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How the coupling to the vacuum field of a microcavity changes the nature of a conduction electron in a doped quantum well

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Recent absorption and electroluminescence experiments on microcavities embedding quantum wells have shown strong coupling between a cavity photon mode and the transition between two conduction subbands, being the lowest one filled with a dense two-dimensional electron gas. Through a non-perturbative theory, here we show that the nature of an electron in the excited conduction subband can be profoundly modified by the strong interaction with the cavity vacuum field. For wavevectors larger than the Fermi one, the electron spectral function has a non-trivial structure reminiscent of a Fano resonance, resulting from the coupling between the bare electron and the continuum of cavity polariton modes. We show how these electron states can be selectively excited by resonant electron tunneling from a narrow-band injector and their impact on ultrahigh efficiency polariton electroluminescence in the low excitation regime.

(February 28, 2008)

Cavity quantum electrodynamics is presently the center of many fascinating investigations in a wide variety of interesting systems, including ultracold atoms in optical cavities on a chip[1], Cooper pair quantum boxes in microwave resonators[2] and semiconductor nanostructures in microcavities[3]. In the strong coupling regime, the eigenstates of a cavity system are a coherent mixing of photonic and electronic excitations. This occurs when the light-matter interaction, quantified by the so-called vacuum Rabi frequency, is dominant with respect to loss mechanisms for the cavity photon field and for the electronic transition. Recently, the strong coupling regime has been experimentally demonstrated[4, 5, 6, 7, 8, 9, 10] in semiconductor microcavities embedding doped quantum wells. The active electronic transition is between two conduction subbands, where a dense two-dimensional electron gas populates the lowest one. Large vacuum Rabi frequencies can be achieved thanks to the giant collective dipole associated to the dense electron gas and even an unusual ultra-strong coupling regime can be reached[11, 12], paving the way to a new class of cavity quantum vacuum radiation effects[11, 13]. In the strong coupling regime, the absorption spectra of such microcavities reveal resonances associated to the cavity polariton normal modes. A cavity polariton is a linear superposition of a cavity photon and a bright intersubband excitation, i.e. the quantum of the electronic polarization field. Remarkably, intersubband transitions[14] are the heart of the so-called quantum cascade unipolar light emitting diodes and lasers[15, 16, 17, 18]. In these semiconductor heterostructures, electrons are resonantly injected into an excited conduction subband of the active quantum wells through electron tunneling from injector contacts. In contrast to the usual bipolar optoelectronic devices, the vertical transport occurs in the conduction band, while the valence band remains completely filled. Very recent electroluminescence experiments[9] on a microcavity-embedded quantum cascade structure have shown that it is possible to observe intersubband cavity polaritons in emission after electrical excitation.

An important question to address is whether the strong interaction with the cavity quantum field can modify not only the dynamics of the bosonic intersubband polarization field (a standard issue for cavity polaritons), but also the nature of fermionic carriers in the excited conduction subband as well as the resonant electron tunneling processes. Here, we present a non-perturbative theoretical solution to this fundamental problem. Our theory permits to calculate the spectral function of the quasi-electrons coupled to the cavity vacuum field. We demonstrate that it is non-trivially modified by the interactions leading to superluminescent electronic states. The electronic eigenstates originate from a Fano-like coupling between the bare electron state and the continuum of cavity polariton modes. Our theory demonstrates that resonant electron tunnelling from a narrow-band injector contact can selectively excite dressed states with very different radiative properties. Our results suggest that with narrow-band tunneling injection, it is possible to achieve electrically-driven emission with ultrahigh quantum efficiency.

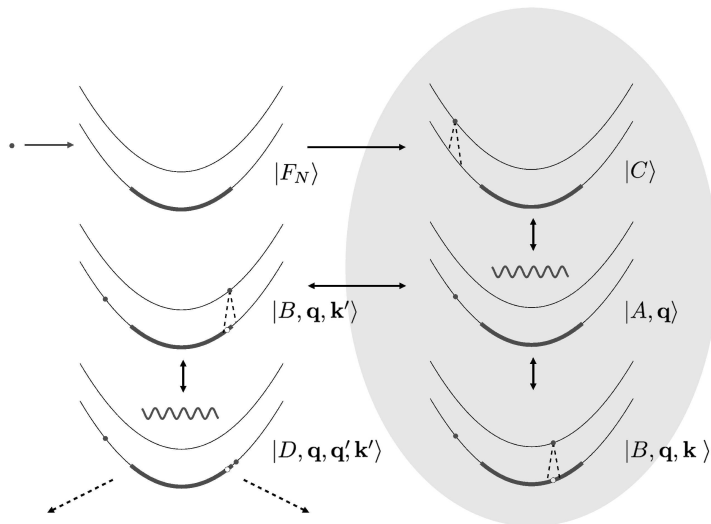


FIG. 1: Sketch of the relevant electron dynamics in the two quantum well conduction subbands with parallel parabolic energy-momentum dispersions. The ground state with N electrons is the standard Fermi sea $|F_N\rangle$. The injection (e.g., through electron tunneling) of an additional electron in the second subband creates the state $|C\rangle = c_{2\mathbf{k}}^\dagger |F_N\rangle$, which, in presence of light-matter interaction, is not an eigenstate. The dashed lines sketch the photon emission cone. Spontaneous emission of a cavity photon couples the $|C\rangle$ state to the states $|A, \mathbf{q}\rangle = a_{\mathbf{q}}^\dagger c_{1\mathbf{k}-\mathbf{q}}^\dagger |F_N\rangle$. Reabsorption of the emitted cavity photon can couple back to the $|C\rangle$ state or to the states $|B, \mathbf{q}, \mathbf{k}'\rangle = c_{2\mathbf{k}'+\mathbf{q}}^\dagger c_{1\mathbf{k}'} c_{1\mathbf{k}-\mathbf{q}}^\dagger |F_N\rangle$. Spontaneous emission can couple some of the $|B\rangle$ states to the states $|D, \mathbf{q}, \mathbf{q}', \mathbf{k}'\rangle = a_{\mathbf{q}'}^\dagger c_{1\mathbf{k}'+\mathbf{q}-\mathbf{q}'}^\dagger c_{1\mathbf{k}'} c_{1\mathbf{k}-\mathbf{q}}^\dagger |F_N\rangle$. However, this is a marginal coupling, because it can occur only close to the border of the Fermi sea.

