Degree of the dominant cycles in no-wait robotic cells

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1 Introduction

In surface treatment lines, products are treated by several machines. The machines contain chemical baths like acids which affect the electrical or mechanical properties of the products. This kind of line is used, for instance, for galvanoplasty or circuit board assembly. The products are mounted on carriers and transported from a machine to another one by a robot. For quality reasons, the time a part remains on a machine (or processing times) is fixed (no-wait constraint).

The m machines of the line are denoted by M_1, $M_2 \dots M_m$. The loading machine, M_0, and the unloading machine, M_{m+1}, have infinite capacity whereas the other machines of the line can only contain one carrier at a time. The robot takes δ time units to travel between two consecutive machines. Thus the travel time from M_i to M_j is $|i-j|\delta$ time units.

The line produces a family of similar parts. The problem is to find the best cyclic robot moves which minimizes the cycle time or equivalently maximizes the throughput. We define a k-cycle as a production cycle where exactly k products enter and k products leave the line. Agnetis [1] conjectured that there always exists an optimal cycle with degree less than $m-1$ and proved this conjecture for 3-machine lines. We present bounds on the degree of the optimal cycles.

The robot moves are described in terms of activities. Activity i $(i = 0, 1 \dots m)$ consists of the following moves : the robot picks up a carrier from M_i, travels from M_i to M_{i+1} and loads the carrier onto M_{i+1}. For simplicity, we consider balanced lines *i.e.* the processing time of the parts is the same on all machines ($p_i = p$). To describe a k-cycle, we use the following notation. The symbol $\prod$ means a concatenation of activities. For instance the sequence of activities 01234 will be written $\prod_{i=0}^{4} i$.

2 Unbounded k

The following theorem indicates that k is not bounded *a priori*.

Theorem 1 *For any k, there exists an instance for which a k-cycle is optimal and strictly dominates all k'-cycles with $k' < k$.*

The idea of the proof is to show that the following k-cycle C_k^* is optimal for $m \geq 2k-1$ and $p \in [4(k-1)\delta, 4(k-1)\delta + 2\delta[$ and that there is no optimal k'-cycle with $k' < k$.

$$C_k^* = \prod_{i=1}^{k}\left[0\prod_{j=1}^{i}(i-j+1)\right] * \prod_{i=k+1}^{m}\left[\prod_{j=1}^{k}(i-j+1)\right] * \prod_{i=1}^{k-1}\left[\prod_{j=1}^{k-i}(m-j+1)\right] \tag{1}$$

For this special instance, the cycle time of C_k^* is $T(C_k^*) = [(m+k-1)p + 2(m+2k-1)\delta]/k$. For example, in a four-machine line with $p \in [4\delta; 6\delta[$, the 2-cycle (0102132434) is optimal, and its cycle time is $5/2p + 7\delta$. We can deduce from the preceding theorem the following result.

Corollary 1 *In a m-machine line, for any $k \leq \frac{m+1}{2}$ there exists a strictly dominant k-cycle.*

Therefore, in order to find the best k-cycle one has to consider cycles of degree $O(m)$.

3 $(m-1)-$cycles

In the previous section we proved that the degree of the optimal cycle can take all values between 1 and $\frac{m+1}{2}$. We conjecture that the following $(m-1)$-cycle $C^*_{(m-1)}$ dominates all k-cycles for a balanced m-machine line with $p \in [4(m-1)\delta - 2\delta; 4(m-1)\delta[$.

$$C^*_{m-1} = \prod_{i=2}^{m} 0 \left[\prod_{\substack{j=0 \\ j \neq m-i}}^{m-2} (m-j)\, 1 \right] * \prod_{j=0}^{m-2} (m-j) \tag{2}$$

In [2], we proved this conjecture for $m = 2$ and 3. We now prove that this $(m-1)-$cycle is optimal for $m = 4$ and that, for $m = 5$, it strictly dominates all k-cycles with $k \leq 4$.

4 Conclusion

In a no-wait robotic cell, we proved that the smallest degree of an optimal cycle, κ, can take any value between 1 and $\frac{m+1}{2}$. We also proved that κ can be equal to $m-1$ for $m = 4$ and that there exists instances for which $\kappa \geq m-1$ for $m = 5$. The next step will be to prove, as conjectured by Agnetis, that $\kappa \leq m-1$ for any instance.

References

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