

Wave drag on a sphere in a strongly stratified flow

Bruno Voisin

LEGI, CNRS–UJF–INPG, BP 53, 38041 Grenoble Cedex 9, France;

Bruno.Voisin@hmg.inpg.fr

Lee-wave drag, namely the wave drag on an obstacle fixed in a stratified flow, is an important feature of geophysical fluid dynamics, with application for example to subgrid-scale dissipation in meso-scale numerical simulation. We consider the weakly and strongly stratified limits, corresponding respectively to high and small internal Froude numbers F . The analysis is developed for the simplest obstacle, a sphere of radius a , and the simplest flow, of uniform velocity U at infinity, in an unbounded uniformly stratified fluid of buoyancy frequency N , with $F = U/(Na)$.

Greenslade (2000) has recently summarized the available experimental data (Mason 1977; Lofquist & Purtell 1984) and proposed two asymptotic models. When $F \gg 1$ the flow is essentially that, irrotational, of a homogeneous fluid, wave generation acting as a small perturbation; the obstacle may be represented by the same distribution of monopoles as for irrotational flow, with wave drag

$$C_D \sim \frac{\ln F + \frac{7}{4} - \gamma}{4F^4} \approx \frac{\ln F + 1.17}{4F^4} \quad \text{for } F \gg 1,$$

where γ is the Euler constant (Gorodtsov & Teodorovich 1982). When $F \ll 1$ the flow is essentially horizontal, wave generation being due to small regions, of height aF , with vertical motion, located at the top and bottom of the obstacle (Hunt *et al.* 1997); based on geometrical considerations and on fits to experimental data, Greenslade (2000) obtained

$$C_D \approx 0.73F^{3/2} \quad (\text{Mason fit}) \quad \text{or} \quad 0.86F^{3/2} \quad (\text{Lofquist \& Purtell fit}) \quad \text{for } F \ll 1.$$

We argue that, in the strongly stratified régime, in spite of the inherent non-linearity of the flow, wave generation is indeed a linear process, the small regions at the top (respectively bottom) of the sphere acting as flat paraboloidal obstacles, of height aF and much larger length $2a(2F)^{1/2}$, lying on horizontal surfaces — the so-called dividing streamsurfaces — below (respectively above) which the motion is purely horizontal (figure 1). The representation of such obstacles is a horizontal distribution of monopoles of strength proportional to the local streamwise slope, with drag

$$C_D \sim \frac{2^{11/2}}{15\pi} F^{3/2} \approx 0.96F^{3/2} \quad \text{for } F \ll 1.$$

The mathematical analysis involves the expression of the drag as a weighted double integral of the distribution's spectrum squared, and the asymptotic evaluation of this integral by Mellin–Barnes theory. The same representation can be used for calculating the waves themselves.

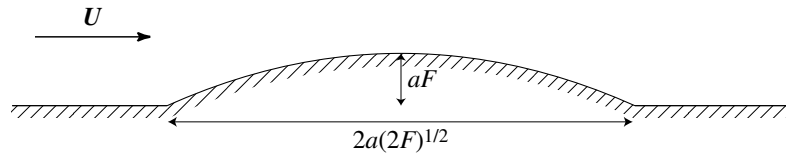


Figure 1. Upper wave generation region for $F \ll 1$, in the streamwise vertical plane through the centre of the sphere.

Gorodtsov, V. A. & Teodorovich, E. V. (1982). *Fluid Dyn.* **17**, 893–898.

Greenslade, M. D. (2000). *J. Fluid Mech.* **418**, 339–350.

Hunt, J. C. R., Feng, Y., Linden, P. F., Greenslade, M. D. & Mobbs, S. D. (1997). *Il Nuovo Cimento C* **20**, 261–272.

Lofquist, K. E. B. & Purtell, L. P. (1984). *J. Fluid Mech.* **148**, 271–284.

Mason, P. J. (1977). *Geophys. Astrophys. Fluid Dyn.* **8**, 137–154.