I. FORMALISM

In order to describe the system under study, we consider the following second quantization Hamiltonian:

$$H = \sum_{\mathbf{k}} \hbar\omega_1(k) c_{1,\mathbf{k}}^\dagger c_{1,\mathbf{k}} + \sum_{\mathbf{k}} \hbar\omega_2(k) c_{2,\mathbf{k}}^\dagger c_{2,\mathbf{k}} + \sum_{\mathbf{k}} \hbar\omega_c(q) a_{\mathbf{q}}^\dagger a_{\mathbf{q}} \quad (1)$$

$$+ \sum_{\mathbf{k}, \mathbf{q}} \hbar\chi(q) a_{\mathbf{q}} c_{1,\mathbf{k}} c_{2,\mathbf{k}+\mathbf{q}}^\dagger + \sum_{\mathbf{k}, \mathbf{q}} \hbar\chi^*(q) a_{\mathbf{q}}^\dagger c_{2,\mathbf{k}+\mathbf{q}} c_{1,\mathbf{k}}^\dagger,$$

where $\hbar\omega_1(k) = \frac{\hbar^2 \mathbf{k}^2}{2m^*}$ and $\hbar\omega_2(k) = \hbar\omega_{12} + \frac{\hbar^2 \mathbf{k}^2}{2m^*}$ are the energy dispersions of the two fundamental quantum well conduction subbands as a function of the in-plane wavevector \mathbf{k} , m^* being the effective mass. The corresponding electron creation fermionic operators are $c_{1,\mathbf{k}}^\dagger$ and $c_{2,\mathbf{k}}^\dagger$. $\omega_c(q)$ is the frequency dispersion of a cavity photonic branch and $a_{\mathbf{q}}^\dagger$ is the corresponding photon creation operator. Due to the selection rules of intersubband transitions, we omit the photon polarization, which is assumed to be Transverse Magnetic (TM). Being all the interactions spin-conserving, we can omit the electron spin. For simplicity, we consider only a photonic branch, which is quasi-resonant with the intersubband transition, while other cavity modes are supposed to be off-resonance. The interaction between the cavity photon field and the two electronic conduction subbands is quantified by the coupling constant

$$\chi(q) = \sqrt{\frac{\omega_{12}^2 d_{12}^2}{\hbar\epsilon_0\epsilon_r L_{cav} S \omega_c(q)} \frac{q^2}{(\pi/L_{cav})^2 + q^2}}, \quad (2)$$

where c is the light speed, d_{12} is the intersubband transition dipole, ϵ_r is the cavity dielectric constant and S is the sample area. Here, we have considered the simple case of a $\lambda/2$ -cavity, where π/L_{cav} is the cavity photon quantized vector along the growth direction. The geometrical factor $\frac{q^2}{(\pi/L_{cav})^2 + q^2}$ is due to the TM-polarization nature of the intersubband transition. Here, for simplicity, we are considering the case of just a single quantum well coupled to the cavity quantum field. Notice that in the Hamiltonian in Eq. (1) the anti-resonant terms of the light-matter interaction have not been included. Therefore, here we can describe the strong coupling regime for the two subband system, but not all the peculiar features of the ultrastrong coupling limit [11, 12, 13].

If we wish to study the injection of one quasi-electron at zero temperatures, we need to consider the following electron spectral function:

$$A_j^+(\mathbf{k}, \omega) = \sum_{\zeta} |\langle \zeta | c_{j,\mathbf{k}}^\dagger | F_N \rangle|^2 \delta(\omega - \omega_{\zeta}), \quad (3)$$

where $|F_N\rangle$ is the N -particle Fermi sea state, i.e. the electronic ground state. $j = 1, 2$ is the conduction subband index. The full ground state of the Hamiltonian in Eq. (1) is $|F_N\rangle|0\rangle$ where $|0\rangle$ is the cavity photon vacuum. Through the paper we will omit to explicitly write the $|0\rangle$.

The two-dimensional density of electrons is N/S and the corresponding Fermi level energy is ϵ_F . The index ζ labels the excited $(N+1)$ -particle eigenstates when one electron has been added to the Fermi sea. From the expression in Eq. (3), it is clear that in order to evaluate the spectral function, we need to determine the overlap between the state $c_{j,\mathbf{k}}^\dagger |F_N\rangle$ and each $(N+1)$ -electron eigenstate $|\zeta\rangle$, where $\hbar\omega_{\zeta}$ is the corresponding quasi-electron energy in presence of interactions. In the case of a non-interacting electron gas, the spectral functions are simply $A_2^+(\mathbf{k}, \omega) = \delta(\omega - \omega_2(k))$ and $A_1^+(\mathbf{k}, \omega) = \delta(\omega - \omega_2(k))\theta(k - k_F)$, where k_F is the Fermi wavevector and $\theta(x)$ is the Heaviside function. In this work, we will prove that due to the interactions with the cavity photon quantum field described in Eq. (1), the electron spectral function $A_2^+(\mathbf{k}, \omega)$ can be profoundly modified. In order to do that, we need to determine in a nonperturbative way the eigenstates of H .

