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# Cyclic production for the robotic balanced no-wait flow shop

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**Abstract.** In a no-wait robotic flow-shop, the parts are transferred from a machine to another one by a robot and the time a part can remain on a machine is fixed. The objective is to maximize the throughput rate of the cell, i.e. to find optimal cyclic robot moves. In this paper, we consider identical parts in the balanced case (all processing times are equal). We present a conjecture which gives the structure of the optimal production cycle and prove it for several cases. These results confirm Agnetis conjecture which claims that, in a no-wait robotic cell, the degree of the dominant cycles can be bounded by the number of machines minus one (which simplifies the search of an optimal production cycle).

**Keywords:** Hoist Scheduling Problem, no-wait flow shop, flow-shop scheduling, optimal cycles.

## 1 Introduction

In surface treatment lines, products are immersed in several tanks. The tanks contain chemical baths like acids which affect the electrical or mechanical properties of the products. This kind of line is used, for instance, for galvanoplasty or circuit board assembly. The products are mounted on carriers and transported from a tank to another one by a hoist. The time a part can stay in a tank is upper and lower bounded. The lower bound indicates the minimum time for a correct treatment. The upper limit is justified by the chemical properties of the baths. For instance, a product should not remain too long in an acid bath or the quantity of precious metal deposited should not be too important in order to minimize the costs.

A classical objective is to find the cyclic hoist moves which yield the maximum throughput. This problem is usually called Cyclic Hoist Scheduling Problem (CHSP). We shall restrict the problem to the production of identical parts. (Lei & Wang 1989) showed that the CHSP is NP-complete. Different methods were proposed to solve this problem: Constraint Logic Programming (Baptiste, Legeard, Manier & Varnier 1996), Genetic Algorithm (Lim 1997) or Branch and Bound Algorithm (Ng 1996). A survey on those problems was proposed by (Baptiste, Bloch & Varnier 2001).

The problem with unbounded processing windows (infinite upper bound) is usually called the Robotic Cell Scheduling Problem (RCSP). It was first studied by (Sethi, Sriskandarajah, Sorger, Blazewicz & Kubiak 1992). For quality reasons, it can be better for all products to remain exactly the same time in tank  $T_i$  (for all  $i$ ). This constraint can be modelled by a no-wait constraint in every tank (equal lower and upper bound). For those zero-width processing window problems, (Agnetais 2000) describes a conjecture which bounds the degree of the optimal cycles and proves

it for two and three-tank lines. A survey on those problems was proposed by (Crama, Kats, van de Klundert & Levner 2000).

In this article we will study the no-wait case with equal treatment times in every tank (balanced line). We propose a conjecture on the optimal cycle (section 3-4) and prove special cases of this conjecture (section 5). Then (section 6) we describe in details the case  $m = 5$ .

## 2 Problem presentation

Figure 1 shows a surface treatment line with one hoist and  $m$  treatment tanks denoted by  $T_1, T_2 \dots T_m$ . The parts are mounted on carriers which are moved from a bath to another one by a hoist which travels along a rail. Tank  $T_0$  (the loading tank) and tank  $T_{m+1}$  (the unloading tank) have infinite capacity whereas the other tanks of the line can only contain one carrier at a time. The line represents a flow-shop with the hoist as the material handling device. All products are identical. A carrier is picked up at  $T_0$  and transferred in succession to  $T_1, T_2$  etc. until it finally reaches the output station,  $T_{m+1}$ . The time the products remain in the baths is fixed (no-wait constraint).

