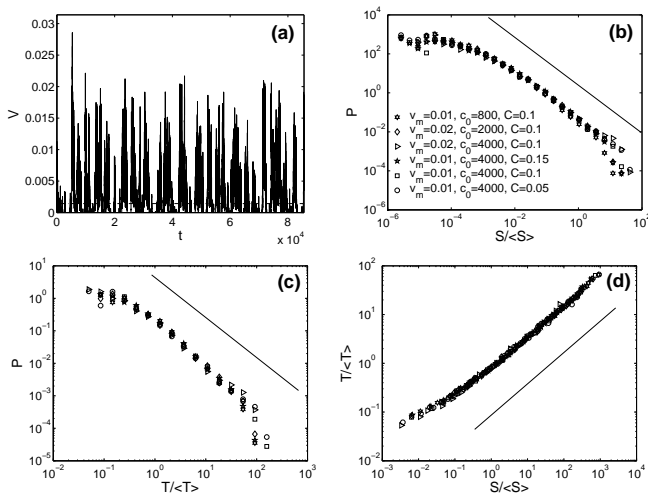


FIG. 3: (a): Typical time evolution of the spatially averaged crack front velocity $v(t) = \langle v(t, x) \rangle_x$. The dash-dot horizontal line corresponds to $v_r = C\max(v)$ with $C = 0.1$ used to define the bursts. Distribution of the normalized burst duration T , normalized burst size S , and the scaling of T with S are plotted in (b), (c) and (d), respectively. The various symbols corresponds to various values of v_m , c_0 and C as specified in the inset of Fig. b. In these three graphs, the straight lines correspond to $P(T) \propto T^{-\tau}$, $P(S) \propto S^{-\alpha}$ and $T \propto S^a$ with the critical exponents $\tau = 1.43$, $\alpha = 1.25$, and $a = 0.58$ predicted by RG approach.

plex failure situations, from nanostructured materials to earthquakes. Moreover, RG calculations performed on a different system belonging to the same universality class, – the motion of domain walls in disordered ferromagnets –, have been used to predict scaling laws for the statistics of the spatially averaged instantaneous crack velocity. It would be interesting to check these predictions experimentally. Work in this direction is under progress [24]. Let us note finally that this description has been derived within the quasi-static approximation. To understand how to include the dynamic stress transfers through acoustic waves occurring as a *dynamically* growing crack is interacting with the material disorder [25] will represent interesting challenges for future investigations.

We thank K.J. Måløy, C. Guerra, C.L. Rountree, L. Barbier, E. Bouchaud, F. Clari, P.-P. Cortet and L. Vanel for useful advices, helpful discussions, and a critical reading of the manuscript. D.B. acknowledges funding from French ANR through grant No. ANR-05-JCJC-0088. S.S. was supported by the NFR Petromax Program No. 163472/S30.

* Electronic address: daniel.bonamy@cea.fr

[1] M. J. Alava, P. K. V. V. Nukala, and S. Zapperi, Adv.

Phys. **55**, 349 (2006).