The Hamiltonian in Eq. (1) commutes with the number of total fermions $\hat{N}_F = \sum_{j=1,2} \sum_{\mathbf{k}} c_{j,\mathbf{k}}^\dagger c_{j,\mathbf{k}}$, the total in plane wave-vector operator $\hat{\mathbf{K}} = \sum_{j=1,2} \sum_{\mathbf{k}} \mathbf{k} c_{j,\mathbf{k}}^\dagger c_{j,\mathbf{k}}$ and the excitation number operator $\hat{Q} = \sum_{\mathbf{k}} a_{\mathbf{k}}^\dagger a_{\mathbf{k}} + c_{2,\mathbf{k}}^\dagger c_{2,\mathbf{k}}$. Hence, a generic eigenstate $|\zeta\rangle$ of H is also an eigenstate of these three operators and is labeled by the corresponding eigenvalues $N_{\zeta} = \langle \zeta | \hat{N}_F | \zeta \rangle$, $\mathbf{K}_{\zeta} = \langle \zeta | \hat{\mathbf{K}} | \zeta \rangle$ and $Q_{\zeta} = \langle \zeta | \hat{Q} | \zeta \rangle$. We will thus identify an eigenstate of H in the subspace ($\hat{N}_F = N$, $\hat{\mathbf{K}} = \mathbf{K}$, $\hat{Q} = Q$) as $|N, \mathbf{K}, Q, \zeta\rangle$, where the index ζ now runs over all the eigenstates of the subspace.

The states obtained by applying electron creation or destruction operators on the eigenstates $|N, \mathbf{K}, Q, \zeta\rangle$ are still eigenstates of \hat{N}_F , $\hat{\mathbf{K}}$ and \hat{Q} . The state $c_{1,\mathbf{k}}^\dagger |N, \mathbf{K}, Q, \zeta\rangle$ is labeled by the quantum numbers $(N+1, \mathbf{K} + \mathbf{k}, Q)$; $c_{1,\mathbf{k}} |N, \mathbf{K}, Q, \zeta\rangle$ by $(N-1, \mathbf{K} - \mathbf{k}, Q)$; $c_{2,\mathbf{k}}^\dagger |N, \mathbf{K}, Q, \zeta\rangle$ by $(N+1, \mathbf{K} + \mathbf{k}, Q+1)$; $c_{2,\mathbf{k}} |N, \mathbf{K}, Q, \zeta\rangle$ by $(N-1, \mathbf{K} - \mathbf{k}, Q-1)$. Having the ground state $|F_N\rangle$ quantum numbers $(N, \mathbf{0}, 0)$, in order to evaluate the spectral function $A_2^+(\mathbf{k}, \omega)$ we need to diagonalize the problem in the $(N+1, \mathbf{k}, 1)$ subspace, while for $A_1^+(\mathbf{k}, \omega)$ we are interested in the $(N+1, \mathbf{k}, 0)$ space.

II. METHODS AND DIAGONALIZATIONS

Let us start with the determination of the spectral function of the first conduction subband. The $(N+1, \mathbf{k}, 0)$ subspace is the space obtained acting over the vacuum with $N+1$ first subband creation operators in such a way that the total momentum is \mathbf{k} . All these states are eigenstates of H in Eq. (1) and thus the only states relevant for the spectral function have the form $c_{1,\mathbf{k}}^\dagger |F_N\rangle$ with $k > k_F$ and $\hbar\omega_1(\mathbf{k})$ is corresponding energy. In other words, the spectral function associated to the first subband is the same as in absence of light-matter interaction, namely

$$A_1^+(\mathbf{k}, \omega) = \delta(\hbar\omega - \hbar\omega_1(k))\theta(k_F - k). \quad (4)$$

The situation is completely different for the second subband electron spectral function, which can be non-trivially modified by the interaction with the cavity vacuum field. The $(N+1, \mathbf{k}, 1)$ Hilbert subspace is spanned by vectors of the form: (i) $c_{2,\mathbf{k}_0}^\dagger \prod_{j=1}^N c_{1,\mathbf{k}_j}^\dagger |0\rangle$, where $|0\rangle$ is the empty conduction band state and $\sum_{j=1}^N \mathbf{k}_j = \mathbf{k} - \mathbf{k}_0$; (ii) $a_{\mathbf{q}_0}^\dagger \prod_{j=1}^{N+1} c_{1,\mathbf{k}_j}^\dagger |0\rangle$ with $\sum_{j=1}^{N+1} \mathbf{k}_j = \mathbf{k} - \mathbf{q}_0$. For a large number of electrons, the exact diagonalization of the Hamiltonian in Eq. (1) is a formidable, if not unmanageable task. Here, we show that by a judicious approximation, we can considerably simplify the diagonalization problem, keeping the relevant non-perturbative physics. Namely, we claim that the elements of $(N+1, \mathbf{k}, 1)$ subspace can be well approximated by vectors of the form:

$$|N+1, \mathbf{k}, 1, \zeta\rangle = \left\{ \mu_{\zeta} c_{2,\mathbf{k}}^\dagger + \sum_{\mathbf{q}} \left[\alpha_{\zeta}(\mathbf{q}) a_{\mathbf{q}}^\dagger c_{1,\mathbf{k}-\mathbf{q}}^\dagger + \sum_{|\mathbf{k}'| < k_F} \beta_{\zeta}(\mathbf{q}, \mathbf{k}') c_{2,\mathbf{k}'+\mathbf{q}}^\dagger c_{1,\mathbf{k}-\mathbf{q}}^\dagger \right] \right\} |F_N\rangle. \quad (5)$$

To understand the origin of our approximation, let us consider the time evolution picture sketched in Fig. 1. Suppose that initially the system is in its ground state $|F_N\rangle$. After injection of one electron the state of the system is $|C\rangle = c_{2,\mathbf{k}}^\dagger |F_N\rangle$. If \mathbf{k} is well inside the Fermi sphere, this state is an eigenstate of the full Hamiltonian. In fact, due to Pauli