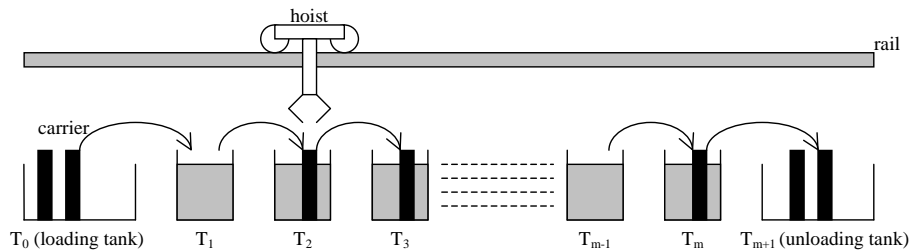


Figure 1: A surface treatment line

The time a part has to remain in tank  $T_i$  is denoted by  $p_i$ . A line is *balanced* if all processing times are equal ( $p_i = p$  for all  $1 \leq i \leq m$ ). The travel time of the hoist between two consecutive tanks is  $\delta$ . The travel times are additive. Hence, the hoist takes  $|i - j|\delta$  time units to travel from  $T_i$  to  $T_j$ . The loading and unloading times can be neglected compared to the travel time.

The hoist moves are described in term of activities. Activity  $i$  ( $i = 0, 1 \dots m$ ) consists in the following moves:

- the hoist picks up a carrier from  $T_i$ ;
- the hoist travels from  $T_i$  to  $T_{i+1}$ ;
- the hoist loads the carrier onto  $T_{i+1}$ .

We consider cyclic moves of the hoist and define the  $k$ -cycles as follows:

**Definition 1** A  $k$ -cycle  $C_k$  is a sequence of activities in which each activity occurs exactly  $k$  times, in which there is exactly one occurrence of activity  $(i - 1)$  and one occurrence of activity  $(i + 1)$  between two consecutive (in a cyclic sense) occurrences of activity  $i$  ( $i = 1, 2 \dots m - 1$ ).

Denote by  $T(C_k)$  the relative cycle time of the  $k$ -cycle  $C_k$ , i.e. the total time for the execution of  $C_k$  divided by  $k$ . Thus a  $k$ -cycle  $C_k$  is optimal if it maximizes the throughput rate or equivalently minimizes the relative cycle time  $T(C_k)$  over the set of all possible  $k$ -cycles ( $k = 1, 2, 3 \dots$ ).

**Definition 2** Let  $S$  and  $S'$  be two sets of production cycles.  $S$  dominates  $S'$  if, for any instance, one has the following property: for every  $k'$ -cycle  $C_{k'}$  of  $S'$ , there exists a  $k$ -cycle  $C_k$  in  $S$  verifying  $T(C_k) \leq T(C_{k'})$ .

The description of the cycles in term of activities can be tedious. To simplify this description we use the symbol  $\prod$  which defines the concatenation of the activities. For example the cycle (3210) will be written  $\prod_{i=0}^3(3-i)$ . The movements of the robot during the cycles can be represented as in Figure 2 :

- the loaded robot moves are the thick line;
- the empty robot moves are the dotted line.

Figure 2 represents the movements of the robot while executing the 2-cycle (02132301) in a three-tank line. In this figure, we do not represent the waiting times but only the robot moves.

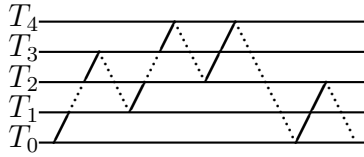


Figure 2: Representation of the cycle (02132301)

### 3 Conjecture on the optimal cycles

In this section we propose a conjecture on the structure of the optimal cycles for a balanced  $m$ -tank line. This conjecture has already been proved for  $m = 3$  (Mangione, Brauner & Penz 2003a) and  $m = 4$  (Mangione, Brauner & Penz 2003b). The cycles  $C_a(\alpha)$ ,  $C_b(\alpha)$ ,  $C_c(\alpha)$  and  $C_e$  are described in detail in section 4.