- [2] E. Bouchaud, J. Phys. Condens. Matter **9**, 4319 (1997).
 [3] L. Ponsou, D. Bonamy, and E. Bouchaud, Phys. Rev. Lett. **96**, 035506 (2006). L. Ponsou, D. Bonamy, and L. Barbier, Phys. Rev. B **74**, 184205 (2006).
 [4] D. Bonamy et al., Phys. Rev. Lett. **97**, 135504 (2006).
 [5] A.-L. Barabasi and H. E. Stanley, *Fractal concepts in surface growth* (Cambridge University Press, 1995).
 [6] J. P. Sethna, K. A. Dahmen, and C. R. Myers, Nature **410**, 242 (2001).
 [7] K. J. Måløy and J. Schmittbuhl, Phys. Rev. Lett. **87**, 105502 (2001); S. Santucci, L. Vanel, and S. Ciliberto, Phys. Rev. Lett. **93**, 095505 (2004); S. Santucci, L. Vanel, and S. Ciliberto, Eur. Phys. J. -Special Topics **146**, 341 (2007); A. Marchenko, D. Fichou, D. Bonamy, and E. Bouchaud, Appl. Phys. Lett. **89**, 093124 (2006).
 [8] A. Garcimartin, A. Guarino, L. Bellon, and S. Ciliberto, Phys. Rev. Lett. **79**, 3202 (1997); J. Davidsen, S. Stanchits, and G. Dresen, Phys. Rev. Lett. **98**, 125502 (2007); J. Koivisto, J. Rosti, and M. J. Alava, Phys. Rev. Lett. **99**, 145504 (2007).
 [9] B. Gutenberg and C. F. Richter, *Seismicity of the earth and associated phenomena*. (Princeton University Press, 1954); A. Corral, Phys. Rev. Lett. **92**, 108501 (2004).
 [10] K. J. Måløy, S. Santucci, J. Schmittbuhl, and R. Toussaint, Phys. Rev. Lett. **96**, 045501 (2006); S. Santucci, K.-J. Måløy, R. Toussaint, and J. Schmittbuhl, in *Dynamics of Complex Interconnected Biosensor Systems*, edited by A. T. Skjeltorp (Kluwer, Amsterdam, 2006).
 [11] B. Lawn, *Fracture of brittle solids* (Cambridge University Press, 1993).
 [12] J.-W. Obreimoff, Proc. Roy. Soc. London A **127**, 290 (1930).
 [13] J. Schmittbuhl, S. Roux, J. P. Vilotte, and K. J. Måløy, Phys. Rev. Lett. **74**, 1787 (1995); S. Ramanathan, D. Ertas, and D. S. Fisher, Phys. Rev. Lett. **79**, 873 (1997).
 [14] H. Gao and J. R. Rice, J. Appl. Mech. **56**, 828 (1989).
 [15] G. Bertotti, G. D. G, and A. Magni, J. Appl. Phys. **75**, 5490 (1994); S. Zapperi, P. Cizeau, G. Durin, and H. E. Stanley, Phys. Rev. B **58**, 6353 (1998).
 [16] G. Durin, S. Zapperi, Phys. Rev. Lett. **84**, 4705 (2000).
 [17] D. Ertas and M. Kardar, Phys. Rev. E **49**, R2532 (1994).
 [18] E. Rolley, C. Guthmann, R. Gombrowicz, and V. Repain, Phys. Rev. Lett. **80**, 2865 (1998).
 [19] O. Narayan and D. S. Fisher, Phys. Rev. B **48**, 7030 (1993); H. Leschhorn, T. Nattermann, S. Stepanow and L. H. Tang, Ann. Physik **6**, 134 (1997);
 [20] P. Bak, C. Tang, and K. Wiesenfeld, Phys. Rev. Lett. **59**, 381 (1987).
 [21] A. Tanguy, M. Gounelle and S. Roux, Phys. Rev. E **58**, 1577 (1998); A. Rosso and W. Krauth, Phys. Rev. E **65**, 025101(R) (2002);
 [22] S. Santucci et al, in preparation (2008).
 [23] J. Schmittbuhl and K. J. Måløy, Phys. Rev. Lett. **78**, 3888 (1997). A. Delaplace, J. Schmittbuhl, and K. J. Måløy, Phys. Rev. E **60**, 1337 (1999).
 [24] S. Santucci et al, in preparation (2008).
 [25] K. Ravi-Chandar and W. Knauss, Int. J. Frac. **26**, 189 (1984); E. Sharon, E. Cohen and J. Fineberg, Nature **65**, 025101(R) (2002); D. Bonamy and K. Ravi-Chandar, Phys. Rev. Lett. **91**, 235502 (2003); D. Bonamy and K. Ravi-Chandar, Int. J. Frac. **134**, 1 (2005).