blocking, the electron in the second subband can not radiatively relax into the first subband. Instead, when $k > k_F$, an injected electron can radiatively decay, so it can emit a photon and fall into the first subband. After the first emission the state will have the form $|A, \mathbf{q}\rangle = a_{\mathbf{q}}^\dagger c_{1, \mathbf{k}-\mathbf{q}}^\dagger |F_N\rangle$. If the cavity system is closed and only the light-matter interaction is considered, the emitted photon will be reabsorbed. The system can evolve back to the state $|C\rangle$ or into one vector of the form $|B, \mathbf{q}, \mathbf{k}'\rangle = c_{2, \mathbf{k}'+\mathbf{q}}^\dagger c_{1, \mathbf{k}'}^\dagger c_{1, \mathbf{k}-\mathbf{q}}^\dagger |F_N\rangle$. If \mathbf{k}' is well inside the Fermi sea, when the second subband electron decay, it will go back to state $|A, \mathbf{q}\rangle$. If \mathbf{k}' is not too far from the border of the Fermi sea, on the contrary, it can evolve into a state of the form $|D, \mathbf{q}, \mathbf{q}', \mathbf{k}'\rangle = a_{\mathbf{q}'}^\dagger c_{1, \mathbf{k}'+\mathbf{q}-\mathbf{q}'}^\dagger c_{1, \mathbf{k}'}^\dagger c_{1, \mathbf{k}-\mathbf{q}}^\dagger |F_N\rangle$. The probability of going into one of the states $|D, \mathbf{q}, \mathbf{q}', \mathbf{k}'\rangle$ is very small compared to probability of going into one of the $|A, \mathbf{q}\rangle$ states. Indeed, it is approximatively given by the ratio between the resonant photonic wave-vector q_{res} ($\omega_{cav}(q_{res}) = \omega_{12}$) and the Fermi wave-vector k_F , typically of the order of 10^{-2} . We can thus neglect these states and look for vectors of the form shown in Eq. (5). In such a subspace, H has the following matrix representation:

$$\mathcal{H}_{N+1, \mathbf{k}, 1} = \hbar \begin{pmatrix} \omega_2(k) & v(q) & v(q') & v(q'') & \cdots \\ v(q)^T & M(\mathbf{q}) & 0 & 0 & \cdots \\ v(q')^T & 0 & M(\mathbf{q}') & 0 & \cdots \\ v(q'')^T & 0 & 0 & M(\mathbf{q}'') & \ddots \\ \vdots & \vdots & \vdots & \ddots & \ddots \end{pmatrix}$$

where $M(\mathbf{q})$ is the Hamiltonian matrix block in the subspace spanned by $\{|A, \mathbf{q}\rangle, |B, \mathbf{q}, \mathbf{k}'\rangle, |B, \mathbf{q}, \mathbf{k}''\rangle, \dots\}$

$$M(\mathbf{q}) = \begin{pmatrix} \omega_c(q) + \omega_1(|\mathbf{k} - \mathbf{q}|) & \chi^*(q) & \chi^*(q) & \cdots \\ \chi(q) & \omega_2(|\mathbf{k}' + \mathbf{q}|) - \omega_1(k') + \omega_1(|\mathbf{k} - \mathbf{q}|) & 0 & \cdots \\ \chi(q) & 0 & \omega_2(|\mathbf{k}'' + \mathbf{q}|) - \omega_1(k'') + \omega_1(|\mathbf{k} - \mathbf{q}|) & \ddots \\ \vdots & \vdots & \ddots & \ddots \end{pmatrix}$$

and $v(q) = [\chi(q) 0 0 \dots]$. Since the typical wavevector q of the resonantly coupled cavity photon mode is much smaller than k_F , we can perform the standard approximation $\omega_2(|\mathbf{k}' + \mathbf{q}|) - \omega_1(k') \simeq \omega_2(k') - \omega_1(k') = \omega_{12}$. This way, we can exactly diagonalize each of the $M(\mathbf{q})$. We have two bright electronic states (i.e., with a photonic mixing component)

$$|\pm, \mathbf{q}\rangle = \frac{(\omega_\pm(q) - \omega_{12})|A, \mathbf{q}\rangle + \chi(q) \sum_{\mathbf{k}} |B, \mathbf{q}, \mathbf{k}\rangle}{\sqrt{(\omega_\pm(q) - \omega_{12})^2 + |\chi(q)|^2 N}}, \quad (6)$$

with energies $\hbar\omega_1(k) + \hbar\omega_\pm(q)$, where

$$\omega_\pm(q) = \frac{\omega_c(q) + \omega_{12}}{2} \pm \sqrt{\left(\frac{\omega_c(q) - \omega_{12}}{2}\right)^2 + N|\chi(q)|^2}. \quad (7)$$

Note that $\hbar\omega_\pm(q)$ are the energies of the two branches of intersubband cavity polaritons [11].

The other orthogonal states are dark (no photonic component), with eigenvalues $\omega_1(k) + \omega_{12}$ and eigenvectors

$$|l, \mathbf{q}\rangle = \frac{\sum_{\mathbf{k}} \beta_l(\mathbf{q}, \mathbf{k}) |B, \mathbf{q}, \mathbf{k}\rangle}{\sqrt{N}} \quad (8)$$

where the $\beta_l(\mathbf{q}, \mathbf{k})$ are such that $\sum_{\mathbf{k}} \beta_l(\mathbf{q}, \mathbf{k}) = 0$ and $\sum_{\mathbf{k}} \beta_l(\mathbf{q}, \mathbf{k}) \beta_{l'}^*(\mathbf{q}, \mathbf{k}) = \delta_{l, l'}$. Since $\langle l, \mathbf{q} | H c_{2, \mathbf{k}}^\dagger | F_N \rangle = 0$, the dark states $|l, \mathbf{q}\rangle$ are also eigenstates of the matrix $\mathcal{H}_{N+1, \mathbf{k}, 1}$ and do not contribute to the electron spectral function, because they have zero overlap with the state $c_{2, \mathbf{k}}^\dagger | F_N \rangle = 0$. In contrast, this is not the case for the bright eigenstates of each block $M(\mathbf{q})$, as we find:

$$\langle \pm, \mathbf{q} | H c_{2, \mathbf{k}}^\dagger | F_N \rangle = \frac{\chi(q)^*(\omega_\pm(q) - \omega_{12})}{\sqrt{(\omega_\pm(q) - \omega_{12})^2 + |\chi(q)|^2 N}} = J_\pm^*(q). \quad (9)$$