**Conjecture 1** In a balanced no-wait robotic cell with  $m$  odd, the following items describe an optimal cycle.

- for  $p \in [4(k-1)\delta, 4k\delta[$  and  $k \leq \frac{m-1}{2}$ , the  $k$ -cycle  $C_a(k)$  is optimal;
- for  $p \in [4(k-1)\delta, 4(k-1)\delta + 2\delta[$  and  $k = \frac{m+1}{2}$ , the  $k$ -cycle  $C_a(k)$  is optimal;
- for  $p \in [4(k-1)\delta, 4(k-1)\delta + 2\delta[$  and  $\frac{m+1}{2} < k \leq m-1$ , the  $k$ -cycle  $C_b(k)$  is optimal.
- for  $p \in [4(k-1)\delta + 2\delta, 4k\delta[$  and  $\frac{m+1}{2} \leq k \leq m-1$ , the  $k$ -cycle  $C_c(k)$  is optimal.
- for  $p \geq 4(m-1)\delta$ , the 1-cycle  $C_d = \prod_{i=0}^m(m-i)$  is optimal.

Note that, according to Conjecture 1, for any  $k \leq (m-1)$  there always exist an optimal  $k$ -cycle which confirms Agnetis conjecture (Agnetis 2000).

**Conjecture 2** In a balanced no-wait robotic cell with  $m$  even, the following items describe an optimal cycle.

- for  $p \in [4(k-1)\delta, 4k\delta[$  and  $k \leq \frac{m}{2}$ , the  $k$ -cycle  $C_a(k)$  is optimal;
- for  $p \in [4(k-1)\delta, 4(k-1)\delta + 2\delta[$  and  $\frac{m}{2} < k \leq m-1$ , the 1-cycle  $C_e$  or the  $k$ -cycle  $C_b(k)$  is optimal.
- for  $p \in [4(k-1)\delta + 2\delta, 4k\delta[$  and  $\frac{m}{2} < k \leq m-1$ , the 1-cycle  $C_e$  or the  $k$ -cycle  $C_c(k)$  is optimal.
- for  $p \geq 4(m-1)\delta$ , the 1-cycle  $C_d = \prod_{i=0}^m(m-i)$  is optimal.

## 4 Cycles definition

The  $\alpha$ -cycle  $C_a(\alpha)$  is defined as follows:

$$C_a(\alpha) = \underbrace{\prod_{i=1}^{\alpha} \left[ 0 \prod_{j=1}^i (i-j+1) \right]}_{(1)} * \underbrace{\prod_{i=\alpha+1}^m \left[ \prod_{j=1}^{\alpha} (i-j+1) \right]}_{(2)} * \underbrace{\prod_{i=1}^{\alpha-1} \left[ \prod_{j=1}^{\alpha-i} (m-j+1) \right]}_{(3)}$$

The cycle time of  $C_a(\alpha)$ , when it is feasible, is :

$$T(C_a(\alpha)) = \frac{1}{\alpha} [(m + \alpha - 1)p + 2(m + 2\alpha - 1)\delta]$$

From an empty line, the cycle  $C_a(\alpha)$  puts  $\alpha$  products in the tanks  $T_1, T_2 \dots T_\alpha$ : (1), then transports all these products to the end of the line (2), and then empties the line (3).

For example, for a five-tank line ( $m = 5$ ), if  $p = 7\delta$  (then  $k = 2$ ) the cycle  $C_a(2) = 010213243545$  is optimal. Figure 3 represents the cycle  $C_a(2)$  for  $m = 5$ .