Therefore, the representation of H in the subspace $\{c_{2, \mathbf{k}}^\dagger | F_N \rangle, |+, \mathbf{q}\rangle, |-, \mathbf{q}\rangle, |+, \mathbf{q}'\rangle, |-, \mathbf{q}'\rangle \dots\}$ reads

$$\mathcal{H}'_{N+1, \mathbf{k}, 1} = \hbar \begin{pmatrix} \omega_1(k) + \omega_{12} & J_+^*(q) & J_-^*(q) & J_+^*(q') & J_-^*(q') & \cdots \\ J_+(q) & \omega_1(k) + \omega_+(q) & 0 & 0 & 0 & \cdots \\ J_-(q) & 0 & \omega_1(k) + \omega_-(q) & 0 & 0 & \cdots \\ J_+(q') & 0 & 0 & \omega_1(k) + \omega_+(q') & 0 & \cdots \\ J_-(q') & 0 & 0 & 0 & \omega_1(k) + \omega_-(q') & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

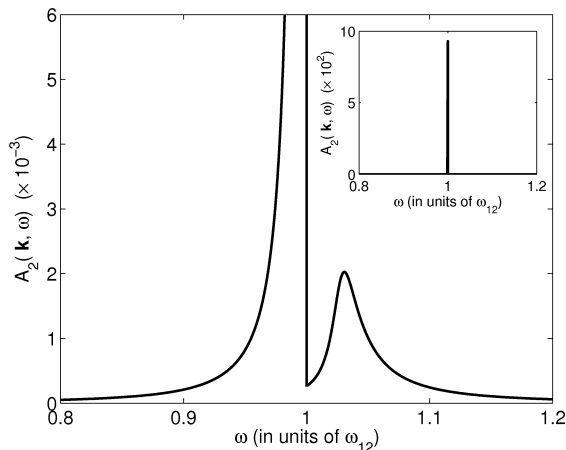


FIG. 2: Electron spectral function $A_2^+(\mathbf{k}, \omega)$ for the second subband, for all $k > k_F$. Inset: the same quantity on a larger vertical scale. Coupling parameter: $\Omega_0(q_{res}) = \chi(q_{res})\sqrt{N} = 0.1\omega_{12}$.

Hence, here we have demonstrated that the bare electron state $c_{2,\mathbf{k}}^\dagger|F_N\rangle$ is coupled to the continuum of the polariton modes with all the different wavevectors \mathbf{q} . Since the polariton frequencies ω_\pm and the coupling J_\pm depend only on the modulus of \mathbf{q} , we can further simplify the diagonalization problem by introducing the 'annular' bright states

$$|\pm, q\rangle = \frac{1}{\sqrt{Lq}} \sum_{|\mathbf{q}|=q} |\pm, \mathbf{q}\rangle, \quad (10)$$

where $L = \sqrt{S}$ and $2\pi/L$ is the linear density of modes in reciprocal space. All annular states are coupled to $c_{2,\mathbf{k}}^\dagger|F_N\rangle$. Instead, all the orthogonal linear combinations of $|\pm, \mathbf{q}\rangle$ (with $|\mathbf{q}|=q$) are uncoupled and therefore do not contribute to the electron spectral function. The representation of H in the subspace $\{c_{2,\mathbf{k}}^\dagger|F_N\rangle, |+, q\rangle, |-, q\rangle, |+, q'\rangle, |-, q'\rangle, \dots\}$ reads

$$\mathcal{H}''_{N+1,\mathbf{k},1} = \hbar \begin{pmatrix} \omega_1(k) + \omega_{12} & J_+^*(q)\sqrt{Lq} & J_-^*(q)\sqrt{Lq} & J_+^*(q')\sqrt{Lq'} & J_-^*(q')\sqrt{Lq'} & \dots \\ J_+(q)\sqrt{Lq} & \omega_1(k) + \omega_+(q) & 0 & 0 & 0 & \dots \\ J_-(q)\sqrt{Lq} & 0 & \omega_1(k) + \omega_-(q) & 0 & 0 & \dots \\ J_+(q')\sqrt{Lq'} & 0 & 0 & \omega_1(k) + \omega_+(q') & 0 & \dots \\ J_-(q')\sqrt{Lq'} & 0 & 0 & 0 & \hbar\omega_1(k) + \omega_-(q') & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots & \ddots \end{pmatrix} \quad (11)$$

Hence, in the subspace $(N+1, \mathbf{k}, 1)$, we have found that eigenstates of H with a finite overlap with the bare electron have the form

$$|N+1, \mathbf{k}, 1, \zeta\rangle = \mu_\zeta c_{2,\mathbf{k}}^\dagger|F_N\rangle + \sum_{q,\sigma=\pm} \lambda_{\zeta,\sigma,q} |\sigma, q\rangle. \quad (12)$$

The coefficients μ_ζ and $\lambda_{\zeta,\sigma,q}$ as well as the corresponding energy eigenvalue $\hbar\omega_\zeta$ can be calculated through a numerical diagonalization of the matrix in Eq. (11). In conclusion, the spectral function of the second subband reads

$$A_2^+(\mathbf{k}, \omega) = \sum_{\zeta} |\mu_\zeta|^2 \delta(\omega - \omega_\zeta) \theta(k - k_F) + \delta(\omega - \omega_2(k)) \theta(k_F - k). \quad (13)$$

In Fig. 2, we show numerical results using a vacuum Rabi frequency $\Omega_0(q_{res}) = |\chi(q_{res})|\sqrt{N} = 0.1\omega_{12}$. As it appears from Eq. (13), the broadening of the spectral function is intrinsic, being associated to the continuum spectrum of frequencies ω_ζ corresponding to the dressed electronic states and given by the eigenvalues of the infinite matrix in Eq. (11). At each frequency ω_ζ , the magnitude of the spectral function is given by the spectral weight $|\mu_\zeta|^2$, depending on the overlap between the dressed state $|N+1, \mathbf{k}, 1, \zeta\rangle$ and the bare electron state $c_{2,\mathbf{k}}^\dagger|F_N\rangle$. As shown in Eq. (11), the electronic eigenstates of the system are given by the Fano-like coupling between the bare electron state and the continuum of cavity polariton excitations. Indeed, the pronounced dip around $\omega = \omega_{12}$ in the spectral function is a quantum interference feature, typical of a Fano resonance[19].

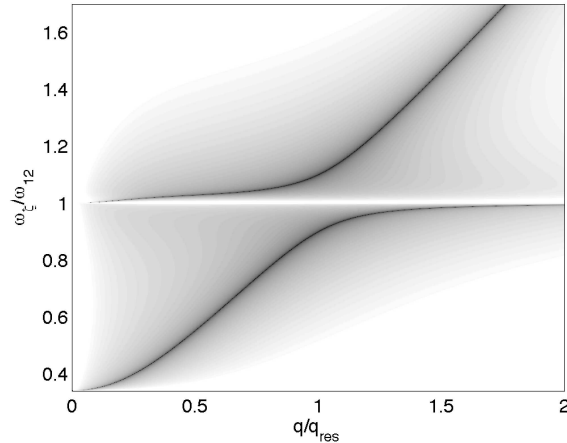


FIG. 3: Logarithmic plot of the absolute value of the radiative matrix element $\alpha_\zeta(\mathbf{q}) = \langle N+1, \mathbf{k}, 1, \zeta | a_{\mathbf{q}}^\dagger c_{1, \mathbf{k}-\mathbf{q}}^\dagger | F_N \rangle$, responsible for the radiative lifetime of the electronic eigenstates. The image is shown in saturated colors to improve readability. The dispersion of the two cavity polariton branches is apparent.

III. TUNNELING COUPLING, LOSSES AND ELECTROLUMINESCENCE

The states $|N+1, \mathbf{k}, 1, \zeta\rangle$ have been obtained by diagonalizing the Hamiltonian in Eq. (1), which takes into account only the coupling between the two-subband electronic system and the microcavity photon quantum field. If, as we have assumed, the light-matter interaction is the strongest one, all other residual couplings can be treated perturbatively. These residual interactions include, e.g., the coupling to the extracavity fields, the interaction with contacts, phonon and impurity scattering as well as Coulomb electron-electron interaction[20].

The dressed states $|N+1, \mathbf{k}, 1, \zeta\rangle$ can be excited, e.g., by resonant electron tunneling. The tunneling coupling with an electronic contact is able to excite the ground state $(N, \mathbf{0}, 0)$ upon injection of an electron. Treating the tunneling coupling perturbatively, we find that the tunneling injection rate reads

$$\Gamma_{inj}(\mathbf{k}, \zeta) = \frac{2\pi}{\hbar} |\mu_\zeta|^2 |V_{tc}(\mathbf{k})|^2 \rho_{inj}(\omega_\zeta) n_F(\omega_\zeta), \quad (14)$$

where $V_{tc}(\mathbf{k})$ is the tunneling coupling matrix element, $\rho_{inj}(\omega)$ is the density of electronic states inside the contact and $n_f(\omega)$ its Fermi distribution. In quantum cascade structures[15, 16, 17, 18], the quantity $|V_{tc}(\mathbf{k})|^2 \rho_{inj}(\omega)$ can be quantum engineered by judiciously choosing the appropriate tunneling barrier and by modifying the properties of the injection heterostructure. The spectral bandwidth of the injector contact is given by the width of $|V_{tc}(\mathbf{k})|^2 \rho_{inj}(\omega) n_F(\omega)$.

The finite transmission of the cavity mirrors is responsible for a finite lifetime for the cavity photons. Consequently, the dressed states $|N+1, \mathbf{k}, 1, \zeta\rangle$ have a finite radiative lifetime. By using the Fermi golden rule and a quasi-mode coupling to the extracavity field, we find that the radiative lifetime $\tau_{r, \mathbf{k}, \zeta}$ is given by the expression

$$\frac{1}{\tau_{r, \mathbf{k}, \zeta}} = \frac{2\pi}{\hbar} \sum_{\mathbf{q}, q_z} |\langle N+1, \mathbf{k}, 1, \zeta | a_{\mathbf{q}}^\dagger c_{1, \mathbf{k}-\mathbf{q}}^\dagger | F_N \rangle|^2 |V_{qm}(\mathbf{q}, q_z)|^2 \delta(\hbar\omega_\zeta - \hbar\omega_{ph}(\mathbf{q}, q_z)) \theta(k - k_F), \quad (15)$$

where $V_{qm}(\mathbf{q}, q_z)$ describes the quasi-mode coupling matrix element and $\omega_{ph}(\mathbf{q}, q_z)$ the extracavity photon frequency dispersion. The Heaviside function in Eq. (15) is due to the Pauli blocking that, in the linear regime we are considering, does not allow for any electron injected into the second subband with $k < k_F$ to radiatively decay.

The matrix elements in Eq. (15) can be calculated by the completeness relation $\sum_{q, j=\pm} |j, q\rangle \langle j, q| + c_{2, \mathbf{k}}^\dagger |F_N\rangle \langle F_N| c_{2, \mathbf{k}} = I$ in the space $(N+1, \mathbf{k}, 1)$. The result reads

$$\langle N+1, \mathbf{k}, 1, \zeta | a_{\mathbf{q}}^\dagger c_{1, \mathbf{k}-\mathbf{q}}^\dagger | F_N \rangle = \sum_{\sigma=\pm} \frac{\lambda_{\zeta, \sigma, q} (\omega_\sigma(q) - \omega_{12})}{\sqrt{(\omega_\sigma(q) - \omega_{12})^2 + N\chi(q)^2} \sqrt{Lq}}. \quad (16)$$

Fig. 3 depicts a logarithmic plot of the radiative matrix element in Eq. (16). The matrix element is strongly peaked at the frequencies ω_\pm of the two cavity polariton branches.