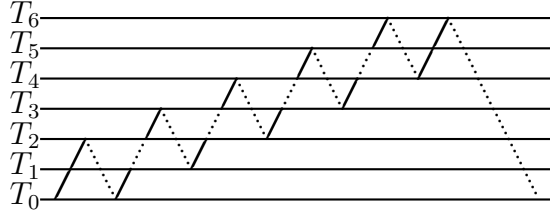


Figure 3:  $C_a(2)$  for  $m = 5$

The  $\alpha$ -cycles  $C_b(\alpha)$  are defined as follows:

$$C_b(\alpha) = \prod_{i=0}^{2(\alpha-1)-m} [i(i-1) \dots 0 \ m(m-1) \dots (m - (2(\alpha-1) - m) + i)] \quad (1)$$

$$\prod_{i=2\alpha-m-1}^{\alpha-1} [i(i-1) \dots 0] \quad (2)$$

$$\prod_{i=\alpha}^m [i(i-1) \dots (i-\alpha+1)] \quad (3)$$

$$\prod_{i=\alpha-2}^{2\alpha-m-1} [m(m-1) \dots (m-i)] \quad (4)$$

$$C_b(\alpha) = \prod_{i=0}^{2(\alpha-1)-m} \left[ \prod_{j=0}^i (i-j) * \prod_{j=0}^{2(\alpha-1)-m-i} (m-j) \right] * \prod_{i=2\alpha-m-1}^{\alpha-1} \left[ \prod_{j=0}^i (i-j) \right] * \prod_{i=\alpha}^m \left[ \prod_{j=0}^{\alpha-1} (i-j) \right] * \prod_{i=\alpha-2}^{2\alpha-m-1} \left[ \prod_{j=0}^i (m-j) \right] \quad (5)$$

The cycle  $C_b(\alpha)$  is composed of the following moves :

- (1) :  $\alpha$  products begin to enter the line while the previous ones leave it;

- (2) : The  $\alpha$  products finish to enter the line;
- (3) : The  $\alpha$  products are treated until the first one arrives at the last tank;
- (4) : The products leave the line until enough are out for the hoist to enter new ones.

Expression (5) is another way to express  $C_b(\alpha)$ . The cycle time of  $C_b(\alpha)$  is  $T(C_b(\alpha)) = \frac{1}{\alpha}[(2m - \alpha)p + 4m\delta]$ .

For example in a five-tank line if  $p = 13\delta$  then  $k = 4$  and the dominant cycle is the cycle  $C_b(4)$ . Part (1) of  $C_b(4)$  is the sequence 054105, part (2) is the sequence 2103210, part (3) is the sequence 4321 5432 and part (4) is the sequence 543. The cycle  $C_b(4) = 054105210321043215432543$  is represented on Figure 4.

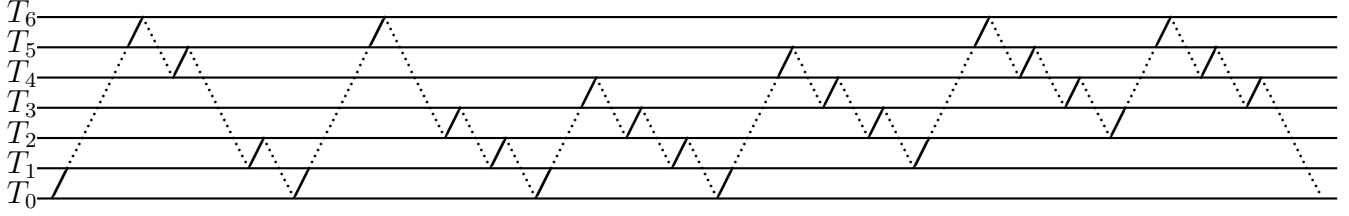


Figure 4:  $C_b(4)$  for  $m = 5$  and  $p \in [12\delta; 14\delta[$

The  $\alpha$ -cycle  $C_c(\alpha)$  is defined as follows:

$$C_c(\alpha) = \prod_{i=0}^{2\alpha-m-1} [i(i-1) \dots 0 \ m(m-1) \dots (m - (2\alpha - m - 1) + i)] \\ \prod_{i=2\alpha-m}^{\alpha-1} [i(i-1) \dots 0] \\ \prod_{i=\alpha}^m [i(i-1) \dots (i - \alpha + 1)] \\ \prod_{i=\alpha-2}^{2\alpha-m} [m(m-1) \dots (m - i)]$$