Having calculated the tunneling injection rate and the radiative lifetime for the different states, we are able to evaluate the electroluminescence spectra. It is convenient to introduce the normalized photon emission distribution corresponding to each eigenstate $|N + 1, \mathbf{k}, 1, \zeta\rangle$, namely

$$L(\mathbf{q}, \zeta) = \mathcal{N} \sum_{q_z} |\langle N + 1, \mathbf{k}, 1, \zeta | a_{\mathbf{q}}^\dagger c_{1, \mathbf{k}-\mathbf{q}}^\dagger | F_N \rangle|^2 |V_{qm}(\mathbf{q}, q_z)|^2 \delta(\hbar\omega_\zeta - \hbar\omega_{ph}(\mathbf{q}, q_z)), \quad (17)$$

where the normalization \mathcal{N} is fixed by imposing $\sum_{\mathbf{q}} L(\mathbf{q}, \zeta) = 1$. The number of photons with in-plane wave-vector \mathbf{q} and frequency ω emitted per unit time is

$$N_{ph}(\mathbf{q}, \omega) = \frac{1}{\pi} \sum_{\mathbf{k}, \zeta} \Gamma_{inj}(\mathbf{k}, \zeta) L(\mathbf{q}, \zeta) \frac{1/\tau_{r, \mathbf{k}, \zeta}}{(\omega - \omega_\zeta)^2 + (1/\tau_{r, \mathbf{k}, \zeta} + 1/\tau_{nr, \mathbf{k}, \zeta})^2}, \quad (18)$$

where the last factor accounts for the Lorentzian broadening due to radiative and non-radiative processes. $\tau_{nr, \mathbf{k}, \zeta}$ is the non-radiative lifetime of the electronic excitations (e.g., due to scattering with phonons, impurities, other electrons) and $\Gamma_{inj}(\mathbf{k}, \zeta)$ is given by Eq. (14). Results for the electroluminescent spectra are shown in Fig. 4. The picture in panel (a) correspond to the case of a broadband injector (width equal to ω_{12} , centered at ω_{12}). In this case, electroluminescence is finite in a broad region of the cavity polariton dispersions[21]. Instead, the other panels correspond to the case of a narrow-band injector (width $0.05\omega_{12}$) centered respectively at ω_{12} (b), $0.8\omega_{12}$ (c) and $1.2\omega_{12}$ (d). In this case, polariton electroluminescence occurs in a small frequency window, implying that the cavity polariton excitations can be selectively excited by the resonant electron tunneling process, in agreement with what suggested by recent experiments [9].

In free-space, it is well known that the quantum efficiency of electroluminescent devices based on intersubband transitions is poor ($\approx 10^{-5}$ in the mid-infrared, $\approx 10^{-8}$ in the far infrared) due to the slow radiative recombination rate of long wavelength transitions. In contrast, in the microcavity case, the efficiency of the emission from an excited state $|N + 1, \mathbf{k}, 1, \zeta\rangle$ is given by $(1 + \tau_{r, \mathbf{k}, \zeta}/\tau_{nr, \mathbf{k}, \zeta})^{-1}$. Being $1/\tau_{nr, \mathbf{k}, \zeta}$ essentially proportional to the matter component of the excitation and $1/\tau_{r, \mathbf{k}, \zeta}$ to its photonic fraction, it is possible to obtain a quantum efficiency approaching unity by selectively injecting electrons into dressed states with a high photonic fraction. In particular, this is achievable by avoiding injection resonant with the central peak of the electron spectral function in Fig.2, which corresponds to states with strong overlap with the bare electron state. Indeed, the strong coupling to the vacuum field of a planar microcavity can make ultraefficient the electroluminescence of an intersubband transition excited by resonant electron tunneling.

To summarize, through a non-perturbative quantum theoretical solution, we have shown how the nature of a conduction subband electron can be profoundly modified by the coupling to the vacuum field of a planar microcavity. The dressed electronic states originate from the Fano-like coupling between the bare electronic states and the continuum of cavity polariton modes. We have proven that these states can be selectively excited by resonant electron tunneling and that the use of narrow-band injector may give rise to ultrahigh efficient electroluminescence. From a general point of view, we have shown in the considered system the fascinating link between cavity quantum electrodynamics and electronic transport, paving the way to exciting progress in fundamental quantum optoelectronics.

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- [1] Y. Colombe, T. Steinmetz, G. Dubois, L. Linke, D. Hunger, and J. Reichel, *Strong atom-field coupling for Bose-Einstein condensates in an optical cavity on a chip*, Nature **450**, 272-276 (2007).
 - [2] A. Wallraff, D. I. Schuster, A. Blais, L. Frunzio, R.-S. Huang, J. Majer, S. Kumar, S. M. Girvin and R. J. Schoelkopf, *Strong coupling of a single photon to a superconducting qubit using circuit quantum electrodynamics*, Nature **431**, 162 (2004).
 - [3] K. Hennessy, A. Badolato, M. Winger, D. Gerace, M. Atature, S. Gulde, S. Falt, E. L. Hu, and A. Imamoglu, *Quantum nature of a strongly coupled single quantum dot-cavity system*, Nature **445**, 896 (2007).
 - [4] D. Dini, R. Kohler, A. Tredicucci, G. Biasiol, and L. Sorba, *Microcavity Polariton Splitting of Intersubband Transitions*, Phys. Rev. Lett. **90**, 116401 (2003).
 - [5] A. A. Anappara, A. Tredicucci, G. Biasiol, L. Sorba, *Electrical control of polariton coupling in intersubband microcavities*, Appl. Phys. Lett. **87**, 051105 (2005).