$$C_c(\alpha) = \prod_{i=0}^{2\alpha-m-1} \left[ \prod_{j=0}^i (i-j) * \prod_{j=0}^{2\alpha-m-1-i} (m-j) \right] * \prod_{i=2\alpha-m}^{\alpha-1} \left[ \prod_{j=0}^i (i-j) \right] * \prod_{i=\alpha}^m \left[ \prod_{j=0}^{\alpha-1} (i-j) \right] * \prod_{i=\alpha-2}^{2\alpha-m} \left[ \prod_{j=0}^i (m-j) \right]$$

The cycle time of  $C_c(\alpha)$  is  $T(C_c(\alpha)) = \frac{1}{\alpha}[(2m - \alpha - 1)p + (4m - 2)\delta]$ .

The 1-cycle  $C_e$  begins with a product in every two tanks and the robot transports a product from  $T_0$  to  $T_1$ , then transports a product from  $T_2$  to  $T_3$ , etc... When all products are moved, the robot goes back to tank  $T_1$  and moves again every products in the line. This cycle can be defined as follows :

$$C_e = \prod_{i=0}^{m/2} 2i * \prod_{i=0}^{m/2-1} (2i + 1)$$

The cycle time of  $C_e$  is :  $T(C_e) = 2\frac{m-1}{m}p + 4\delta$ .

Figure 5 is an example of  $C_e$  for a four machine line.

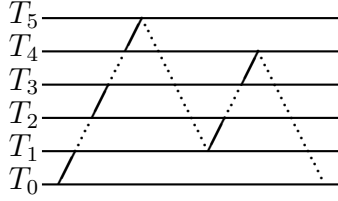


Figure 5: Example of  $C_e$  for  $m = 4$

## 5 Proof of some cases of the conjecture

In this section, we prove parts of Conjecture 1 and Conjecture 2.

### 5.1 Optimal cycles for $k \leq \frac{m+2}{4}$

**Theorem 1** For  $p \in [4(k-1)\delta, 4k\delta[$  and  $k \leq \frac{m+2}{4}$  the  $k$ -cycle  $C_a(k)$  is optimal.

**Proof:**

If  $p \in [4(k-1)\delta, 4k\delta[$ , then, at the same time, the maximum number of products on the line is  $k$ . Moreover the maximum distance between two products is strictly smaller than  $\frac{m+1}{2}$  i.e., when a product leaves the line, all products on the line are at a machine with index greater than  $\frac{m+1}{2}$ . Therefore, from this moment, no products can enter the line as long as a product is still on the line. Hence the line has to be completely emptied during every cycle.

It implies that in every  $\alpha$ -cycle the  $\alpha$  products which enter the line are exactly the  $\alpha$  products that leave the line.

Brauner and Finke (Brauner & Finke 2001) proved that, during the execution of a cycle, if the line is emptied more than once then, the cycle is dominated by the best sub-cycle. The value of  $p$  and the previous remark impose that the line has to be emptied, at least, every  $k$  products which means that for a  $\alpha$ -cycle ( $\alpha > k$ ) the line must be emptied at least twice. Therefore the  $\alpha$ -cycles with  $\alpha > k$  are dominated by a  $\beta$ -cycles with  $\beta \leq k$ .

Consider an  $\alpha$ -cycle  $C(\alpha)$  with  $\alpha \leq k$ . This cycle moves at most  $\alpha$  products at the same time. Then we can deduce a lower bound for  $T(C(\alpha))$  :

$$\alpha T(C(\alpha)) \geq \underbrace{mp + 2(m+1)\delta}_{\text{the first product crosses the line}} + \underbrace{(p+4\delta)(\alpha-1)}_{\text{exit of the } (\alpha-1) \text{ other products}}$$

Then

$$T(C(\alpha)) \geq \frac{1}{\alpha} [(m+\alpha-1)p + 2(m+2\alpha-1)\delta] = f(\alpha)$$

$$f(\alpha) = p + 4\delta + (m-1)\frac{p+2\delta}{\alpha}$$

Since  $f$  is a decreasing function, it is minimum for  $\alpha$  maximum. But we know that, for  $\alpha > k$ , the  $\alpha$ -cycles are dominated. Therefore  $f$  is minimum for  $\alpha = k$ . Since  $T(C_a(k))$  reaches this lower bound,  $C_a(k)$  is dominant. □

## 5.2 Optimal cycles for $p \geq 4(m - 1)\delta$

For  $p \geq 4(m - 1)\delta$  the cycle  $C_d = \prod_m^{i=0} (m - i)$  is dominant with a cycle time of  $p + 4\delta$  (Crama et al. 2000).

Remains to prove the other items of Conjecture 1 and Conjecture 2.

## 6 Example for $m = 5$

To illustrate the previous results we consider the example of  $m = 5$ . In order to compute the optimal cycles, use the following method. If  $p = 13\delta$ , then the value of the division of  $p$  by 4 is 3, which means that the dominating cycle is a  $k$ -cycle ( $k = (3 + 1) = 4$ ). As the remainder of this division is strictly lower than 2 then the dominant cycle is a  $C_b$  type cycle. The optimal cycle for  $p = 13\delta$  is the cycle  $C_b(4)$  (figure 10).

The dominant cycles are given in Table 1 and illustrated in Figures 6 - 12. The grey cells still have to be proved.

Table 1: Dominant cycles for  $m = 5$

p	$[0, 4\delta[$	$[4\delta, 8\delta[$	$[8\delta, 10\delta[$	$[10\delta, 12\delta[$	$[12\delta, 14\delta[$	$[14\delta, 16\delta[$	$\geq 16\delta$
Cycle	$C_a(1)$	$C_a(2)$	$C_a(3)$	$C_c(3)$	$C_b(4)$	$C_c(4)$	$C_d$
Temps de cycle	$5p+12\delta$	$6p/2+8\delta$	$7p/3+20\delta/3$	$6p/3+18\delta/3$	$6p/4+20\delta/4$	$5p/4+18\delta/4$	$p+4\delta$
k	1	2	3	3	4	4	1