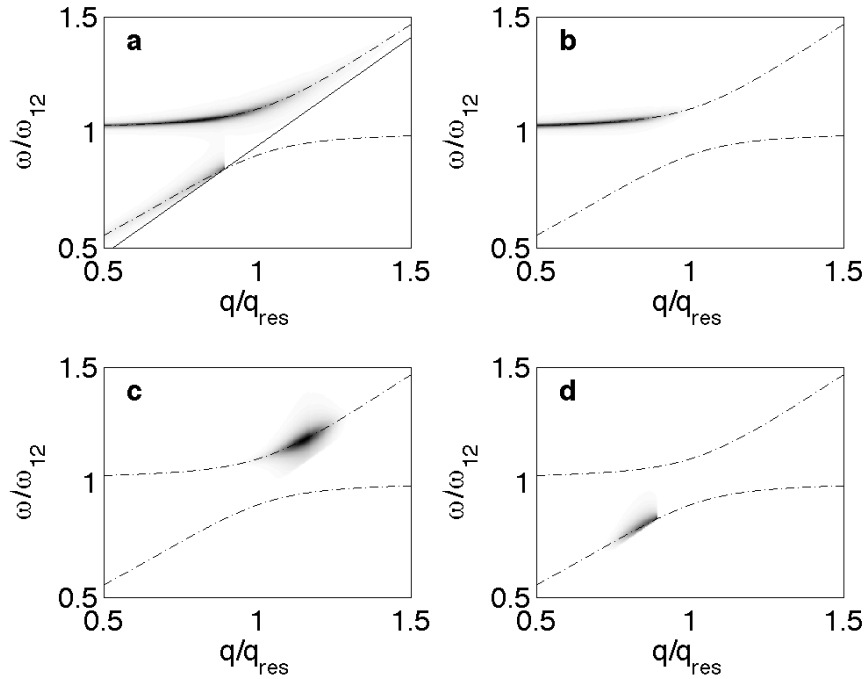


FIG. 4: Extracavity electroluminescence spectra $N_{ph}(\mathbf{q}, \omega)$. Panel (a): the case of a broadband injector (bandwidth equal to ω_{12} , centered at ω_{12}). The other panels show the results for a narrow-band injector (width $0.05\omega_{12}$) centered respectively at ω_{12} (b), $0.8\omega_{12}$ (c) and $1.2\omega_{12}$. The non-radiative relaxation rate $1/\tau_{nr}$ has been taken equal to $0.005\omega_{12}$. In all panels, the dashed-dotted lines are the frequency dispersions $\omega_{\pm}(q)$ of the two cavity polariton branches. In the first panel the solid line represents the edge of the light cone [10].

- [6] A. A. Anappara, A. Tredicucci, F. Beltram, G. Biasiol, L. Sorba, *Tunnel-assisted manipulation of intersubband polaritons in asymmetric coupled quantum wells*, Appl. Phys. Lett. **89**, 171109 (2006).
- [7] R. Colombelli, C. Ciuti, Y. Chassagneux, C. Sirtori, *Quantum cascade intersubband polariton light emitters*, Semicond. Sci. Technol. **20**, 985 (2005).
- [8] L. Sapienza, A. Vasanelli, C. Ciuti, C. Manquest, C. Sirtori, R. Colombelli, and U. Gennser, *Photovoltaic probe of cavity polaritons in a quantum cascade structure*, Appl. Phys. Lett. **90**, 201101 (2007).
- [9] L. Sapienza, A. Vasanelli, R. Colombelli, C. Ciuti, Y. Chassagneux, C. Manquest, U. Gennser, C. Sirtori, *Electrically injected cavity polaritons*, Phys. Rev. Lett., in press; preprint arXiv:0709.3894
- [10] A. A. Anappara, A. Tredicucci, F. Beltram, G. Biasiol, L. Sorba, S. De Liberato, C. Ciuti, *Cavity polaritons from excited-subband transitions*, Appl. Phys. Lett. **91**, 231118 (2007)
- [11] C. Ciuti, G. Bastard, I. Carusotto, *Quantum vacuum properties of the intersubband cavity polariton field*, Phys. Rev. B **72**, 115303 (2005).
- [12] C. Ciuti, I. Carusotto, *Input-output theory of cavities in the ultrastrong coupling regime: The case of time-independent cavity parameters*, Phys. Rev. A **74**, 033811 (2006).
- [13] S. De Liberato, C. Ciuti, I. Carusotto, *Quantum Vacuum Radiation Spectra from a Semiconductor Microcavity with a Time-Modulated Vacuum Rabi Frequency*, Phys. Rev. Lett. **98**, 103602 (2007).
- [14] *Intersubband Transitions in Quantum Wells: Physics and Device Applications I and II*, edited by H. C. Liu and F. Capasso, Semiconductors and Semimetals Vol. 62 (Academic Press, San Diego, 2000).
- [15] J. Faist, F. Capasso, D.L. Sivco, C. Sirtori, A. L. Hutchinson, A.Y. Cho, *Quantum Cascade Laser*, Science **264**, 553 (1994).
- [16] R. Köhler, A. Tredicucci, F. Beltram, H.E. Beere, E.H. Linfield, A.G. Davies, D.A. Ritchie, R.C. Iotti, F. Rossi, *Terahertz semiconductor-heterostructure laser*, Nature **417**, 156 (2002).
- [17] R. Colombelli, K. Srinivasan, M. Troccoli, O. Painter, C. F. Gmachl, D. M. Tennant, A. M. Sergent, D. L. Sivco, A. Y. Cho, F. Capasso, *Quantum Cascade Surface-Emitting Photonic Crystal Laser*, Science **302**, 1374 (2003).
- [18] R. Terazzi, T. Gresch, M. Giovannini, N. Hoyler, N. Sekine, J. Faist, *Bloch gain in quantum cascade lasers*, Nature Phys. **3**, 329 (2007).
- [19] U. Fano, *Effects of Configuration Interaction on Intensities and Phase Shifts*, Phys. Rev. **124**, 1866 (1961)
- [20] D. E. Nikonov, A. Imamoglu, L. V. Butov, and H. Schmidt, *Collective Intersubband Excitations in Quantum Wells: Coulomb Interaction versus Subband Dispersion*, Phys. Rev. Lett. **79**, 4633 (1997).

- [21] S. De Liberato and C. Ciuti, *Quantum model of microcavity intersubband electroluminescent devices*, preprint arXiv:0711.3146