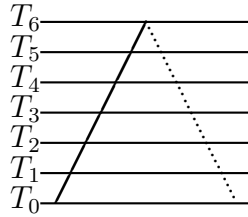


Figure 6:  $C_a(1)$  for  $m = 5$  and  $p < 4\delta$

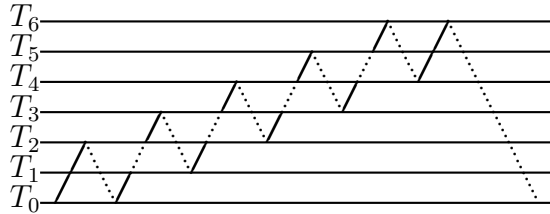


Figure 7:  $C_a(2)$  for  $m = 5$  and  $p \in [4\delta; 8\delta[$



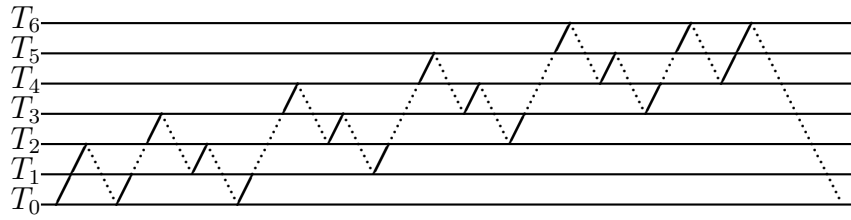


Figure 8:  $C_a(3)$  for  $m = 5$  and  $p \in [8\delta, 10\delta[$

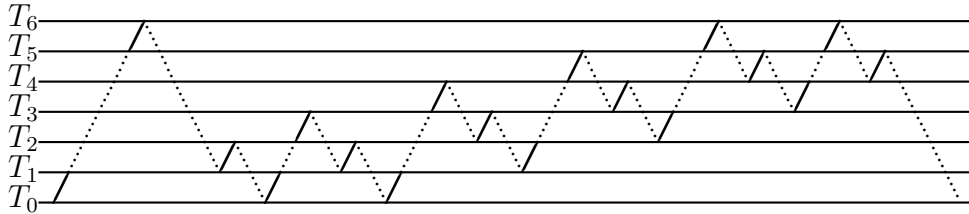


Figure 9:  $C_c(3)$  for  $m = 5$  and  $p \in [10\delta; 12\delta[$

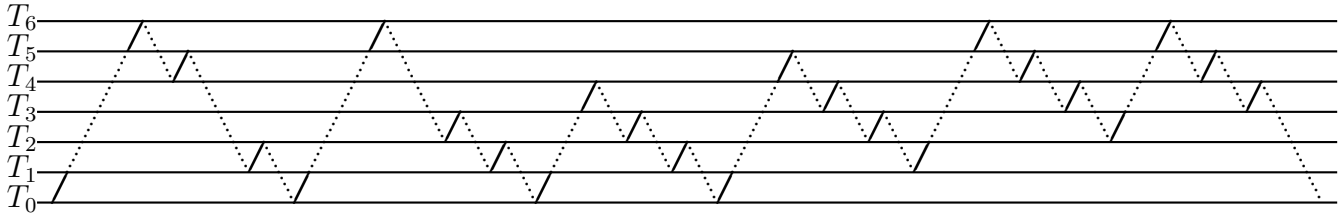


Figure 10:  $C_b(4)$  for  $m = 5$  and  $p \in [12\delta; 14\delta[$

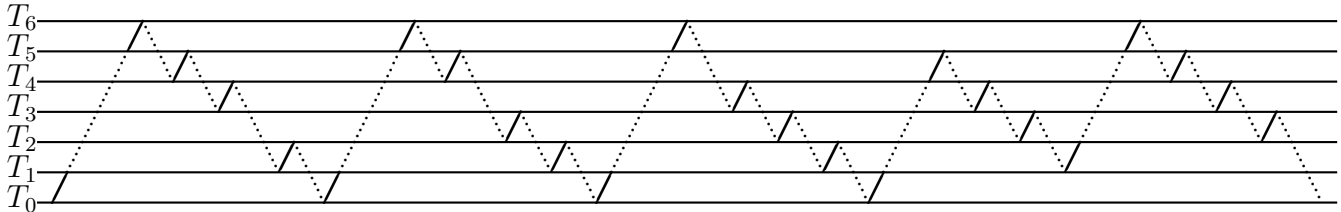


Figure 11:  $C_c(4)$  for  $m = 5$  and  $p \in [14\delta; 16\delta[$

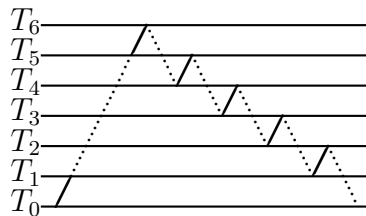


Figure 12:  $C_d$  for  $m = 5$  and  $p \geq 16\delta$

## 7 Conclusion

In this paper, we studied the robotic flow shop. We concentrated our works on the balanced no-wait case (all treatment times are fixed and equal). We proposed a conjecture on the structure of the optimal cyclic robot moves and proved the optimality with a lower bound method for some values of the treatment time  $p$ . Those results confirm the conjecture by Agnetis which claims that, in a no-wait robotic cell, the  $k$ -cycles with  $k \leq m - 1$  are dominant. The next objectives will be to prove the conjecture for any value of  $p$  and to analyze the extension of this conjecture to unbalanced lines.

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